

Lecture 10

Carrier Action

Please use this power point as a guide to reading the textbook.

n and p

non-degenerate

$$n = N_c \exp(-\beta(E_c - E_F))$$

$$p = N_v \exp(-\beta(E_F - E_v))$$

$$np = n_i p_i$$

Law of mass action

$$n_i = p_i = N_c \exp(-\beta(E_c - E_i)) = N_v \exp(-\beta(E_i - E_v))$$

$$n_i = \sqrt{N_c N_v} e^{-E_G \beta / 2}$$

$$N_c = n_i \exp(\beta(E_c - E_i))$$

$$N_v = n_i \exp(\beta(E_i - E_v))$$

$$n = n_i \exp(\beta(E_F - E_i))$$

$$p = p_i \exp(\beta(E_i - E_F))$$

E_F for
intrinsic
semicond.

$$N_c = 2 \left(\frac{m_n^* kT}{2\pi \hbar^2} \right)^{3/2} \quad N_v = 2 \left(\frac{m_p^* kT}{2\pi \hbar^2} \right)^{3/2}$$

$$\beta = 1/k_B T$$

n and p

- Charge neutrality

$$p - n + N_D^+ - N_A^- = 0$$

$$p - n + N_D - N_A = 0$$

ionized donor
 At RT \approx same \Rightarrow
 total donor
 acceptor densities

- Carrier concentration

$$\frac{n_i^2}{n} - n + N_D - N_A = 0$$

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

For
 p
 swap
 $N_D \leftrightarrow N_A$

n and p

- Important regime

$N_D \gg N_A, n_i$ (n-type, moderate T)

majority

$\rightarrow n \approx N_D, p \approx n_i^2 / N_D$

minority

$\approx RT$
or less

$N_A \gg N_D, n_i$ (p-type, moderate T)

majority

$\rightarrow p \approx N_A, n \approx n_i^2 / N_A$

S_i at RT $n_i \approx 10^{10} / \text{cm}^3$

- majority carrier $\gtrsim 10^{16} / \text{cm}^3$
- \rightarrow minority carrier $\lesssim 10^6 / \text{cm}^3$

Fermi level

$$n = n_i \exp(\beta(E_F - E_i))$$

$$p = p_i \exp(\beta(E_i - E_F))$$

$$E_i = \frac{E_c + E_v}{2} + \frac{3}{4}k_B T \ln \left(\frac{m_p^*}{m_n^*} \right)$$

$$E_F = E_i + k_B T \ln \left(\frac{n}{n_i} \right) = E_i - k_B T \ln \left(\frac{p}{p_i} \right) \quad n_i = p_i$$

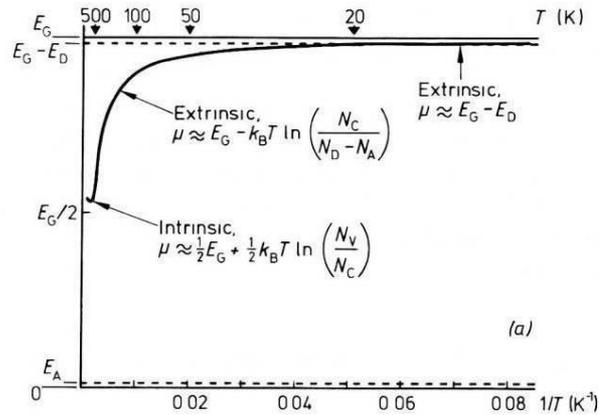
We know n, p (previous two slides), and so we know E_F completely!

For example n -type, moderate T : $E_F \approx E_i + k_B T \ln \left(\frac{N_D}{n_i} \right)$

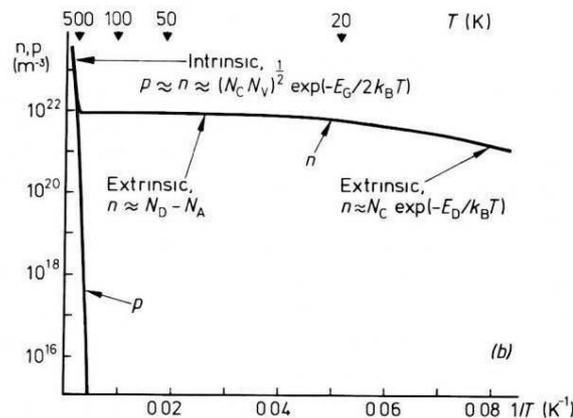
p type, moderate T : $E_F \approx E_i - k_B T \ln \left(\frac{N_A}{n_i} \right)$

Example

n-type



← Fermi level



← n, p

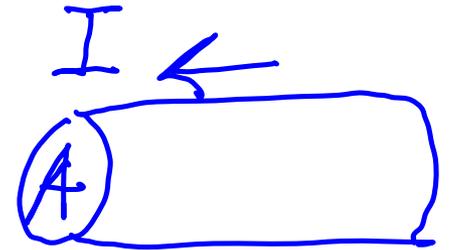
Fig. 5.6 Variations of (a) the Fermi level μ and (b) the electron and hole concentrations (note the logarithmic scale) with $1/T$ for an n-type semiconductor containing a significant number of acceptor impurities. The figure was calculated for a germanium semiconductor with $N_D = 10^{22} \text{ m}^{-3}$, $E_D = 0.012 \text{ eV}$, $N_A = 10^{21} \text{ m}^{-3}$ and $E_A = 0.010 \text{ eV}$; the scale at the top shows temperature values for this case

From Hoo K & Hall

Current Density

- The Current Density

$$J = \frac{I}{A}$$



- Current can be large for large cross-section wire (large “gauge” wire).
- The intrinsic quantity to compare different materials with is then Current per cross-section, i.e. the current density.
- Unit ? $\frac{C}{sm^2}$

Ohm's Law

Current and voltage

- $V = IR$

R = resistance (Ω)

- $I = V/R = GV$

G = conductance ($\frac{1}{\Omega} = S$)

- $J = \sigma E$

σ = conductivity (S / m)

$$R = \frac{l}{\sigma A} \cdot \frac{d}{A}$$

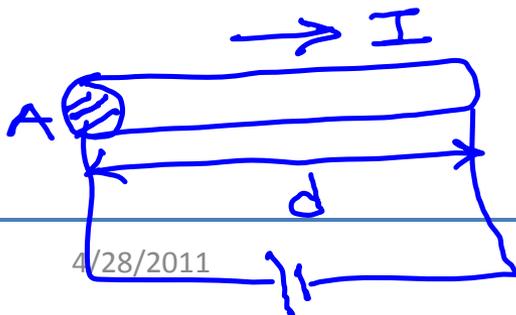
$$E = V/d, \quad J = I/A$$

- $E = J/\sigma = \rho J$ ρ = resistivity (Ωm)

Common unit = $\mu\Omega cm$, $m\Omega cm$

$$G = \frac{1}{R}$$

$$\sigma = \frac{1}{\rho}$$



Multiple Carriers

- If there are two types of carriers then

A. their resistivities add up

B. their conductivities add up

Mobility

- Conditions for a good conduction of current

1. Each charge carrier is efficient.

Mobility (μ_n or μ_p)

2. Lots of charge carriers.

Density of charge carriers (n or p)

3. High electric field

intrinsic
affects conductivity

$$\sigma = e\mu_n n + e\mu_p p$$

- The current density

$$J = (e\mu_n n + e\mu_p p) E$$

$$\text{unit} = \frac{C}{m^2 s} \frac{m^3}{C} \frac{m}{V} = \frac{m^2}{Vs}$$

$$\text{standard unit} = \frac{cm^2}{Vs}$$

Drift Velocity

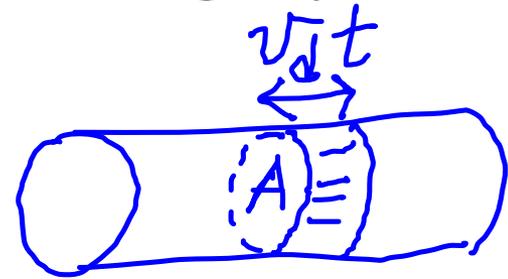
- Drift velocity
 - The average velocity gain of charge carriers due to the electric field.
 - Quite smaller, usually, than the average speed of charge carriers at equilibrium.

- $J = nev_{d,n} + pev_{d,p}$

- Comparing with the last page

$$\mu = v_d/E$$

microscopic origin of μ



$$I = Av_d t ne/t$$

Drift Velocity

- A. Is on the order of the actual average speed of each charge carrier in magnitude
- B. Is much greater than the actual average speed of each charge carrier in magnitude
- C. Is generally much less than the actual average speed of each charge carrier in magnitude

“Drude” Theory of Conduction

- Consider the carrier distribution as a entity whose drift velocity has a “decay time” of τ , like a radioactive particle that decays. $\tau =$ **relaxation time** due to scattering of the carrier by impurity, lattice vibration, other carriers etc.

- $\frac{dv_d}{dt} + \frac{v_d}{\tau} = 0$, if left alone $\rightarrow v_d(t) = v_d(0)e^{-t/\tau}$.

- If driven by a field, $m_n^* \left(\frac{dv_d}{dt} + \frac{v_d}{\tau} \right) = eE$. *Further microscopic origin of μ !*

- In steady state: $v_d = \frac{eE\tau}{m_n^*}$ and so $\mu_n = \frac{e\tau}{m_n^*}$.

Mobility

- Mobility is due to the combination of two factors

$$\mu_n = \frac{e\tau_{n,r}}{m_n^*}$$

relaxation
time

$$\mu_p = \frac{e\tau_{p,r}}{m_p^*}$$

effective
mass

$\tau = \tau$ of the book
unusual notation

Multiple Scattering Mechanisms

- If a carrier goes through multiple scattering channels (with impurity, phonons, or other carriers), then

A. their resistivities in different channels add up

B. their conductivities in different channels add up

Matthiessen's rule

- Probabilities add up **for a given carrier (e or h)**, if scattering processes are independent.

$$\frac{1}{\tau} = \left(\frac{1}{\tau}\right)_{\text{impurity}} + \left(\frac{1}{\tau}\right)_{\text{lattice vibration}} + \left(\frac{1}{\tau}\right)_{\text{other carriers}}$$

$$\frac{1}{\mu} = \frac{1}{\mu_{\text{impurity}}} + \frac{1}{\mu_{\text{lattice vibration}}} + \frac{1}{\mu_{\text{other carriers}}}$$

$\frac{dt}{\tau}$: probability of scattering event during dt

$$R = R_{\text{impurity}} + R_{\text{lattice vibration}} + R_{\text{other carriers}}$$

- It is a “*rule*” not a law – which means that it can and does break easily. Scattering processes are not independent.