Due May. 24, Tuesday

Throughout this homework (except the last problem), assume the following, unless stated otherwise.

$$n = n_0 + \Delta n$$
$$p_n = p_{n,0} + \Delta p_n$$

Here the subscript 0 means the "equilibrium," and the subscript n means "n type" (and therefore $p_{n,0} \ll n_0$). Assume a low level injection, i.e. $\Delta n, \Delta p_n \ll n_0$. We assume uniform carrier densities for the equilibrium carrier densities: n_0 and $p_{n,0}$ have no dependence on spatial coordinates. In contrast, the out-of-equilibrium quantities Δn and Δp_n are functions of both position (x) and time (t).

(Everything we do here applies to the p type semiconductor as well. The equations for the p type case can be obtained by (1) the global symbol name change ($p \leftrightarrow n$) and (2) appropriate sign changes for certain terms. You need to consult with the lecture notes or the textbook to know which terms in the carrier equations acquire different signs when going from n type to p type. You will be asked to provide the p-type version of the answer in some places below.)

Problem 1 (20 points) In the limit of low level carrier injection, the following formula is applicable.

$$\left.\frac{\partial p_n}{\partial t}\right|_G = G$$
$$\left.\frac{\partial p_n}{\partial t}\right|_R = -Rnp_n$$

Here, G and R are positive, material-specific, constants that characterize the generation and recombination process, and they are *independent of* n and p_n due to the low level injection assumption. From the above two equations, show that

$$\left.\frac{\partial \Delta p_n}{\partial t}\right|_{RG} = \left.\frac{\partial p_n}{\partial t}\right|_{RG} = \left.\frac{\partial p_n}{\partial t}\right|_{R} + \left.\frac{\partial p_n}{\partial t}\right|_{G} = -\frac{\Delta p_n}{\tau_p}$$

where $\tau_p \equiv (Rn_0)^{-1}$ is the minority carrier lifetime. In deriving this, you should note that (1) $p_{n,0}$ and n_0 are time-independent, being equilibrium quantities, (2) use the fact that in equilibrium the generation rate and the recombination rate cancel each other exactly, and (3) ignore all terms that are higher order than the first order in small densities ($p_{n,0}$, Δp_n and Δn).

Problem 2 (40 points) Let us analyze the "Hall effect" experiment with considerable care. We assume n-type, and consider majority carriers only. In the presence of the electro-magnetic field, the semi-classical equation of motion (LN 6), is written as

$$\hbar \frac{d\vec{k}}{dt} = -e\vec{v}_g \times \vec{B} - e\vec{E}$$

where the symbol $\vec{v}_g$ stands for the group velocity. We consider, as usual, the conduction band electrons in a parabolic band

$$\varepsilon_c(\vec{k}) = E_c + \frac{\hbar^2(\vec{k} - \vec{k}_c)^2}{2m_n^*}$$

(a) Show that the above semi-classical equation of motion can be rewritten as

$$m_n^* \frac{d\vec{v}_g}{dt} = -e\vec{v}_g \times \vec{B} - e\vec{E}$$

(b) Define the *drift velocity*, $\vec{v}_d$, as the average of $\vec{v}_g$ of all the electrons. Show that

$$m_n^* \frac{d\vec{v}_d}{dt} = -e\vec{v}_d \times \vec{B} - e\vec{E}$$

Assume that $\vec{B}$ and $\vec{E}$ are constants.

(c) Introduce the relaxation time τ, which is the time scale in which the drift velocity decays. Specifically, the decay process is according to the following law. In a short time interval from t to $t + dt$, $\vec{v}_d$ can be left intact or it can be reset to zero, relaxing to the equilibrium value, i.e. zero. The probability that the relaxation happens is given by dt/τ. Thus, after statistical average, we can write

$$d\vec{v}_d = -\frac{dt}{\tau}\,\vec{v}_d$$

(Note that this is precisely the Rutherford law of radioactive decay if one substitutes N – the number of radioactive particles – for $\vec{v}_d$.) By including this relaxation term, show that the above equation gets modified to

$$m_n^* \left(\frac{d\vec{v}_d}{dt} + \frac{\vec{v}_d}{\tau} \right) = -e\vec{v}_d \times \vec{B} - e\vec{E}$$

(d) Assume that the following conditions are imposed: $\vec{v}_d = v_x\hat{x}$ and $\vec{B} = B_z\hat{z}$, where v_x and B_z are *constants*. There are two non-zero components of $\vec{E}$. Find them. Discuss the meaning of the equation for E_x (cf. LN 10: look for "Drude").

(e) The conditions just imposed correspond to a steady current flow along the x direction and an applied magnetic field along the z direction. This set-up corresponds to a Hall effect experiment, where the resulting transverse voltage E_y is measured as an outcome. The Hall coefficient R_H is then defined as

$$R_H \equiv \frac{E_y}{J_x B_z}$$

where J_x is the current density ($\vec{J} = -ne\vec{v}_d$). Show that

$$R_H = -\frac{1}{ne}$$

(f) Show that, if we had a p type semi-conductor, then the Hall coefficient would be given by

$$R_H = \frac{1}{pe}$$

(Of course, you do not need to re-derive this. Simply note which of E_y, J_x and B_z changes sign for a p type semiconductor.)

(g) Summarize your work to show that the majority carrier density and the majority carrier mobility can be measured by measuring the Hall coefficient and the conductivity. (As usual, we assume that the minority carrier density is far less than the majority carrier density.) (Note: Conversely, you may also notice that the Hall effect provides a way to measure the magnetic field once the majority carrier density is characterized precisely – a semiconductor device used for such purpose is called a Hall probe.)

Problem 3 (30 points) Recall from the lecture that the majority carrier equation in the absence of the electric field is given by

$$\frac{\partial \Delta n}{\partial t} = -\frac{\Delta n - \Delta p_n}{\tau_{n,D}} + \frac{\lambda_{n,D}^2}{\tau_{n,D}} \frac{\partial^2 \Delta n}{\partial x^2}$$

where

$$\tau_{n,D} = \frac{K_s \epsilon_0}{\mu_n n e} \qquad \text{Dielectric relaxation time } (n \text{ type})$$

$$\lambda_{n,D} = \sqrt{D_n \tau_{n,D}} \qquad \text{Debye length } (n \text{ type})$$

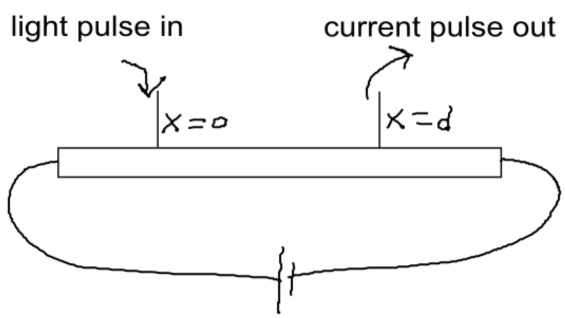
(a) Take $n = 10^{16}$ cm^{-3}, $\mu_n = 2000$ cm^2/(Vs), $K_s = 10$, and $T = 300$ K, to estimate the dielectric relaxation time $\tau_{n,D}$, and the Debye length $\lambda_{n,D}$. (Hint: Use the Einstein relation to calculate the diffusion coefficient D_n.) Show that these scales are much smaller than those of the minority carrier dynamics ($\gtrsim$ ns and $\gtrsim$ μm).

(b) Re-cast the above equation in terms of $\rho = e(\Delta p_n - \Delta n)$. What is the meaning of ρ? (Hint: Since Δp_n, the minority carrier density, has a very slow dynamics involving a very long length scale, compared to the majority carrier dynamics, as found in (a), we can treat Δp_n as a *constant*!)

(c) Obtain the general solution $\rho(t)$ for the above equation assuming no x dependence. Discuss the nature of the solution.

(d) Obtain the general solution $\rho(x)$ for the above equation assuming no t dependence. Discuss the meaning of the solution, using the solution for the following boundary condition as a specific example: $\rho(x \to \infty) = 0$ and the sample exists in the region $x \geq 0$.

Problem 4 (40 points) From problem 2, now we know how to measure the mobility and the carrier density of the majority carriers, a question that you might ask is "how about the minority carriers?" This is the topic of the famous "Haynes-Shockley" experiment. In this experiment, a short pulse of light is applied on one part of the sample, and the resultant current is measured at a distance d away as a function of time, i.e. the shape of the current pulse as it passes the fixed distance d is measured. The experimental conditions are such that the light pulse enters a very small part of the crystal (as in one of the example problems that we discussed in class). Thus, the light pulse "injects" spatially and temporally localized pulses of majority and minority charge carriers. We assume that the light pulse is of moderate intensity so that the injected carrier densities are small.



We assume that the length scale (d) and the time scale of the experiment are much longer than those of the majority carrier dynamics (see the previous problem). As seen in that problem, the majority carrier dynamics occurs, in general, at a much faster rate than the minority carrier dyanmics. One role of the majority carriers is then that of fast neutralizing any charge fluctuation due to the long-lived minority carriers. Another role of the majority carriers is that of current flowing under an external bias. Both these roles are not important for Haynes-Shockley experiment, since the second mechanism just provides a background current, while the first mechanism can be shown to lead to a negligible contribution to the current pulse compared to that due to the minority carriers. Thus, in this problem, we analyze the minority carrier dynamics only.

(a) Write down the minority carrier equation, but assuming that there *is* an externally applied DC electric field E applied to the entire sample. Substitute the following function into the equation, and show that it is *a* solution.

$$\Delta p_n(x,t) = P_i \, \frac{1}{\sqrt{4\pi D_p t}} \exp\left(-\frac{t}{\tau_p}\right) \exp\left(-\frac{(x-\mu_p E t)^2}{4D_p t}\right)$$

Here, P_i is a constant representing the total number of hole carriers injected at $t = 0$. All symbols other than x, t are as defined during the lecture or in the lecture note.

(b) Look up the definition of a "normalized Gaussian function," or "normalized Gaussian distribution function." The "mean," the "variance," and the "standard deviation" are important quantities that characterize the Gaussian distribution function. By "normalized," I mean $\int_{-\infty}^{\infty} dx g(x) = 1$, where $g(x)$ is a normalized Gaussian function. Show that, for a fixed value of $t \ll \tau_p$, (i) the solution in (a) is a normalized Gaussian function up to a multiplicative constant, (ii) the mean value of x is $\mu_p E t$, (iii) the standard deviation of x is $\sqrt{2D_p t}$, and (iv) the total number of injected minority carrier density at any given time t, $\int_{-\infty}^{\infty} dx \Delta p_n$, is given by P_i.

(c) Now, at *any* time value (not just $t \ll \tau_p$), explain what the solution in (a) means physically, and how this experiment can be used to probe the minority carrier lifetime, the minority carrier mobility, the minority carrier diffusion coefficient, and the validity of the Einstein relation.

(d) Based on your answer for the first part of (c), provide a physical argument as to why the solution of (a) may be taken as *the* solution describing the Haynes-Shockley experiment? (Keep in mind that the solution (a) is merely one of many kinds of possible solutions for the minority equation.)

(e) Assume that $\mu_p = 500$ cm^2/(Vs), $d = 1$ cm, and $E = 10$ V/cm. Consider two possible samples, one with $\tau_p = 1$ μs and 1 ms. For each sample, answer the following questions. (i) At what time does the peak pass through the point $x = d$? (ii) What fraction of the originally injected holes arrive at $x = d$?

Problem 5 (30 points) Problem 5.4 of the textbook.