

Due May. 5, Thursday.

**Problem 1** (20 points) Let us calculate the density of states (DOS) for the dispersion relation

$$\varepsilon(\vec{k}) = \frac{\hbar^2 k^2}{2m^*}$$

where  $k = |\vec{k}|$  is the magnitude of a three dimensional wave vector  $\vec{k}$ . In the  $\vec{k}$  space, the quantized  $\vec{k}$  values are equally spaced, i.e., the density of  $\vec{k}$  points is constant. This means that the volume in  $\vec{k}$  space is proportional to the number of  $\vec{k}$  points, i.e. the number of states. Thus, the total number of states,  $N(E)$ , from energy  $\varepsilon(\vec{k}) = 0$  to  $\varepsilon(\vec{k}) = E$  is given by

$$N(E) \propto k^3$$

where

$$E = \frac{\hbar^2 k^2}{2m^*}$$

In other words,  $N(E) = Ak^3$  where  $A$  is a constant.

- Show, from the above, that  $\text{DOS} \propto \sqrt{E}$ . This proves the energy dependences of Eq. T2.6a and Eq. T2.7b.
- Show that  $\text{DOS} \propto E/|\vec{v}_g|$ . Here,  $\hbar\vec{v}_g = \nabla_{\vec{k}}\varepsilon(\vec{k})$  where  $\nabla_{\vec{k}} = \hat{x}\frac{\partial}{\partial k_x} + \hat{y}\frac{\partial}{\partial k_y} + \hat{z}\frac{\partial}{\partial k_z}$  (the gradient in the  $\vec{k}$  space).
- (10 points, extra credit) Given the fact that the  $\vec{k}$ -space volume per  $\vec{k}$ -point is given by  $(2\pi)^3/V$ , where  $V$  is the volume of the sample, find the constant  $A$  in  $N(E) = Ak^3$ . Using this result, show that the DOS (per volume) is given by

$$\frac{\sqrt{2m^*3E}}{\pi^2\hbar^3}$$

This proves both Eq. T2.6a and Eq. T2.7b, completely. [Do not forget the spin degeneracy when you count the number of states.]

**Problem 2** (20 points) Problem 2.17 of the textbook.

**Problem 3** (20 points) Problem 2.18 of the textbook.