Due Apr. 28, Thursday.

Problem 1 (20 points) Consider a wave, whose amplitude $\psi(x,t)$ is given by

$$\psi(x,t) = \int_{-\infty}^{\infty} d\xi\, g(\xi) \exp[i(\xi x - \omega_\xi t)]$$

where $g(\xi)$ is a function of the wave vector variable ξ, as is the frequency ω_ξ. If $g(\xi)$ is a function that is sharply peaked around a certain wave vector k, then we call $\psi(x,t)$ a **wave packet**. Thus, for a wave packet, the function ω_ξ can be approximated as $\omega_\xi \approx \omega_k + v_g \zeta$, where $\zeta \equiv \xi - k$ and $v_g \equiv d\omega_k/dk$ (**group velocity**). (a) Using this approximation, and a change of integration variable, prove that

$$\psi(x,t) = A(x - v_g t)\ \exp[i(kx - \omega_k t)]$$

where

$$A(z) = \int_{-\infty}^{\infty} d\zeta\ g(k+\zeta)\ \exp[i\zeta z]$$

Here, $A(z)$ can be interpreted as an "envelope function." (b) Show that the probability distribution function $|\psi(x,t)|^2$ propagates at the group velocity v_g.

Problem 2 (20 points) Assume that a DC E field of magnitude 100 V/cm is applied to the $-x$ direction. Let us assume that the relaxation time τ is 1 pico-second, a typical time scale in a semiconductor. During time τ, how much ~~crystal momentum~~ wave vector (in Å^{-1}) does an electron gain? How does it compare with a typical unit cell size $\sim 1\text{Å}^{-1}$ in k space?

(wave vector $\times\, \hbar$ = crystal momentum)

Problem 3 (20 points) Given the tight binding band $\varepsilon(k) = -2\alpha - 2t\cos ka$, find the expression for the effective mass m_n^* (near the minimum of the band) and m_p^* (near the maximum of the band). How is m^* correlated with t (i.e. when t increases, how does m^* change)? Obtain the values of the effective masses relative to the bare electron mass, assuming $t = 3$ eV and $a = 3$ Å.

Problem 4 (20 points) Pierret 2.9. Include *print-outs* of your source codes and plots for the computation part.

Problem 5 (20 points) Pierret 2.10. Include *print-outs* of your source codes and plots for the computation part.

NOTE

Very substantial help (some example programs and detailed instructions) for the computation parts of the last two problems will be posted on-line soon ($\approx$ this weekend).