Due Apr. 14, Thursday.

Problem 1 (20 points) Quantum Strand: Quantum Tunneling Activity at http://et.portal.concord.org/activities/. **Note**: (1) Please do "Intro and Admin: Activities: Survey" first, if you haven't done it yet. (2) As long as you complete the activities, you will get close to the perfect grade. You can revise your answers any time until the due date.

Problem 2 (20 points) Prove steps going from (4.26) to (4.27) of Lecture Note 4.

Problem 3 (20 points) Derive Eq. (4.32) of Lecture Note 4.

Problem 4 (10 points) Derive Eqs. (4.33) and (4.34) of Lecture Note 4.

Problem 5 (20 points) Consider a hypothetical one dimensional crystal consisting of (1) 65536 H atoms, (2) 65536 He atoms, and (3) 65536 Li atoms (so these are "nano-crytals."). The atoms are arranged on a line, with equal spacing a (lattice constant). Consider the two lowest lying bands arising from Wannier orbitals, whose main characters are $1s$ and $2s$ – i.e. "1s band" and "2s band." We assume that each of these bands is described well by the tight binding band theory, and that the two bands do not overlap in energy (2s band above 1s band, and completed separated from 1s band).

(a) How many unique k points exist for the 1s band? And, for the 2s band?

(b) At zero temperature, how many electrons fill the 1s band and the 2s band? Your answer should be different for each of the three crystals given.

[Note] (1) The number of atoms is chosen to be a power of 2 here, but in fact it can be chosen as any large natural number. (2) The correct answers to these questions apply to the *any* band theory, as long as the two bands do not overlap, not just the tight binding band theory.

Problem 6 (Exra Credit, 20 points) Derive Eqs. (4.35) and (4.36) of Lecture Note 4.