Please record your name in your answer sheet. Return the answer sheet only.

15 minutes.

Consider a two body problem with a central force. The relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$ is governed by the Lagrangian

$$L = \frac{1}{2}\mu|\dot{\vec{r}}|^2 - U(|\vec{r}|)$$

This Lagrangian has a conserved energy E and a conserved angular momentum vector $\vec{L}$. The latter means that the motion is confined in a plane. By taking the z axis to be parallel to $\vec{L}$, and taking the cylindrical coordinate system (r, θ, z), we can take the xy plane as the plane in which the motion occurs. Thus,

$$L = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) - U(r)$$

We assume $l = |\vec{L}|$ is positive (i.e., not zero).

(a) Utilizing the conserved quantity l, the energy E can be expressed as $E =$ the radial kinetic energy (the one that depends on $\dot{r}$) + $U_{eff}(r)$. Find $U_{eff}(r)$.

(b) Make a sketch of $U_{eff}(r)$ for $U(r) = -k/r$ ($k > 0$). Your sketch should show the correct signs and the correct limiting values of U_{eff} as $r \to 0^+$ and $+\infty$. **Important:** In your sketch, indicate the value of r corresponding to a circular motion.

(c) Do the same for $U(r) = kr$ ($k > 0$).