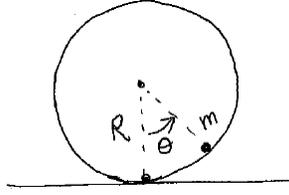


Please record your name in your answer sheet. Return the answer sheet only.

15 minutes.

Consider a “loop the loop” roller coaster problem.



For a roller coaster in a loop, we approximate it as a point particle moving inside a hoop, whose shape is a perfect circle. It is moving in a constant gravitational field, $-g\hat{z}$, where z is the vertical axis. We shall treat the constraint $r = R$ with a Lagrange multiplier $\lambda(t)$.

- (a) Find the Lagrangian, $L(r, \theta)$, where r is the distance from the center, and θ is the angle of rotation. θ is defined in the above diagram. If you define it differently, then you need to include your own definition as a diagram.
- (b) Find the two equations of motion for r and θ , incorporating $\lambda(t)$ in them as necessary.
- (c) What is the physical meaning of $\lambda(t)$? What should be the sign of it, when it is non-zero?
- (d) Show, from (b) and (c), that $v_{T,min} = \sqrt{Rg}$, where v_T is the speed of the roller coaster at top of the loop.