Due Nov. 23, Tuesday.

Problem 1 (20 points) Consider the example that was studied in Lecture 14 and summarized in pages 9 and 10 of LN 14.

(a) Assume that the initial conditions are: $x_1 = 2a$, $x_2 = 0$, and no initial velocities for either mass. Determine D_1, D_2, ϕ_1, ϕ_2 (notation as in LN14-10) and, thus, find $x_1(t)$ and $x_2(t)$ at all times.

(b) We expect, for any initial condition, that the total energy of this system is the sum of the energies for the two normal modes, and that the energy of each normal mode is conserved. So, we expect these general properties to be valid for the solutions $x_1(t)$ and $x_2(t)$ of (a). By direct evaluation of the energy associated with $x_1(t)$ and $x_2(t)$, show that indeed the total energy is constant and is a sum of two parts, each of which is also constant, arising from the two normal modes.

(c) Evaluate the effective mass m_1^* and m_2^* (and thus k_1^* and k_2^* from $\omega_i^2 = k_i^*/m_i^*$), defined at the bottom of L14-4, to show in detail that the energy for each normal mode is indeed consistent with the intrinsic parameters of each normal mode and its amplitude (given by the initial condition).

Problem 2 (30 points) Consider a circular wire in which three identical masses ("beads") can slide without friction. The masses are connected by three identical springs, as shown.
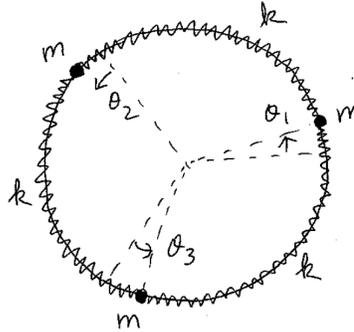
For small angular displacements, $\theta_1, \theta_2, \theta_3$ of the three masses, answer the following questions. You can set the radius of the circle to be 1, for convenience.

(a) Find the eigen-frequencies and the normal modes, by solving the eigenvalue equation
$$\overleftrightarrow{A}\vec{T} = \omega^2 \overleftrightarrow{M}\vec{T}$$

(b) Your answer should show that one of the eigen-frequencies is zero. Explain the physical reason why this is so.

(c) Your answer should show that the modes such as $2\theta_1 = 2\theta_2 = -\theta_3$ and $\theta_1 = -\theta_2, \theta_3 = 0$ are degenerate, i.e. they have the same eigen-frequency. Explain why this is so by considering Newton's law (for one mass is sufficient) specifically for these normal modes.

Problem 3 (20 points) Consider three identical masses ("beads"), each moving along a linear frictionless wire. The wires lie in one plane with a common end point, and are at 120 degrees with respect to each other.
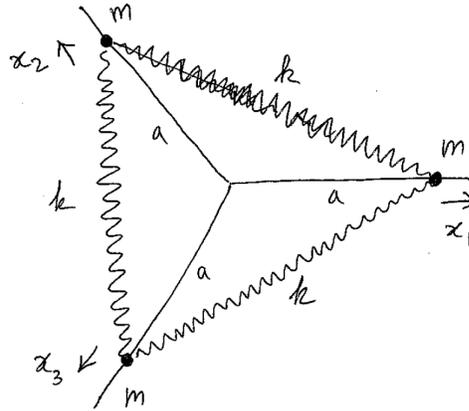
The masses are connected by three identical springs, forming an equilateral triangle when in equilibrium. In equilibrium, each mass is at distance a from the center. Considering small displacements from the equilibrium configuration, calculate the eigen-frequencies and the normal modes.

Problem 4 (10 points) 12.6 of the textbook. [Here, you cannot use our beloved "generalized eigenvalue equation," as there are damping terms. You need to do this problem using the Newtonian method. You have two options. (1) Use the standard method of putting $x_1 = u \exp(i\omega t)$ and $x_2 = v \exp(i\omega t)$ to obtain normal modes. (2) In this "simple and symmetric" case, you can add and subtract two equations of motion to get the normal modes.]

Problem 5 (20 points) 12.18 of the textbook. [Note that the matrix $\overset{\leftrightarrow}{T}$ cannot be taken as an orthogonal matrix for this one.]