This pre-exam shows the type of problems that will be in the midterm. These are just some examples, and do not cover all possible subjects.

Solutions to this pre-exam are not available. However, "your group" can submit, by email, your work on any problem to me. Summarize your work neatly, scan/photograph the summary, and email it to me. "Your group" is defined as either you alone or you and your study mates. Your group cannot be larger than four students. Your group can differ for different problems. If your submitted solution is more than 90 % correct, then that solution will be posted in ecommons. For such a solution, **all members of your group will receive an insured minimum 90 % credit for an equivalent problem (determined by me) in the actual midterm**. You will still have to do all problems in the midterm, but you will be insured against a possible poor performance in an equivalent problem. There is a maximum of two problems for which any given student can be protected by such an insurance. For any problem, if three good solutions are already received by me, then that problem is considered closed and no more solutions will be accepted (unless you believe that you have an unusually brilliant solution, which is always possible). I will announce on ecommons when a problem is closed.

In the actual midterm, you will be required to do a total of four problems.

You can use these formula. If you need other formula, you can try asking.

$$
\begin{aligned}
\sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b \\
\cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\
\sin(2a) &= 2 \sin a \cos a \\
\cos(2a) &= 2\cos^2 a - 1 \; = \; 1 - 2\sin^2 a \\
\sin(\delta) &= \delta + O(\delta^3) \\
\cos(\delta) &= 1 - \frac{\delta^2}{2} + O(\delta^4) \\
(1+\delta)^\alpha &= 1 + \alpha\delta + \frac{1}{2}\alpha(\alpha-1)\delta^2 + O(\delta^3) \\
\ln(1+\delta) &= \cancel{1} + \delta - \frac{1}{2}\delta^2 + O(\delta^3)
\end{aligned}
$$

Show all your work.

Problem 1 Write the general solutions of a simple harmonic oscillator for the three different regimes. Explain why the critical damping condition is the best condition if the goal was to diminish any finite displacement to zero as quickly as possible.

Problem 2 Name the three fundamental kinds of symmetry of space and time. Explain what they are and why they are of fundamental importance.

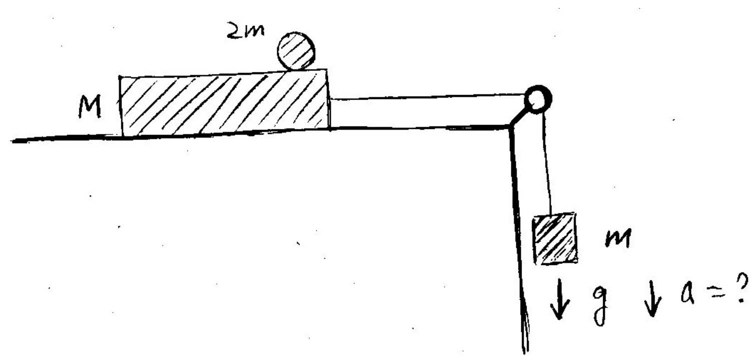
Problem 3 Consider a projectile motion with a small air resistance $-k\vec{v}$. The particle leaves with speed v_0 and at angle θ relative to the horizontal. Calculate the speed of the projectile at which the particle returns to the same height, to first order in air resistance.

Problem 4 Consider a one dimensional simple harmonic oscillator

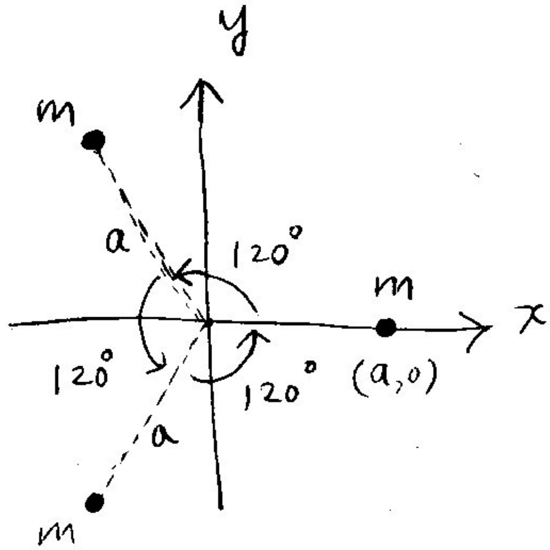
$$m\ddot{x} \;=\; -kx - b\dot{x}$$

Treat the $-b\dot{x}$ term as a perturbation, assuming a weak damping. Calculate, up to first order, the fractional energy lost during one period τ corresponding to the natural frequency of the oscillator. The fractional energy lost = the energy lost divided by the total energy. Relate your answer to the Q factor: $Q \equiv \omega_R/(2\beta)$, with $\omega_R = \sqrt{\omega_0^2 - 2\beta^2}$, $\beta = b/(2m)$ and ω_0 is the natural frequency.

Problem 5 Consider two masses, m, M, attached to each other by a massless string and a frictionless pulley, as shown. The mass M lies on a frictionless surface. On M is a cylinder with mass $2m$ and radius R (so its rotational inertia is mR^2). Initially, all masses are at rest. Then, the mass m is let go and is allowed to drop. As m starts to move, M starts to move as well, and $2m$ starts to roll on M without slipping. What is the acceleration a in terms of g, and m, M? You can use either the Lagrangian formalism (without constraint) or the Newtonian formalism.



Problem 6 As shown in the diagram below three masses are located at distance a from the origin, and at angles 0, 120, and 240 degrees respectively.

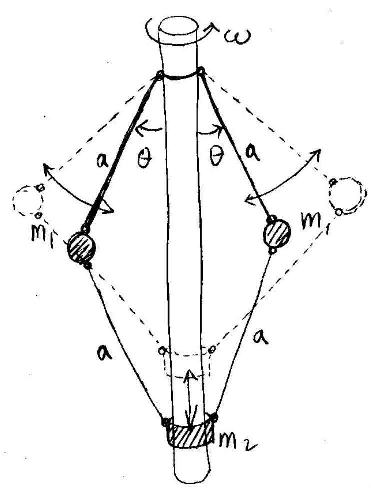


(a) Find the gravitational potential Φ at an in-plane point (x, y).

(b) Expand Φ near the origin, assuming $|x|$, $|y| \ll a$.

(c) Discuss the stability of a small "test mass" placed at the origin with respect to a small displacement in all possible directions including along the z direction (out of the plane). [For the z direction, you can use a simple physical argument.]

Problem 7 Consider two identical masses, m_1, and a mass, m_2, attached as in the following figure. The mass m_2 can slide up and down freely, while each mass m_1 is connected, through friction-free hinges and thin massless rods, to mass m_2 below it and, above it, to a fixed point on the thick shaft. The whole setup is rotating at a constant angular velocity ω.



(a) Find the Lagrangian L of this motion.

(b) Find the Hamiltonian H.

(c) Is H equal to the energy $E = T + U$? Is H conserved? Is E conserved?

(d) Find the effective potential $V(\theta)$ and the effective kinetic energy $K(\theta)$ so that $L = K - V$ and $H = K + V$.

(e) When ω is greater than a certain value ω_c, then there is an equilibrium point at a non-zero θ_e. Find ω_c and θ_e.

(f) Is $\theta = \theta_e$ a stable equilibrium point? Explain.