

# Appendix A

## Nature of Physical Laws

(Especially, Newton's laws)

You may or may not like this, but nature's laws are rather like every day laws (think "law and order"), I say. For one, laws, in comparison to, say, rules, carry a heightened sense of authority. For the most part, they are OK, we believe, and we probably should abide by them. But, hey, never say never, right? Inevitably there are renegades or there are renegade moments even for a good citizen. For sure, one might say, there are many bad kinds of law breakers, crooks or worse, who make the world a bit more stressful than it should be. Well, there are also "good kinds of law breakers," one might argue. These may be those people disobeying laws for the greater good of the community and the mankind. In the end, those "good law-breakers" may improve the society or sometimes bring down an oppressive regime in a revolution.

Well, nature's laws are not that different. We have beautiful laws that we cherish. Newton's laws, quantum mechanics, relativity, etc. Many phenomena seem to fit these laws perfectly. You may call them "good citizens." Documenting these phenomena only would be quite boring, though. Then, you discover phenomena that don't fit. They are renegades. Puzzles. They grey scientists' hair, and they add stress to the science community. Also fun. Something to figure out. At any given time, you may say that there are a whole bunch of phenomena waiting to "uproot" the establishment. In this view, scientists are like "lawyers". Experimentalists discover cases and carefully document them. Then, scientists figure out whether a case is genuine or not. There may be bad law breakers, "useless experiments." A collection of experiments may shine a light on a path to truly new knowledge. These are "good kinds of law breakers." When enough of them emerge, then the law itself can lose some of its power, and the scientific revolution, *a la* Thomas Kuhn, may be in order.

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In learning Classical Mechanics, you should be quite acutely aware of these things. Because that kind of scientific revolution already happened! About 100 years ago. A whole bunch of phenomena simply did not fit what classical mechanics was telling us. A phenomenon as simple as the low temperature specific heat of diamond. And a scientific revolution quickly followed! One of the new laws that emerged is called Quantum Mechanics. (One of the books that I fondly remember reading as an undergraduate student about the development of quantum mechanics is “Thirty Years That Shook Physics: The Story of Quantum Theory” by George Gamow.) Classical mechanics applies only to our visible every day world objects. It fails completely when atomic phenomena are concerned. Then, Quantum Mechanics rules.

Actually, a better way to think is: **Classical Mechanics emerges from Quantum Mechanics when many quantum particles interact with each other.** Let us take an example, and “walk through it.” The example is “walking” itself. We all “know” that the reason why we are able to walk is because of Newton’s third law. We push the ground, and the ground pushes us back! Sounds reasonable, we say! But, wait a second, what *really* happens when the rubber sole of our shoe presses against the floor? Here is Newton’s answer, as I paraphrase it: something must be happening at very small length scales, but I . . . really have no idea . . . what those length scales are and what’s happening, but hey, I discovered that I can describe what is happening perfectly with a vector quantity called “force.” He was simply brilliant! Now, after some 300+ years, we know more than Newton, of course. Here is our answer: a truly enormous number ( $\sim 10^{23}$ ) of electrons and ions go through all kinds of quantum mechanical interactions. However, in this interaction, there is no such thing as “force,” so we can’t begin to talk about Newton’s laws!

But don’t worry – not all is lost. There are some things. E.g., the momentum is conserved, no matter how complicated ways quantum particles interact! Remember that Newton’s third law is really the conservation principle of momentum (and angular momentum in its strong form) for Classical Mechanics. It also codifies the fact that force is “exchanged”, not just exerted one way. Namely “force” is the result of some kind of “interaction.” This characterization – the interaction that conserves momentum – may be thought of as the generalization of Newton’s third law, and it is still valid in quantum mechanics!

Now let us look closely at what happens when we walk. Or, a cleaner, but essentially equivalent, example of two billiard balls that slide without friction, hit each other and then slide without friction with different momenta. If one measures the initial velocities and the final velocities and the masses of these balls, then one would verify the momentum conservation, and thus Newton’s third law. In the initial state or the final state, let us call these two billiard balls two “particles” in the classical mechanics sense. How about the moment when they collide?! The two balls deform! Momentarily they seem melded at the interface! For electrons at the interface, we

truly do not know which electrons belong in which ball, because all electrons are fundamentally indistinguishable! Where do we draw a boundary to distinguish the two balls as two classical particles? Impossible! The two balls exchange electrons! A huge number of electrons interact! . . . quantum mechanically, with no such thing as “force”! In a true sense, this is not the realm of classical mechanics! Even so, Newton so successfully described this situation with the concept of “force” and his third law! In hindsight, then, the “force” is a macroscopic average effect of the collision process that is fundamentally not classical at all! However, we can see from this discussion that, as long as the quantum interaction conserves momentum, Newton’s third law *must* emerge, regardless of other microscopic details! It is in this sense that **Classical Mechanics is an emergent law of Quantum Mechanics.**

It is very important to keep in mind that, if you could separately observe a few electrons or a few ions during the collision process, the concept such as force or Newton’s third law is an utterly useless concept to describe what is happening.