Closed everything. No calculator. If you need other (physics or math) formula, you may ask if it can be given. Please ask if any problem needs to be clarified further. **Good luck!!**

$$\begin{aligned}
\sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b \\
\cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\
\sin(2a) &= 2 \sin a \cos a \\
\cos(2a) &= 2\cos^2 a - 1 \;=\; 1 - 2\sin^2 a \\
\sin\delta &= \delta + O(\delta^3) \\
\cos\delta &= 1 - \frac{1}{2}\delta^2 + O(\delta^4) \\
(1+\delta)^\alpha &= 1 + \alpha\delta + \frac{1}{2}\alpha(\alpha-1)\delta^2 + O(\delta^3) \\
\ln(1+\delta) &= \cancel{1} + \delta - \frac{1}{2}\delta^2 + O(\delta^3) \\
e^\delta &= 1 + \delta + \frac{1}{2}\delta^2 + O(\delta^3)
\end{aligned}$$

Show all your work. Your name and page numbers must be clearly written on your solution sheets. Be neat in writing. Partial credit for an incorrect answer will be given whenever there is a good reason for it (e.g. correct logical steps from an incorrect previous answer). Very little credit may be given for a correct answer, if not properly derived/explained.

Do only one of problems 1-2 (40 points each), and two of problems 3-5 (50 points each). Problem 6 is mandatory (60 points). Extra credit may be given if, and only if, you do the third problem of problems 3-5.

Problem 1 Consider raising a book by height h. Assume that the motion is purely vertical. Initially the book has zero velocity, as it does finally after moving the height h. The book experiences a constant acceleration a ($a > 0$, meaning upwards) in the first half of the motion and $-a$ in the second half of the motion. The gravitational field is constant, $-g$. Calculate the total work done by you on the book, the total work done by the gravity on the book, and the net work, and verify the work energy theorem.

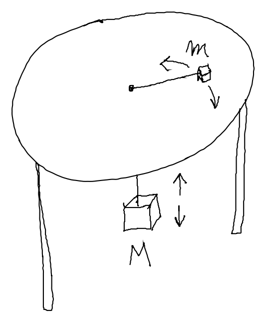
Problem 2 Name two of the three fundamental kinds of symmetry of space and time. Explain what they mean for the reproducibility of experiment, physical laws, what they mean for the Lagrangian of a closed system, and what conserved quantities they lead to.

Problem 3 Consider a simple harmonic oscillator (mass m on spring with the Hooke's law force $-kx$) that is damped by the common friction force, whose magnitude is $\mu_k mg$ (μ_k is a constant, mg the weight). The initial conditions are: $x|_{t=0} = A$ and $\dot{x}|_{t=0} = 0$. Find the value of x at the first turning point encountered in the subsequent motion.

Problem 4 Consider a projectile motion where an object is thrown at an angle θ and with the initial speed v_0 from zero height ($y = 0$). The air friction is given by $-kmv^3$, where the negative sign means opposite to the direction of the velocity. Assume that the air friction is small. Using the perturbation theory, find the speed of the object, v_f, when it comes back down to $y = 0$ again, including only the leading order correction in k. You must express your answer in the form $v_f = v_0(1 - C\alpha)$, where α is the dimensionless perturbation parameter ($\propto k$), which you must identify, and C is a dimensionless quantity that depends only on θ.

Problem 5 Consider a standard simple pendulum consisting of mass m connected to a massless string (length l) hanging down from a fixed pivot point. The surface gravity is g. Find the period of small oscillation. You must show your derivation, which must be concise but complete, starting from the most basic law, be it the Newtonian law of motion (for rotation) or the Lagrangian equation of motion.

Problem 6 A mass m is free to move on a horizontal table without any friction. At the center of the table there is a small hole. A string goes through this hole to connect the mass m to another mass M under the table. M moves up and down only. Assume that the string is short enough so that M never touches the floor, but is long enough so that M never touches the table and m is always away from the small hole. The string is massless, is always taut, and causes no friction whatsoever. The surface gravity is g. Except for (a), (c) (and possibly (b)), you are *not* expected to apply the Lagrangian formalism (unless you are advanced and are *absolutely* sure that you can). In fact, you can do all of (b-e) just by using the good old Newtonian mechanics.



(a) Find the Langrangian L. It is recommended that you take the polar coordinates, r, θ, of the mass m on the plane of the table, as the generalized coordinates.

(b) Is the energy conserved? Explain briefly.

(c) Is there any other conserved quantity? Find it (or them).

(d) Under certain condition, a stable equilibrium can be maintained in the sense that the mass M is stationary. Find such condition, and find the value of r in terms of the conserved quantity (or quantities) that you found above.

(e) Is the above equilibrium stable or not? Explain qualitatively (or quantitatively, if you like) but rigorously.