Generally, the following topics are of great importance.

- Principle of symmetry and conservation
- Perturbation theory
- Work-energy theorem
- Potential energy and SHO
- Lagrangian/Hamiltonian mechanics, effective potential

These example problems show what the midterm may look like in terms of *style*. Solutions to some problems, for which solutions cannot be found in existing homework solutions or lecture notes, will be posted about two days before the actual exam.

In the actual midterm, you will be required to do a total of four problems.

You can use these formula. If you need other formula, you can try asking.

$$
\begin{aligned}
\sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b \\
\cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\
\sin(2a) &= 2 \sin a \cos a \\
\cos(2a) &= 2\cos^2 a - 1 = 1 - 2\sin^2 a \\
\sin(\delta) &= \delta + O(\delta^3) \\
\cos(\delta) &= 1 - \frac{\delta^2}{2} + O(\delta^4) \\
(1+\delta)^\alpha &= 1 + \alpha\delta + \frac{1}{2}\alpha(\alpha-1)\delta^2 + O(\delta^3) \\
\ln(1+\delta) &= \cancel{1} + \delta - \frac{1}{2}\delta^2 + O(\delta^3)
\end{aligned}
$$

Show all your work.

Problem 1 Write the general solutions of a simple harmonic oscillator for the three different regimes. Explain why the critical damping condition is the best condition if the goal was to diminish any finite displacement to zero as quickly as possible.

Problem 2 Name the three fundamental kinds of symmetry of space and time. Explain how they are related to the reproducibility of experiments and the energy, momentum, and angular momentum conservation principles.

Problem 3 State the work-energy theorem and discuss for what forces it is applicable. Derive it from Newton's law, $\vec{F} = m\vec{a} = m d\vec{v}/dt$, and the definition of work.

Problem 4 Consider a projectile motion with a small air resistance $-k\vec{v}$. The particle leaves with speed v_0 and at angle θ relative to the horizontal. Calculate the speed of the projectile at which the particle returns to the same height, to first order in air resistance.

Problem 5 A particle is dropped from rest.

(a) Without air resistance, how long does it take for the particle to drop distance h? Your answer should be a function of h and g, the surface gravitational field.

(b) With air resistance $-mkv$, how long does it take for the particle to drop distance h? Find your answer only up to the leading order correction of the air resistance, assuming that the air resistance is weak. [Hint: You can solve Newton's equation either (1) using the perturbation theory, or (2) exactly and then doing an expansion. Then, to solve for the time corresponding to the distance h, you would need to use the perturbation method.]

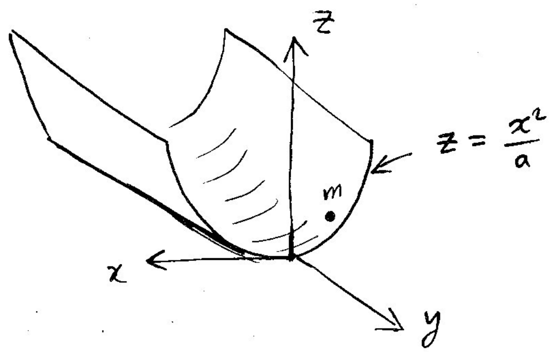
(c) What is the perturbation parameter in part (b)? Express your answer in terms of the answer for part (a) and k. [Hint: It must be dimensionless. Even if you don't have a correct answer for (b), a dimensionless parameter obtained by "an intelligent guess" is acceptable.]

Problem 6 Consider a one dimensional simple harmonic oscillator

$$m\ddot{x} = -kx - b\dot{x}$$

Treat the $-b\dot{x}$ term as a perturbation, assuming a weak damping. Calculate, up to first order, the fractional energy lost during one period τ corresponding to the natural frequency of the oscillator. The fractional energy lost = the energy lost divided by the total energy. Relate your answer to the Q factor: $Q \equiv \omega_R/(2\beta)$, with $\omega_R = \sqrt{\omega_0^2 - 2\beta^2}$, $\beta = b/(2m)$ and ω_0 is the natural frequency.

Problem 7 A particle with mass m moves, without friction, on a "trough" whose shape is given by $z = x^2/a$. There is no friction. There is a constant gravitational field, $\vec{g} = -g\hat{z}$. The following diagram shows a bottom section of the trough. Note that the trough is infinite in length (in y) and height (in z).



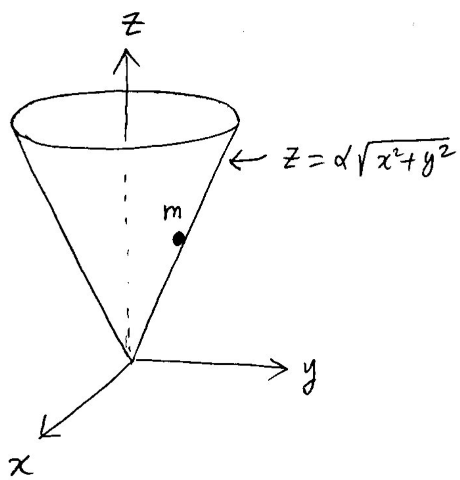
(a) Find the Lagrangian.

(b) Find the equations of motion for x and y.

(c) For small x, find the period of the x motion in terms of g, a. [Hint: $\dot{x}$ or $\ddot{x}$ is $O(x)$. Ignore terms that are higher order than linear in x in the equation of motion. For example, $x^2\ddot{x}$ would be such a term ($O(x^3)$) to ignore.]

(d) What are the conserved quantities among H, p_x, p_y, L_z? (H: Hamiltonian, p_x, p_y: linear momentum along x and y respectively, L_z: the z-component of the angular momentum.) Concisely explain why.

Problem 8 Consider a particle of mass m moving in a cone, whose shape is given by $z = \alpha\sqrt{x^2+y^2}$. There is a constant gravitational field $\vec{g} = -g\hat{z}$.



(a) Find the Lagrangian L.

(b) Find the canonical momenta for ρ and ϕ. Here, ρ and ϕ are the two cylindrical coordinates (ρ, ϕ, z). $x = \rho\cos\phi$, $y = \rho\sin\phi$.

(c) Find the Hamiltonian H.

(d) Discuss any conserved quantities among H, E, and any linear or angular momenta.

(e) Find the effective potential for ρ and discuss all possible equilibrium points and their stability.