Lecture 9 Topics

- Three occupation number plots
- Classical limits
- Fermi Gas
  - Fermi energy
  - Cold metal
  - Contact potential
- Boson Gas
  - Superfluid: He_4

Boltzman Distribution/Probability

Boltzman probability:

$$P(E_n) = \frac{e^{-\frac{E_n}{k_B T}}}{\sum_n e^{-\frac{E_n}{k_B T}}}$$

Average Energy:

$$\bar{E} = \sum E_n P(E_n) = \frac{\sum_n E_n e^{-\frac{E_n}{k_B T}}}{\sum_n e^{-\frac{E_n}{k_B T}}} = \frac{\sum_n E_n \mathcal{N}(E_n)}{\sum_n \mathcal{N}(E_n)} = \frac{\int E\, \mathcal{N}(E) D(E) dE}{\int \mathcal{N}(E) D(E) dE}$$

Occupation number:

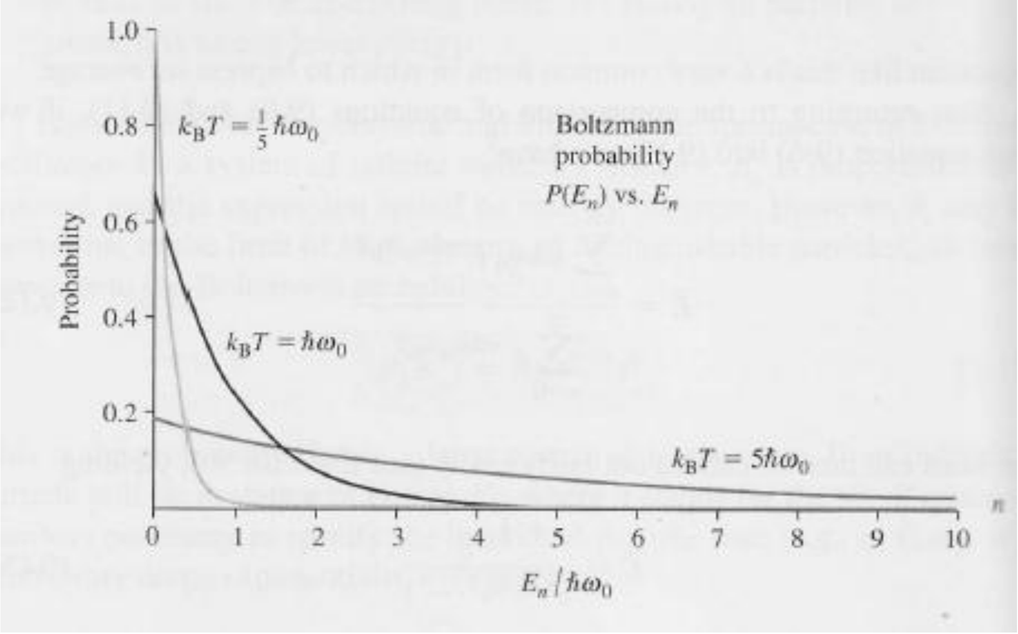
$$\mathcal{N}(E_n)_{Boltman} = N\, P(E_n) = N \frac{e^{-\frac{E_n}{k_B T}}}{\sum_n e^{-\frac{E_n}{k_B T}}}$$

Density of states over (E, E+dE):

$$D(E) \equiv \frac{dn}{dE}$$

→ When E spacing is very close

Boltzmann Probability



Three types of distribution

Distribution	Occupation index	Particles	Identical particles?	Spin	Distingui-shable?	Exclusion principle?
Boltzman	$\frac{1}{Be^{E/k_BT}}$	Classical	Yes	Any spin	Yes	No
Bose-Einstein	$\frac{1}{Be^{E/k_BT}-1}$	Bosons	Yes	0 or integer spin	No	No
Fermi-Dirac	$\frac{1}{Be^{E/k_BT}+1}$	Fermions	Yes	1/2	No	Yes

When N is REALLY large, like 10^{23}

System requirement

- Conservation of particles

$$\sum \mathcal{N}(E_i) = \mathcal{N}(E_1) + \mathcal{N}(E_2) + \cdots + \mathcal{N}(E_k) = N$$

- Conservation of energy

$$\sum \mathcal{N}(E_i) E_i = \mathcal{N}(E_1) E_1 + \mathcal{N}(E_2) E_2 + \cdots + \mathcal{N}(E_k) E_k = E$$

- Example: 4 (a, b, c, d) particles with total energy of

$$2\hbar\omega_0.$$

Maxwell-Boltzman

n	10 possible ways										No. of possibilities where a particle can have n quantum number (#)	Probability of a particle having the n quantum number (P=#/40)	Probable number of particles to have the n quantum number P x 4
2	*a*	*b*	*c*	*d*									
1					*ab*	*ac*	*ad*	*bc*	*bd*	*cd*			
0	*bcd*	*acd*	*abd*	*abc*	*cd*	*bd*	*bc*	*ad*	*ac*	*ab*			
										Total			

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2	*a*	*b*	*c*	*d*							4	0.1	0.4
1					*ab*	*ac*	*ad*	*bc*	*bd*	*cd*	12	0.3	1.2
0	*bcd*	*acd*	*abd*	*abc*	*cd*	*bd*	*bc*	*ad*	*ac*	*ab*	24	0.6	2.4
										Total	40	1.0	4.0

Bose-Einstein

n	10 possible ways										No. of possibilities where a particle can have n quantum number (#)	Probability of a particle having the n quantum number (P=#/40)	Probable number of particles to have the n quantum number P x 4
2	x	x	x	x							4	0.1	0.4
1					xx	xx	xx	xx	xx	xx	12	0.3	1.2
0	xxx	xxx	xxx	xxx	xx	xx	xx	xx	xx	xx	24	0.6	2.4
										Total	40	1.0	4.0

Bose-Einstein

n	Two possible ways		(#)	(P=#/8)	P x 4
2	X		1	0.125	0.50
1		XX	2	0.250	1.00
0	XXX	XX	5	0.625	2.50
		Total	8	1.000	4.00

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Fermi-Dirac

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2	x	x	x	x							4	0.1	0.4
1					xx	xx	xx	xx	xx	xx	12	0.3	1.2
0	xxx	xxx	xxx	xxx	xx	xx	xx	xx	xx	xx	24	0.6	2.4
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Fermi-Dirac

n	1 possible way	(#)	(P=#/8)	P x 4
2		0	0.0	0.0
1	XX	2	0.5	2.0
0	XX	2	0.5	2.0
	Total	4	1.0	4.0

Three Distributions

n	10 possible ways										No. of possibilitie s where a particle can have n quantum number (#)	Probability of a particle having the n quantum number (P=#/40)	Probable number of particles to have the n quantum number P x 4
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Probable Number Plotting

n	Maxwell-Boltzmann	Bose-Einstein	Fermi-Dirac
3	0.0	0.0	0.0
2	0.4	0.5	0.0
1	1.2	1.0	2.0
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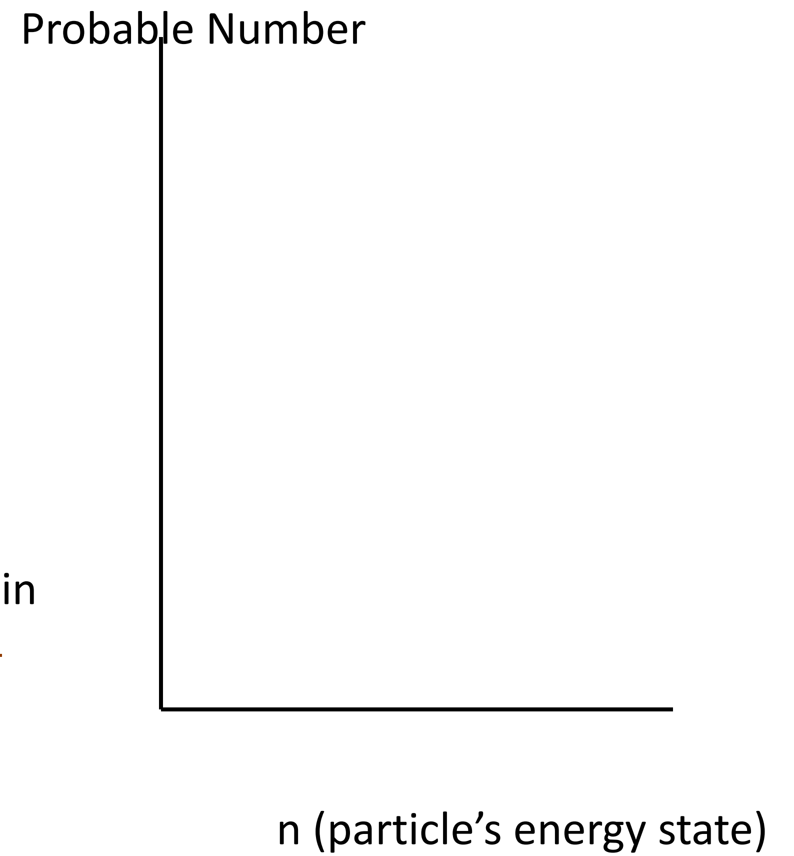

When there are four harmonic oscillators in a system with the system's energy is $2\hbar\omega_0$.

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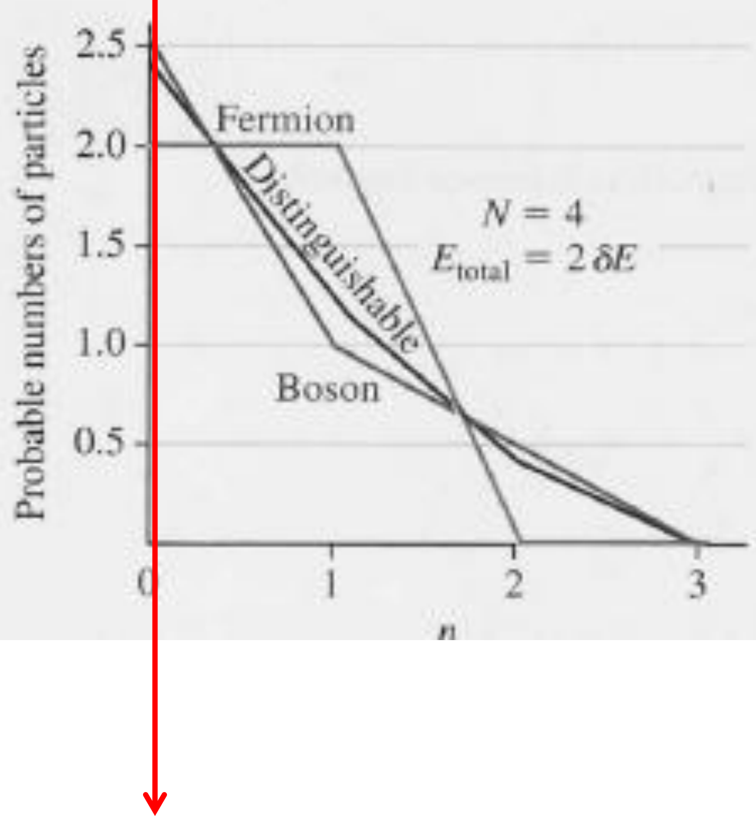
Figure 9.9 The probable number of particles at the allowed energies depends on whether the particles are bosons, fermions, or distinguishable.



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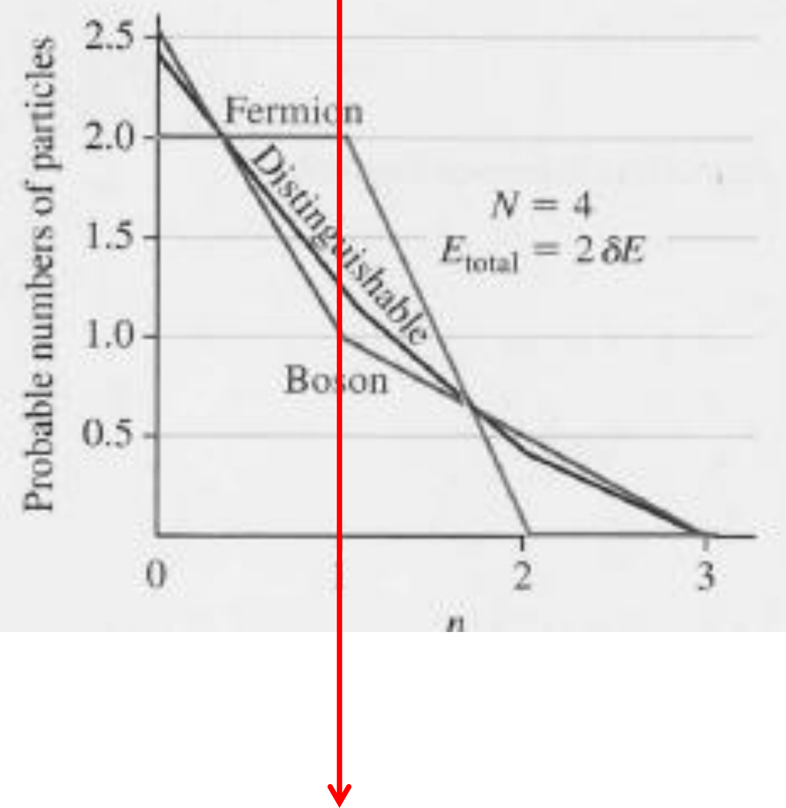

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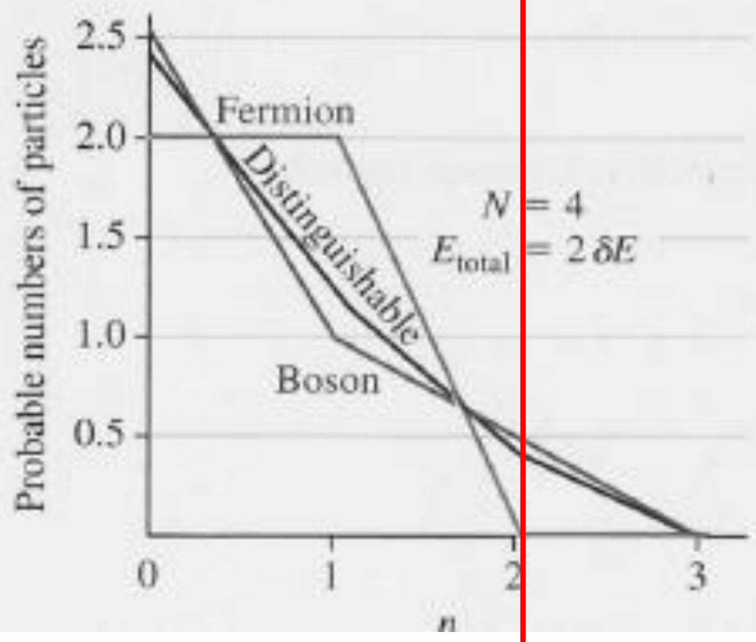

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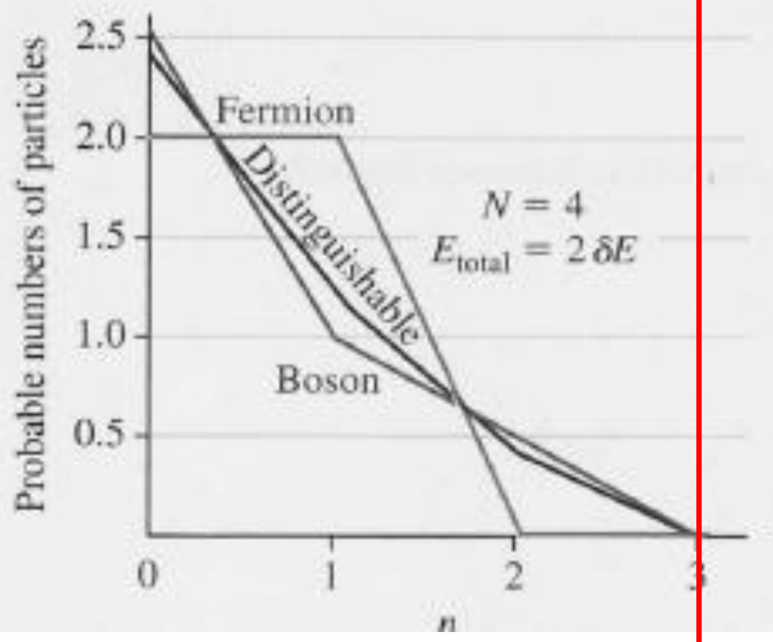
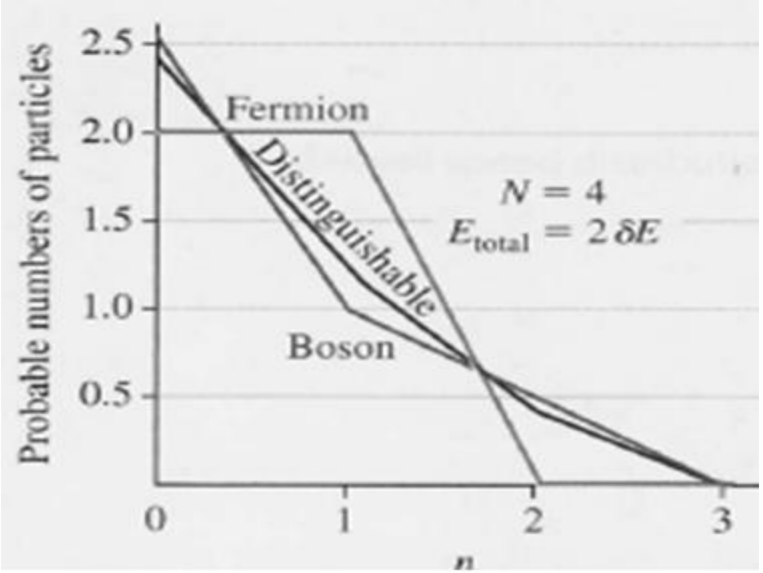
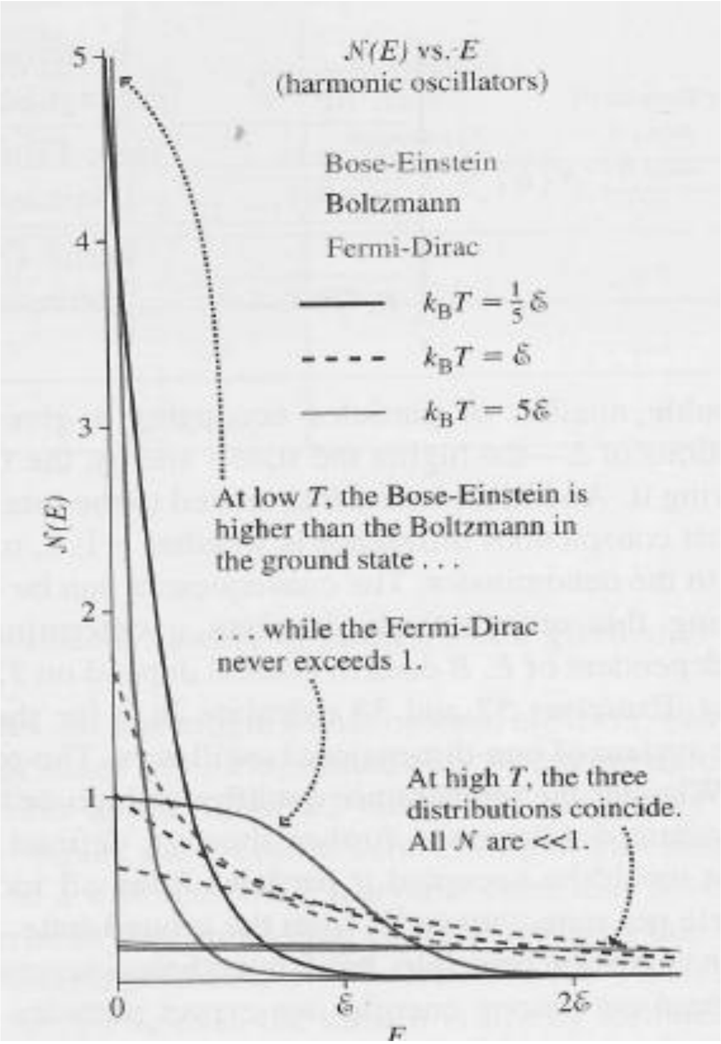


Occupation number distribution comparison

As E increases

$k_B T << E$ → Low temp relative to system E
Solid lines





Occupation number distribution comparison

As E increases

$k_B T << E$ → Low temp relative to system E
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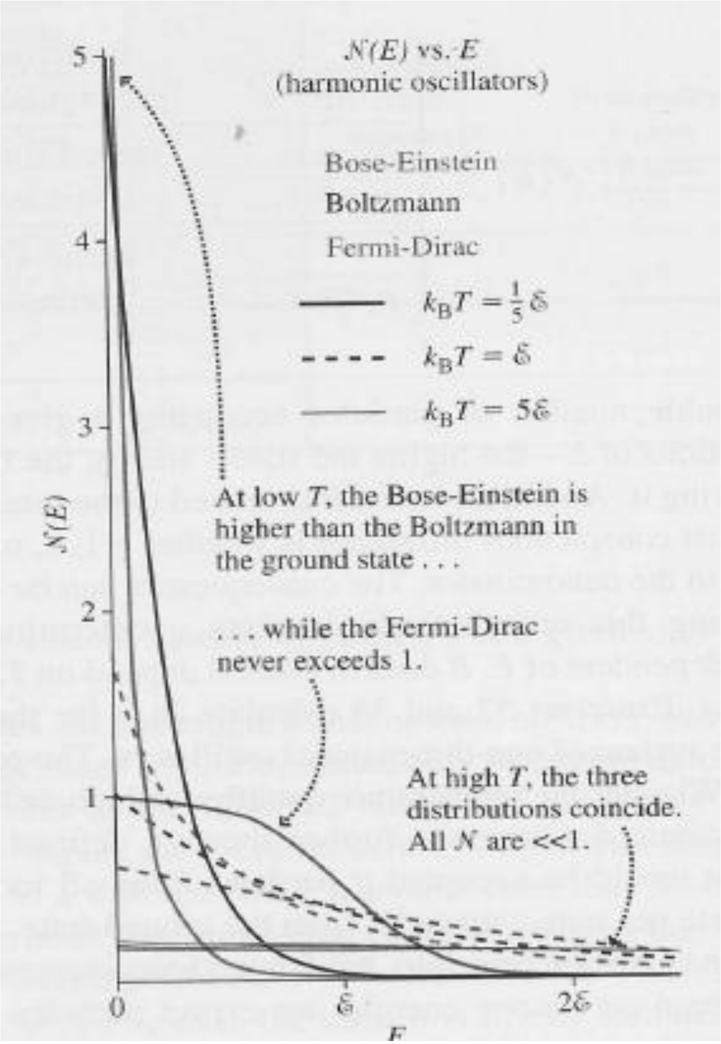
$k_B T \approx E$
Dashed lines



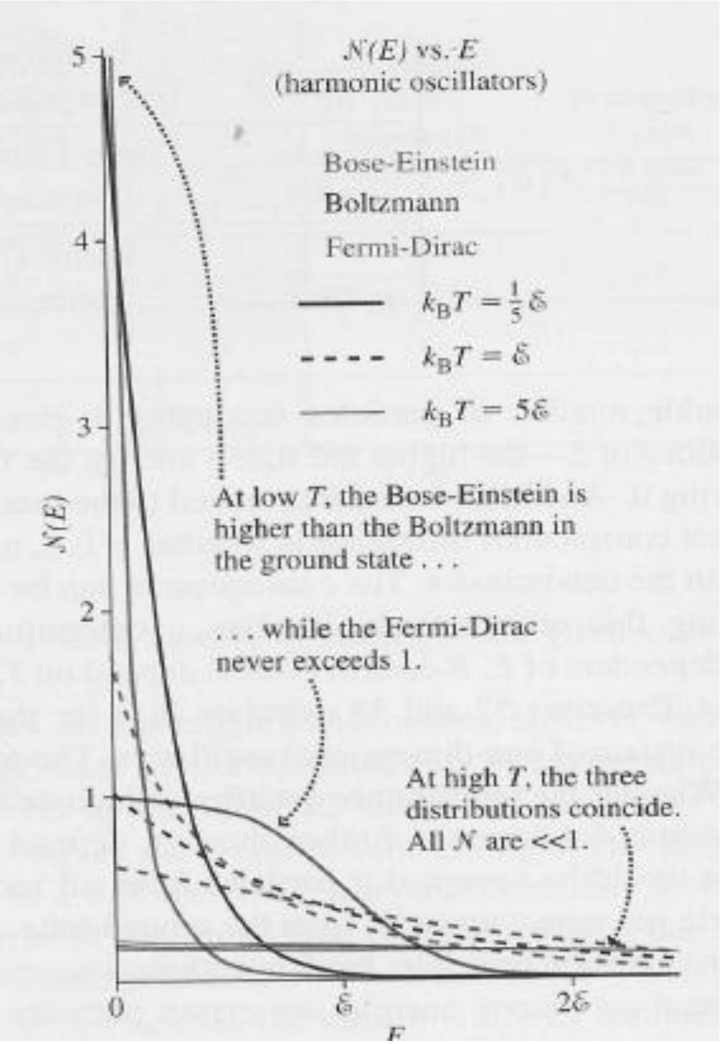
Occupation number distribution comparison

As E increases

$k_BT << E$ → Low temp relative to system E
Solid lines

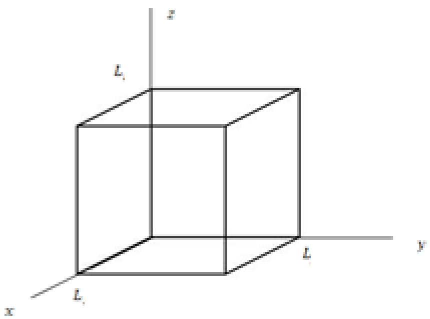
$k_BT \approx E$
Dashed lines

$k_BT >> E$ → High temp relative to system E
Thin gray lines



When to use quantum gas treatment?

- A system of N particles in a 3-D box



The 3D Box

$$U(\vec{x}) = \begin{cases} 0 & 0 < x, y, z < L, \\ \infty & \text{otherwise} \end{cases}$$

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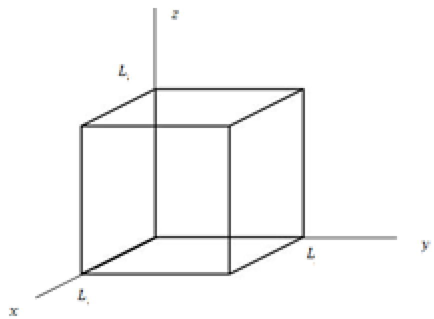


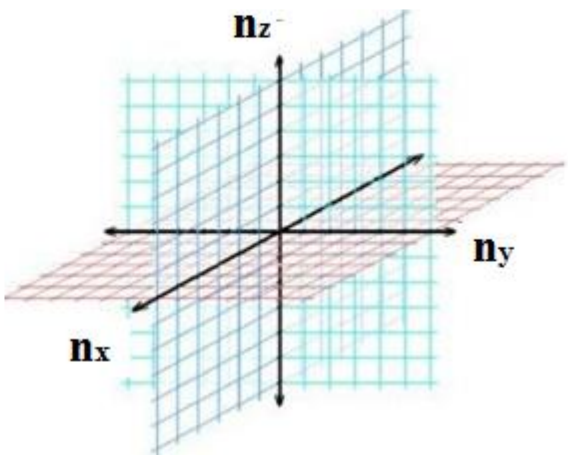
The 3D Box

$$U(\vec{x}) = \begin{cases} 0 & 0 < x, y, z < L, \\ \infty & otherwise \end{cases}$$

$$\psi(x,y,z) = A \sin\frac{n_x \pi x}{L} \sin\frac{n_y \pi y}{L} \sin\frac{n_z \pi z}{L}$$

$$E_{(n_x, n_y, n_z)} = (n_x^2 + n_y^2 + n_z^2)\frac{\pi^2 \hbar^2}{2mL^2}$$

Quantum Number Space



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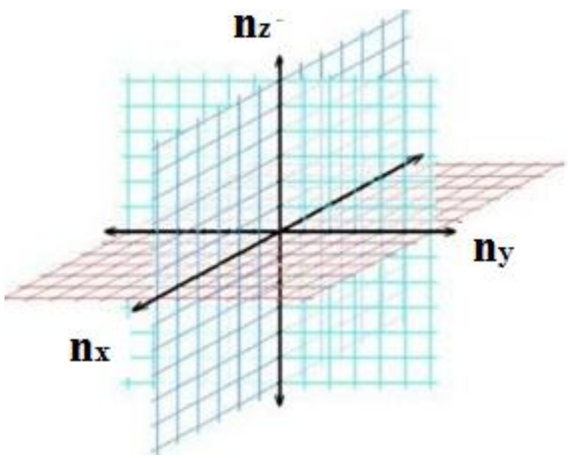

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$$E = \frac{\pi^2\hbar^2 n^2}{2mL^2} \qquad n = \sqrt{\frac{2mL^2}{\pi^2\hbar^2}}\sqrt{E}$$

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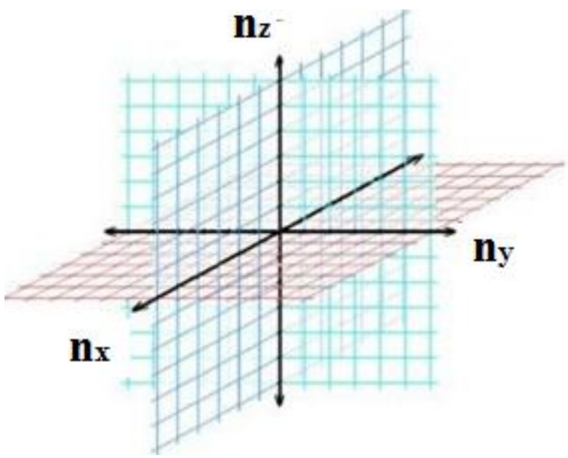

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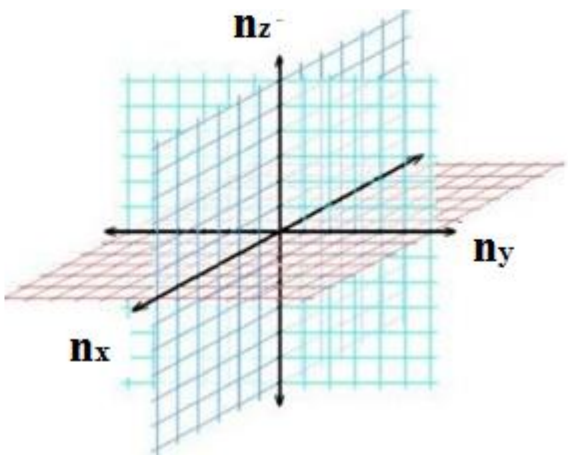

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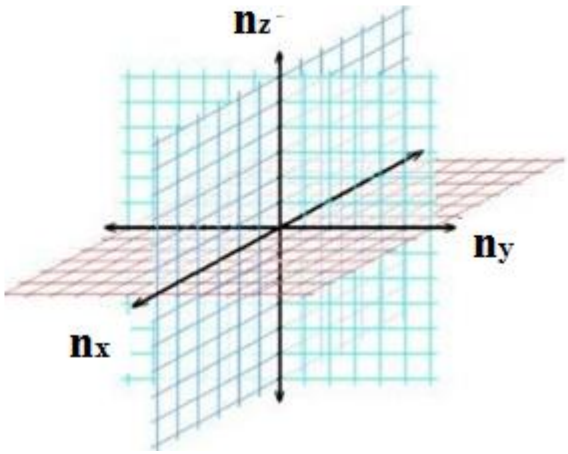
Quantum Number Space



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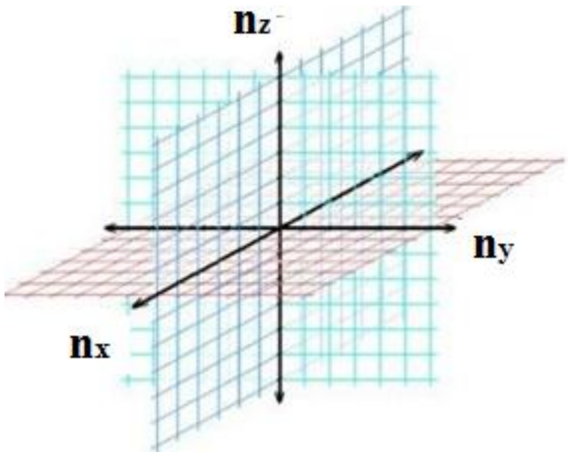

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Particles that obey the Boltzman distribution

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Particles that obey the Bose-Einstein distribution

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Particles that obey the Fermi-Dirac distribution

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Classical Limit: $$\frac{\hbar^3}{(mk_BT)^{3/2}}\left(\frac{N}{V}\right)^1 \ll 1$$

$$D(E) = (2s+1)\frac{m^{3/2}V}{\pi^3\hbar^3\sqrt{2}}\sqrt{E}$$

$$N = \int_0^\infty \mathcal{N}(E)D(E)dE = \frac{m^{3/2}V}{\pi^3\hbar^3\sqrt{2}}\int_0^\infty \frac{\sqrt{E}}{Be^{E/k_BT} \mp 1}dE$$

$$\bar{E} = \frac{\int_0^\infty E\mathcal{N}(E)D(E)dE}{\int_0^\infty \mathcal{N}(E)D(E)dE} = \frac{\int_0^\infty \frac{E^{\frac{3}{2}}}{Be^{\frac{E}{k_BT}} \mp 1}dE}{\int_0^\infty \frac{\sqrt{E}}{Be^{\frac{E}{k_BT}} \mp 1}dE} = \frac{3}{2}k_BT\left[1 \mp \frac{\pi^3\hbar^3\sqrt{2}}{(2s+1)(2\pi m k_BT)^{3/2}}\left(\frac{N}{V}\right)^1 + \cdots\right]$$

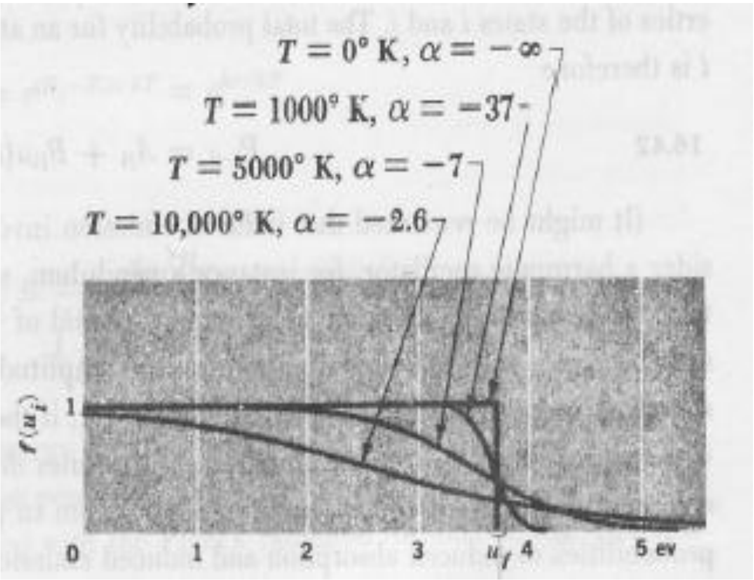
For the ideal gas, this term is
In the order of $\cong 9 \times 10^{-9}$

Classical Limit: $\frac{\hbar^3}{(mk_BT)^{3/2}}\left(\frac{N}{V}\right)^1 \ll 1$ OR $\left(\frac{\lambda}{d}\right)^3 \ll 1$

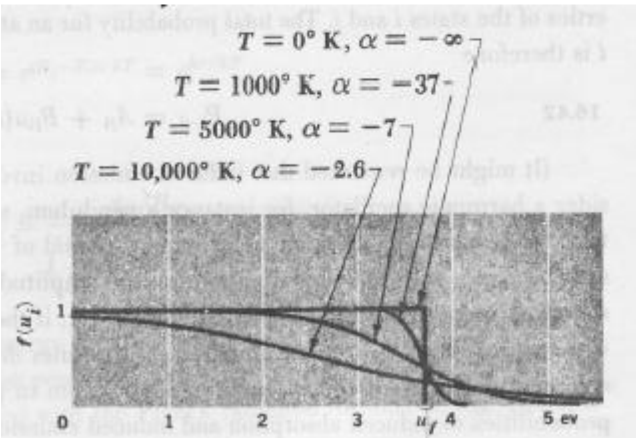
Fermi Energy

$$\mathcal{N}(E_F) = \frac{1}{Be^{E/k_BT}+1} = \frac{1}{2} \quad \text{which makes } B = e^{-\frac{E_F}{k_BT}}$$

$$\mathcal{N}(E_F) = \frac{1}{e^{(E-E_F)/k_BT}+1}$$



Fermi Energy



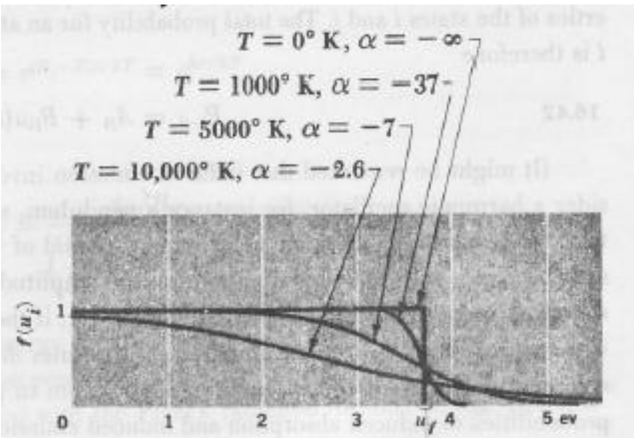
At T=0, electrons fill upto the E_F Level
There are N electrons.

$$N = \int \mathcal{N}(E) D(E) dE$$

$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$

$$D(E) = (2s+1)\frac{m^{3/2}V}{\pi^3 \hbar^3 \sqrt{2}}\sqrt{E}$$

Fermi Energy



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=1 upto E_F

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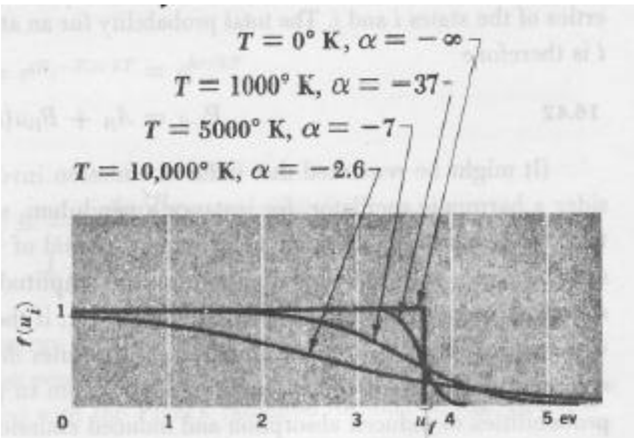
Fermi Energy



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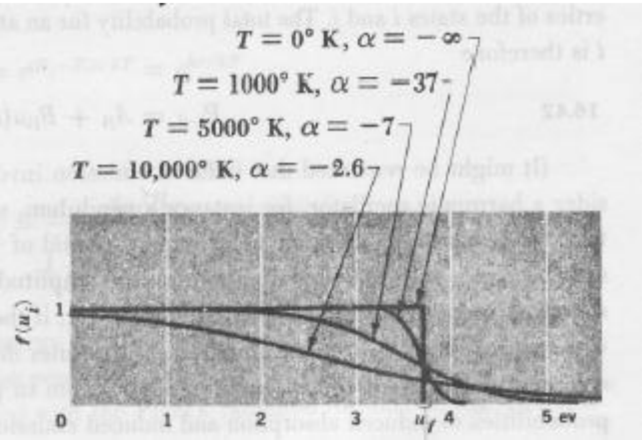
Fermi Energy



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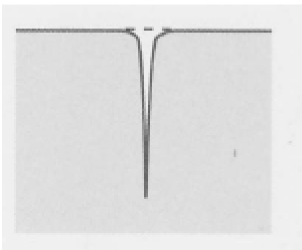
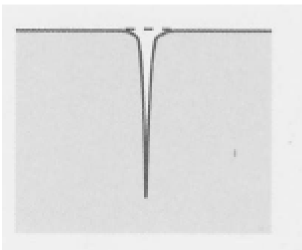
$$E_F = \frac{\pi^2\hbar^2}{m}\left[\frac{3}{(2s+1)\pi\sqrt{2}}\frac{N}{V}\right]^{2/3}$$

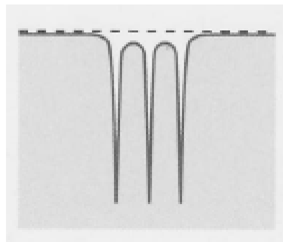
For Silver, E_F is 5.5 eV

Fermi Gas: Conduction Electrons

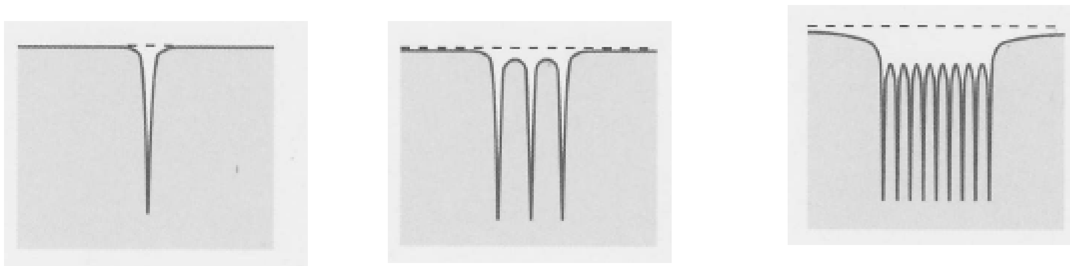


Fermi Gas: Conduction Electrons

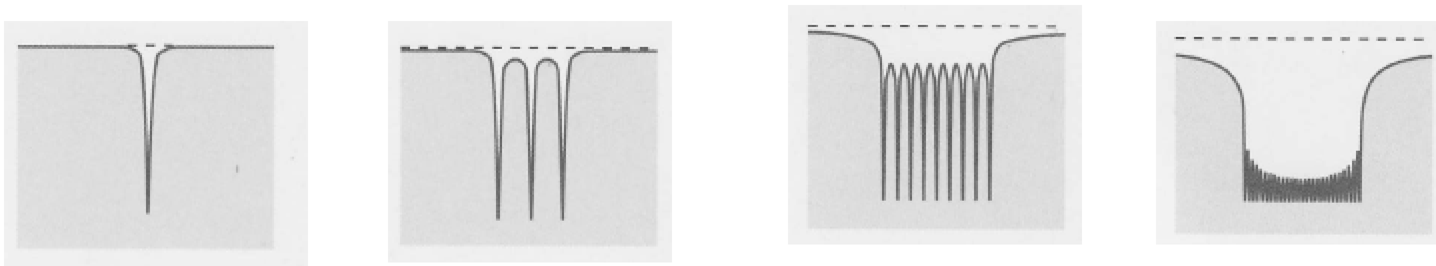




Fermi Gas: Conduction Electrons



Fermi Gas: Conduction Electrons



Fermi Gas: Conduction Electrons

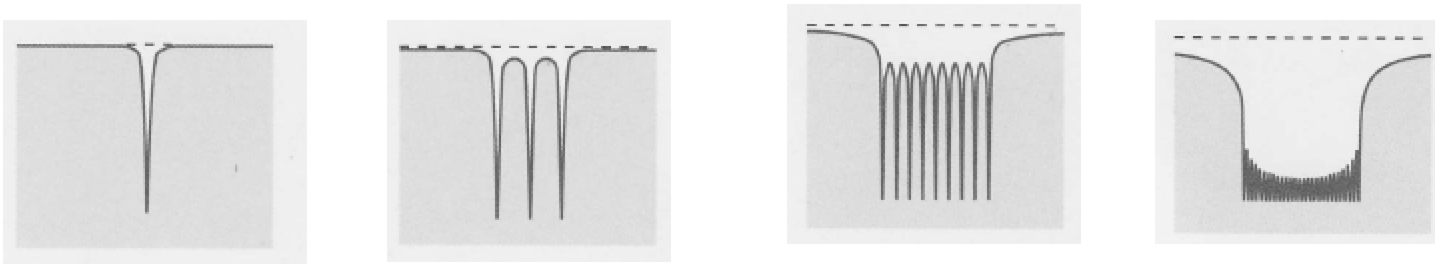
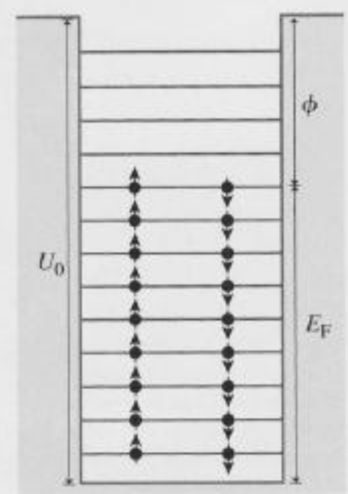
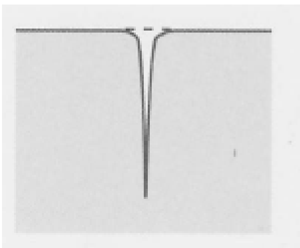
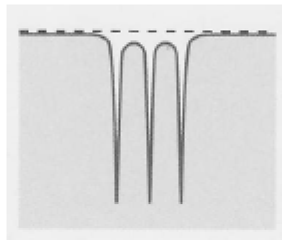


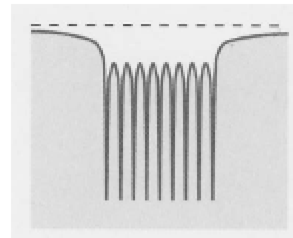
Figure 9.17 Electron energies in a "cold" metal.

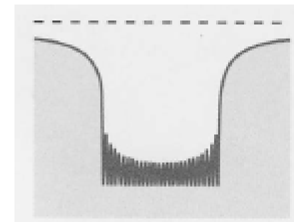


Fermi Gas: Conduction Electrons



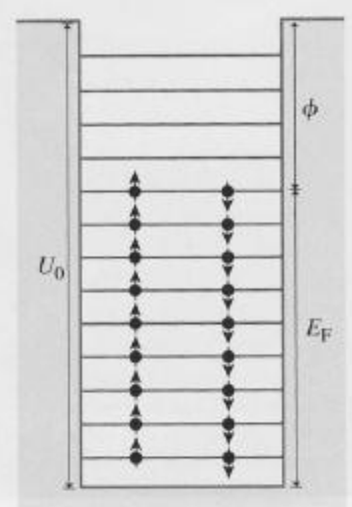




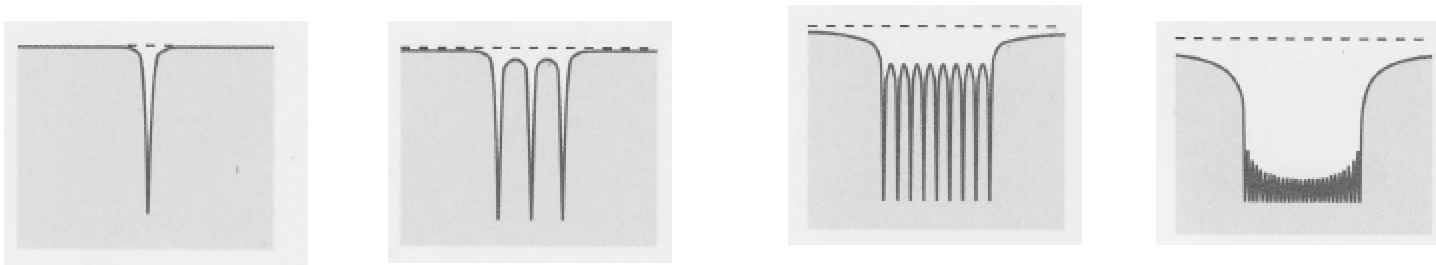


Electrons stack up the energy levels according to Pauli Exclusion Principle.

Figure 9.17 Electron energies in a "cold" metal.



Fermi Gas: Conduction Electrons



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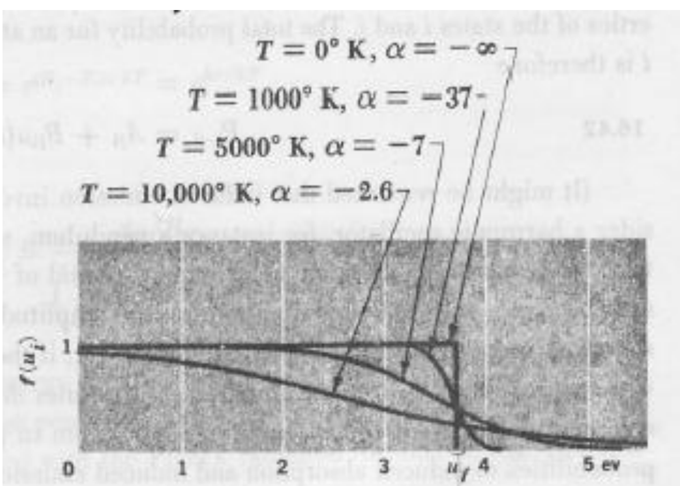

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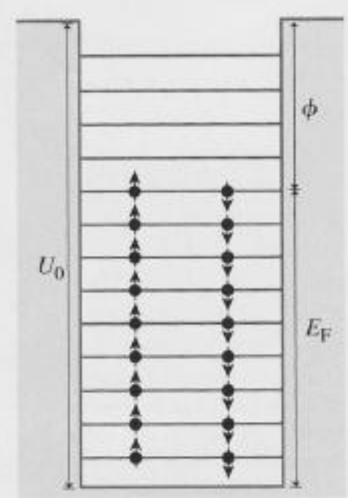
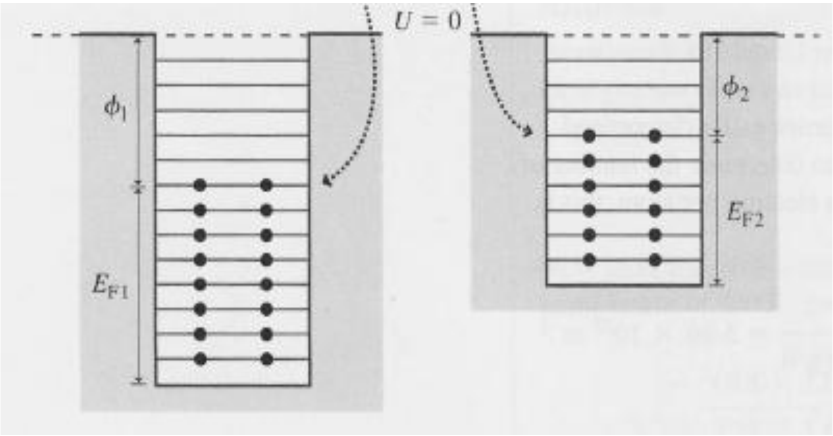


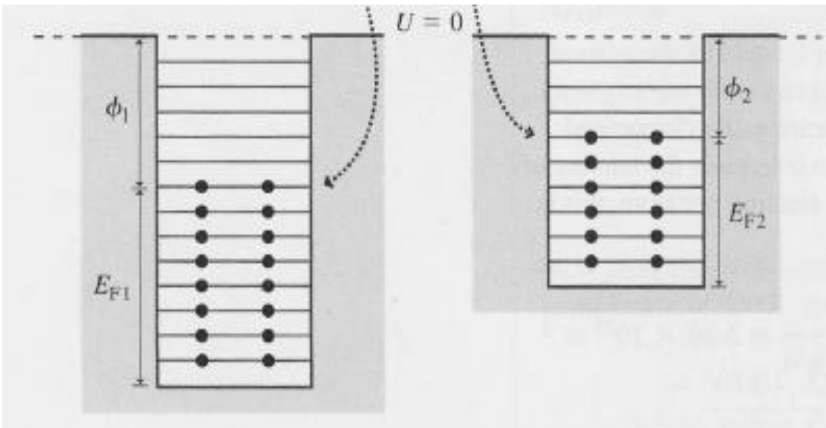
Contact Potential



Work function:

$$\phi = U_0 - E_F$$

Contact Potential



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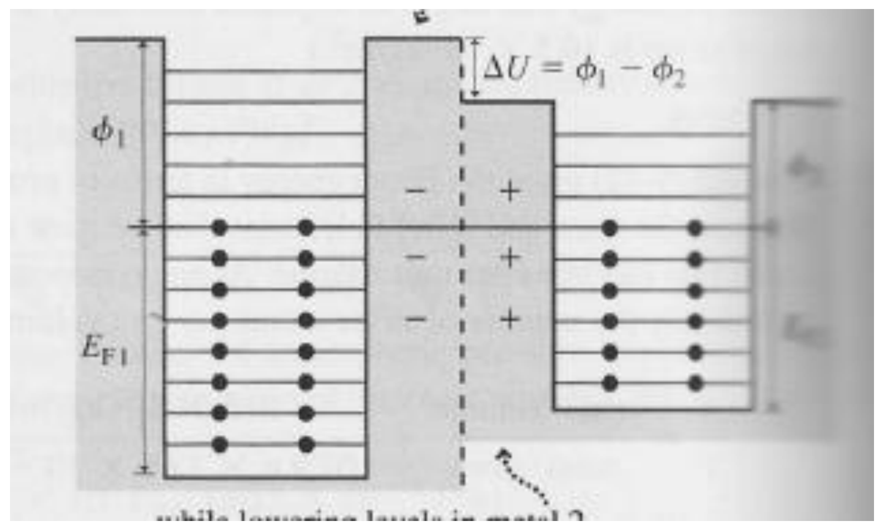
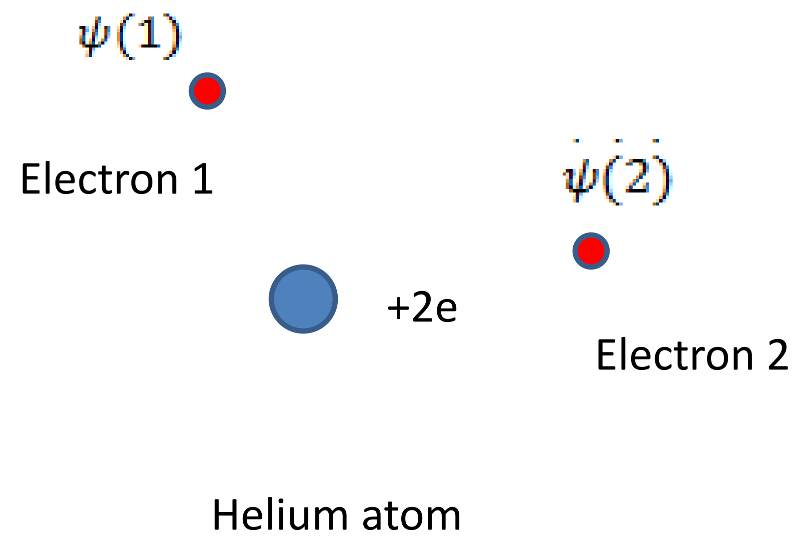
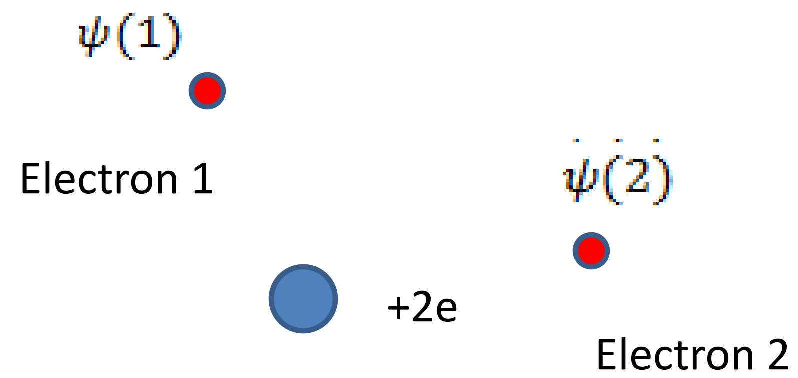


SuperFluid: He_4



SuperFluid: He_4

Is He a Boson or a Fermion?



Helium atom

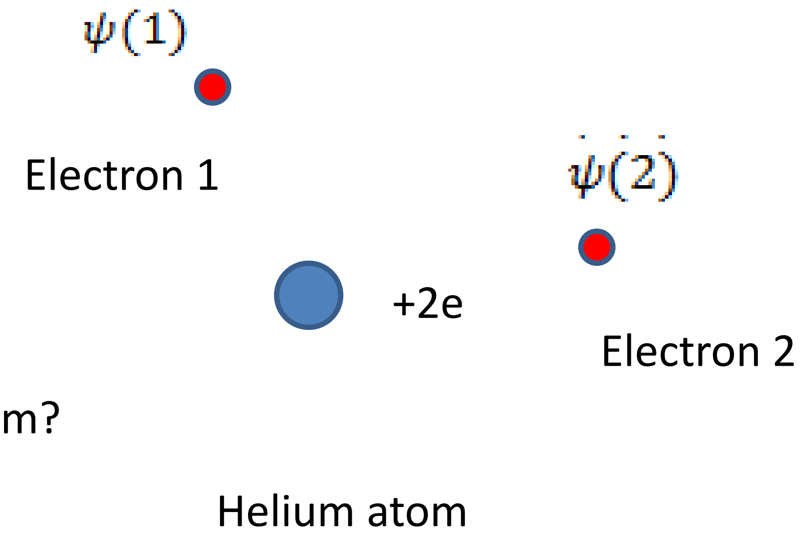
SuperFluid: He_4

He_4 has two electrons of spin 1/2

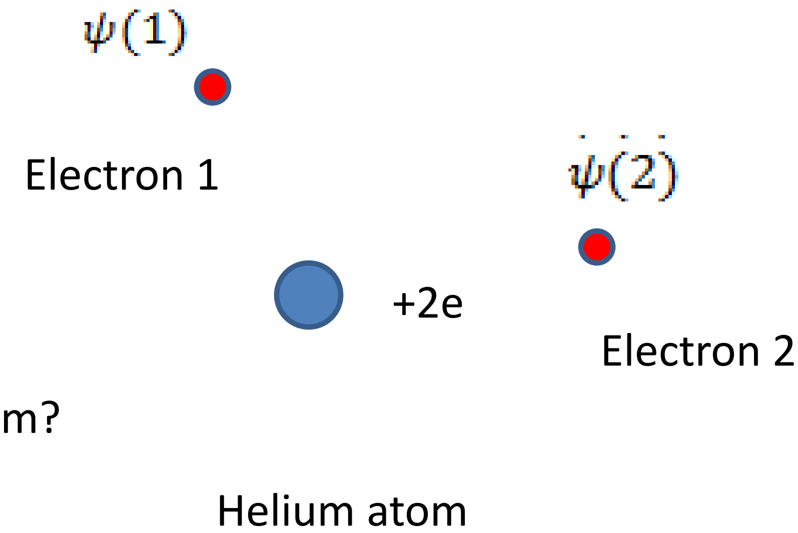
$S_1 = ½$ $S_2 = ½$

What is the total spin angular momentum?

What are m_s possibilities?



SuperFluid: He_4



He_4 has two electrons of spin 1/2

$S_1 = ½$ $S_2 = ½$

What is the total spin angular momentum?
S = ½ + ½ =1   OR S= ½ - ½ =0

What are m_s possibilities?

For S = 0, m_s =0

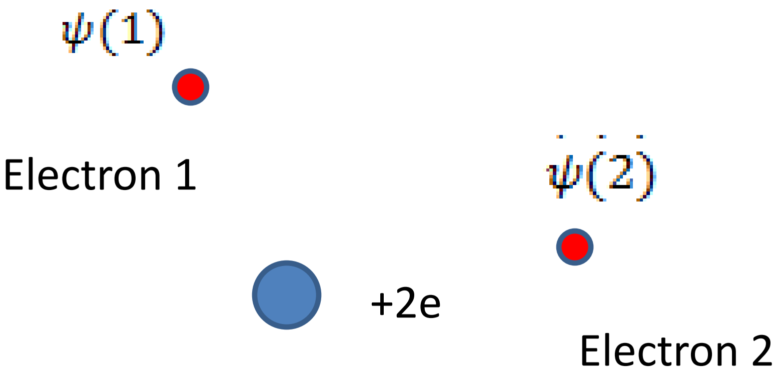
For S = 1, m_s =1

m_s =0

m_s =-1

SuperFluid: He_4

He_4 has two electrons of spin 1/2



Helium atom

$S_1 = ½$ $S_2 = ½$

What is the total spin angular momentum?
S = ½ + ½ =1 OR S= ½ - ½ =0

What are m_s possibilities?

For S = 0, m_s =0 ↑↓ - ↓↑

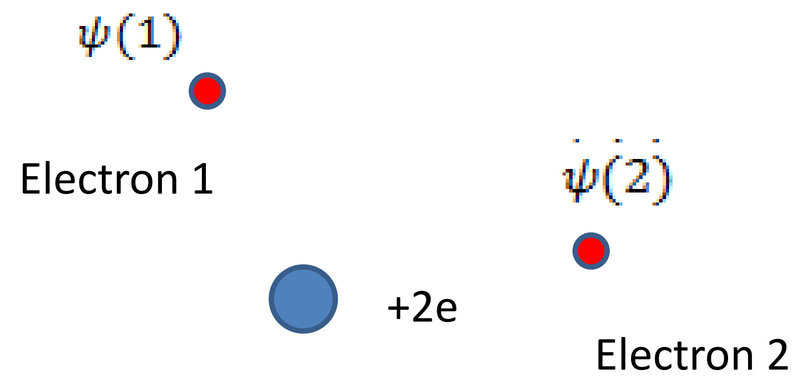
For S = 1, m_s =1 ↑↑

m_s =0 ↑↓ + ↑↓

m_s =-1 ↓↓

SuperFluid: He_4

He_4 has two electrons of spin 1/2



Helium atom

$S_1 = ½$ $S_2 = ½$

What is the total spin angular momentum?

$S = ½ + ½ = 1$ OR $S = ½ - ½ = 0$

What are m_s possibilities?

For $S = 0$, $m_s = 0$ ↑↓ - ↓↑

Singlet anti-symmetric

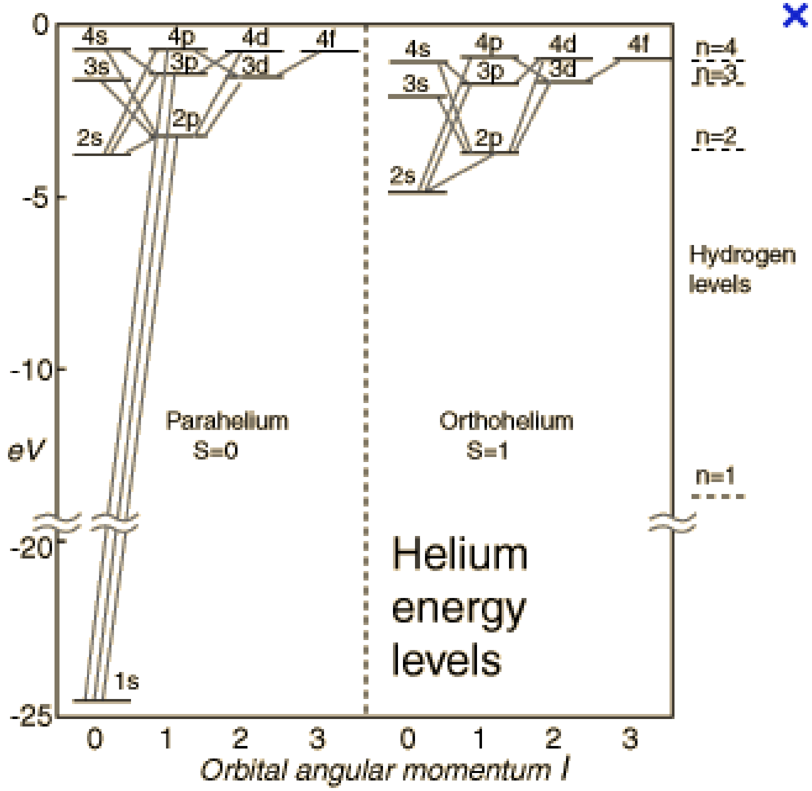
For $S = 1$, $m_s = 1$ ↑↑

$m_s = 0$ ↑↓ + ↑↓

$m_s = -1$ ↓↓

Triplet symmetric

Superfluid: He4



- The Spin O state is more stable for the 1S orbital.

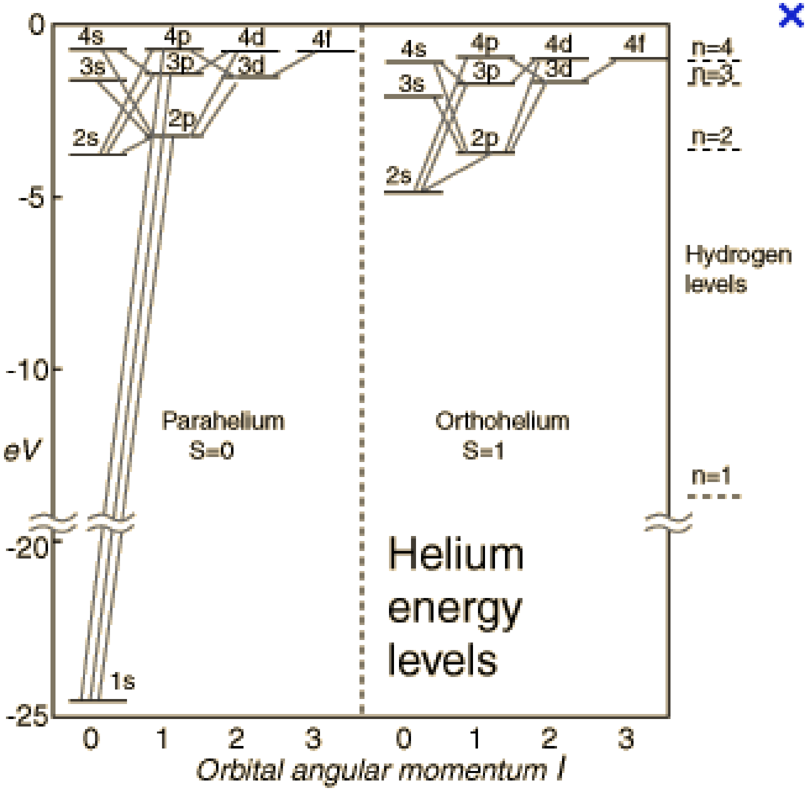
- Since electrons in He4 are considered bosons, they can occupy the same energy state.

- As He_4 cools, more stable energy states are sought by electrons.

- Around 2.2K, most electrons in He4 occupies the lowest possible energy states, meaning sharing the same wave function.

$$\frac{1}{\sqrt{2}}\begin{vmatrix} \psi_{1s}(x_1)\uparrow_1 & \psi_{1s}(x_1)\downarrow_1 \\ \psi_{1s}(x_2)\uparrow_2 & \psi_{1s}(x_2)\downarrow_2 \end{vmatrix}$$

Superfluid: He_4



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- In a system of N He atoms:

$$\sum_{j=1}^{N} e^{-i\vec{k_j}\cdot\vec{x_j}} \begin{vmatrix} \psi_{1s}(x_1)\uparrow_1 & \psi_{1s}(x_1)\downarrow_1 \\ \psi_{1s}(x_2)\uparrow_2 & \psi_{1s}(x_2)\downarrow_2 \end{vmatrix}$$

Power of coherent waves

Power of coherent waves