PH102, 2014W, Lecture Notes: March 6, Thurs, Lecture 18

Radioactive decay:

- Definition: An unstable nucleus (parent nucleus) can spontaneously emit small particles or energies to become a nucleus (daughter nucleus) in a more stable state.
- Energy is conserved in radioactive decay: $Q\ (kinetic\ energy\ released) = (m_i - m_f)c^2$

Three forms of radioactive decay depending upon what is emitted during radioactive decay

Alpha decay

- Emits an alpha particle (He nucleus=2 protons + 2 neutrons)
- This process makes:

$$^{A}_{Z}X(parent) \rightarrow ^{A-4}_{Z-2}Y(daughter) + ^{4}_{2}He$$

  - $Z_{daughter} = Z_{parent} - 2$
  - $N_{daughter} = N_{parent} - 2$
  - $A_{daughter} = A_{parent} - 4$

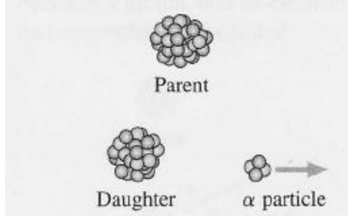


Figure 11.18 Alpha decay.

- Released kinetic energy (Q)

$$Q = (m_{parent} - m_{daughter} - m_{^4_2He})\ c^2$$

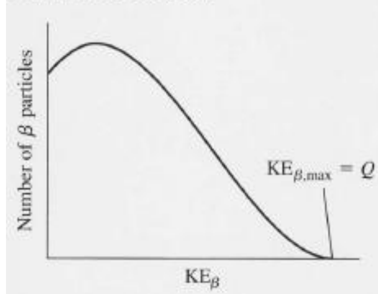
- Example: alpha decay of $^{238}_{92}U$

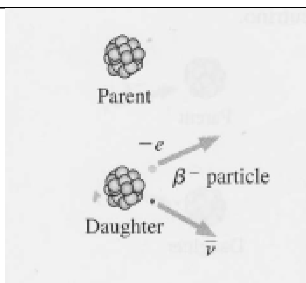
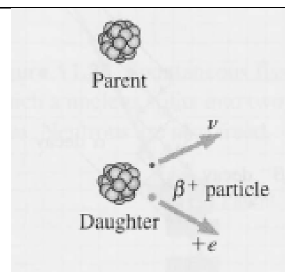
$$^{238}_{92}U \rightarrow\ ^{234}_{90}Th +\ ^{4}_{2}He$$
$$Q = (m_{parent} - m_{daughter} - m_{^4_2He})\ c^2$$
$$= (238.050784 - 234.043593 - 4.002603)uc^2$$
$$= 0.004588 \text{ x } 931.5\ MeV = 4.27\ MeV$$

Since the mass of He is a lot smaller than that of Th, the alpha particle gets almost all the kinetic energy during the alpha decay process.

Beta decay emits temporarily created negatively charged electrons ($\boldsymbol{\beta^-\ particle}$) or positively-charged electrons ($\boldsymbol{\beta^+\ particle}$).
In order to satisfy charge conservation, energy conservation, and angular momentum conservation, a new particle is introduced called neutrinos/antineutrinos.

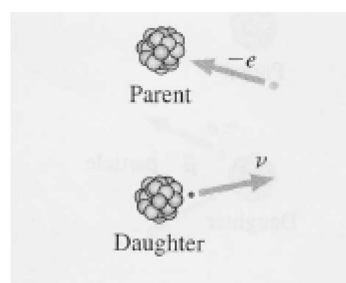
- Charge conservation: since beta particles carry a charge, the charge of the parent nucleus should increase by 1 in β^- decay and decrease by 1 in β^+ decay. As a result, neutrinos' charge should be neutral.
- Angular momentum conservation: since β particles have a spin of ½, to conserve angular momentum, neutrinos should have a spin of ½.
- Energy conservation. Figure 11.19 shows that kinetic energies of β particles emitted during the decay vary greatly from zero to Q (maximum allowed). Therefore, neutrinos should carry portions of kinetic energies with β particles. To allow β particles to have the maximum allowed kinetic energy, neutrinos' mass should be negligible.

Figure 11.19 The mysterious variation in β particle energies.


$\boldsymbol{\beta^-}$ **decay**	$\boldsymbol{\beta^+}$ **decay**
• An electron ($\boldsymbol{\beta^-\ particle}$) is temporarily created and then emitted. • Emits an electron and an anti-neutrino. • Changes a neutron inside the nucleus into a proton.	• Emits a positron and a neutrino • Changes a proton inside the nucleus into a neutron.
${}^{A}_{Z}X(parent) \rightarrow {}_{Z+1}^{\ \ A}Y(daughter) + {}_{-1}^{\ \ \mathbf{0}}\boldsymbol{\beta^-} + \bar{\boldsymbol{\nu}}$ ○ $Z_{daughter} = Z_{parent} + 1$ ○ $N_{daughter} = N_{parent} - 1$ ○ $A_{daughter} = A_{parent}$ Released kinetic energy $Q = (m_{parent} - m_{daughter})\ c^2$	${}^{A}_{Z}X(parent) \rightarrow {}_{Z-1}^{\ \ A}Y(daughter) + {}_{+1}^{\ \ \mathbf{0}}\boldsymbol{\beta^+} + \boldsymbol{\nu}$ ○ $Z_{daughter} = Z_{parent} - 1$ ○ $N_{daughter} = N_{parent} + 1$ ○ $A_{daughter} = A_{parent}$ Released kinetic energy $Q = (m_{parent} - m_{daughter} - 2m_{electron})\ c^2$
Example: ${}^{12}_{5}B \rightarrow {}^{12}_{6}C\ + {}_{-1}^{\ \ \mathbf{0}}\boldsymbol{\beta^-} + \bar{\boldsymbol{\nu}}$ $Q = (12.014352 - 12)uc^2 = 13.4\ MeV$	${}^{12}_{7}N \rightarrow {}^{12}_{6}C\ + {}_{+1}^{\ \ \mathbf{0}}\boldsymbol{\beta^+} + \boldsymbol{\nu}$ $Q = (12.018613 - 12 - 2 \text{ x } 0.0005486)uc^2$ $= 16.3\ MeV$
	

The third form of beta decay is Electron Capture.

- A nucleus with too many protons can change a proton into a neutron by capturing an electron.
- Electron capture is easier than $\boldsymbol{\beta^+}$decay since an electron is already exists for a nucleus to capture.



$${}^{A}_{Z}X(parent) + {}_{-1}^{\ \ \mathbf{0}}\boldsymbol{\beta^-} \rightarrow {}_{Z-1}^{\ \ A}Y(daughter) + \nu$$

- ○ $Z_{daughter} = Z_{parent} - 1$
- ○ $N_{daughter} = N_{parent} + 1$
- ○ $A_{daughter} = A_{parent}$

- Released kinetic energy

$$Q = (m_{parent} - m_{daughter})\ c^2$$

- Example:

$${}^{11}_{6}C + {}_{-1}^{\ \ \mathbf{0}}\boldsymbol{\beta^-} \rightarrow {}^{11}_{5}B\ + \nu$$

$$Q = (11.01143 - 11.009305)uc^2 = 1.97\ MeV$$

Gamma Decay

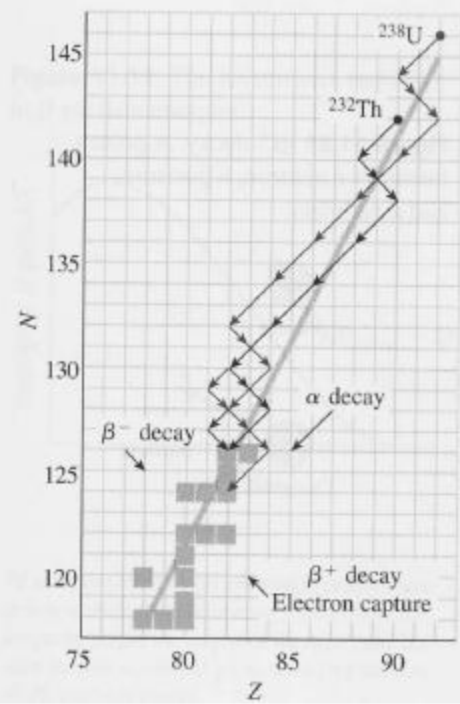
- A nucleus in an excited state emits photons (gamma particles, γ) to go into a lower energy state.
- Gamma decay does not alter N or Z.
- Gamma energies are characteristic of a given isotope, and are thus used to identify the isotope.

Figure 11.24 Gamma decay.

Decay series

- An unstable nucleus can be involved in a series of decays until it finds a stable state.
- We can plot this process on a graph that represents N and Z numbers of each nucleus in the series.
- Figure 11.23 shows such a graph. The gray line represents the line of stability. The figure shows two series: one for U-238 and the other for Th232.
- Alpha decay is shown by an arrow
  - $Z_{daughter} = Z_{parent} - 2$
  - $N_{daughter} = N_{parent} - 2$
- $\boldsymbol{\beta^-}$ **decay**
  - $Z_{daughter} = Z_{parent} + 1$
  - $N_{daughter} = N_{parent} - 1$
- $\boldsymbol{\beta^+}$ **decay and electron capture**
  - $Z_{daughter} = Z_{parent} - 1$
  - $N_{daughter} = N_{parent} + 1$

Figure 11.23 The "directions" of α and β decays, and the decay series of uranium-238 and thorium-232.



Radioactive Decay Law

- For all decays, the rate of decay over time will be proportional to the sample size:

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N \qquad \text{where N=Number of nuclei; } \lambda\text{=decay constant}$$

$$\frac{dN}{N} = -\lambda\, dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$ln\frac{N}{N_0} = -\lambda t$$

$$N = N_0 e^{-\lambda t}$$

- Decay rate R = λN (decays per second)
- A sample of the same nuclei will decay by the same fraction in equal successive intervals of time.

- Half-life ($T_{1/2}$) is defined as the time interval at which half of the sample will decay:

$$\frac{1}{2}N_0 = N_0 e^{-\lambda T_{1/2}}$$

From this relationship, we can calculate decay constant

$$\lambda = \frac{ln2}{T_{1/2}}$$

Figure 11.26 shows the relationship between half life and the number of nuclei remaining over time.

- Half-lives vary widely, from 10^{-22} seconds to 10^{+17} years.

Figure 11.26 Exponential decay.

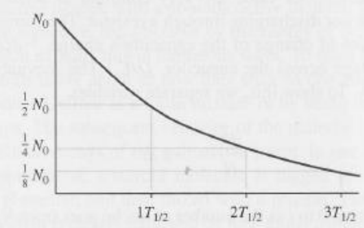


TABLE 11.3 Selected decays

Isotope	Decay Mode	Half-Life
$^{35}_{20}Ca$	β^+	50 ms
$^{3}_{1}H$	β^-	12.3 yr
$^{238}_{92}U$	α	4.5×10^9 yr

Example:
A sample holds 2 μg of tritium.

- Initial decay rate (R)

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} \cdot \frac{sample\ mass}{atomic\ mass\ of\ Tritium}$$

Since Tritium's atomic mass = 3.02 u and 1 u=1.66 x $10^{-27} kg$, $T_{1/2} = 12.3\ yr = 12.3 \text{ x } 3.16 \text{ x } 10^7 sec$

$$\lambda = \frac{\ln 2}{T_{1/2}} = 1.78\ x\ 10^{-9} \quad \text{/sec}$$

$$R = 7.1 \text{ x } 10^8\ decays/sec$$

- Elapsed time before the decay rate falls to 1% of its initial value?

$$N = N_0 e^{-\lambda t}$$

$$\frac{1}{100}N_0 = N_0 e^{-\lambda t}$$

$$t = \frac{-\ln\left(\frac{1}{100}\right)}{\lambda} = 2.6\ x\ 10^9 \text{ sec} = 81.7\ years$$

Carbon-14 dating

- Carbon-14's β^-decay has a half-life of 5730 years.
- Carbon-14 dating only works for formerly living organisms.
- Carbon-14's amount is constantly maintained for living organisms since living organisms exchange Carbon with the environment. Ratio of naturally produced C14/C12=1.3 x 10^{-12}
- When, a living organism dies, it stops the exchange process, thus C-14 in the dead organism decays exponentially.

Example:

What is the age of a fossil sample that contains 6 g of carbon and has a decay rate (R) of 30 decays per minute?

- The sample has 6 g of Carbon. 1.3 x 10^{-12} th of 6g of Carbon should be C-14 at the time when the organism in the sample was alive. This amount of C-14 is subject to the decay. Therefore,
$$N_0(C_{14}) = (1.3x\ 10^{-12}) \cdot \left(\frac{6\ g}{12\ g}\right) \cdot (6.02 \text{ x } 10^{23}) = 3.9 \text{ x } 10^{11}$$
- Decay constant
$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5730\ years\ \cdot 3.16\ x10^7 sec/year} = 3.83 \text{ x } 10^{-12}\ \ /sec$$
- Current decay rate (R)=30 decays/60 seconds=1/2 decays/sec
$$R = N \cdot \lambda = N \cdot 3.83 \text{ x } 10^{-12}\ \ /sec = 0.5\ /sec$$
$$N = 1.31 \text{ x } 10^{11}$$
$$N = N_0 e^{-\lambda t}$$
$$t = -\frac{1}{\lambda}\ln\frac{N}{N_0} = -\frac{1}{3.83 \text{ x } 10^{-12}\ \ /sec}\ln\frac{1.31 \text{ x } 10^{11}}{3.9 \text{ x } 10^{11}} = 2.86 \text{ x } 10^{11}\ sec \sim 9000\ years$$

Nuclear Reactions refer to any occurrences in which nucleons are changed or exchanged between nuclei. Radioactive decay is a spontaneous nuclear reaction. Nuclear reactions can be induced by striking a nucleus with another nucleus.

- Exothermic nuclear reaction:
  - Q > 0 (kinetic energy is released)
  - total mass decreases after reaction, i.e., $m_i > m_f$
  - Example:

    $${}^{10}_{5}B + {}^{1}_{0}n \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He$$

    Released kinetic energy (Q)
    $$= (10.012937 + 1.008665 - 7.016003 - 4.002603)\ uc^2 = 2.79\ MeV$$

- Endothermic nuclear reaction:
  - Q < 0 (kinetic energy is absorbed)
  - total mass increases after reaction. i.e. $m_i < m_f$
  - Example:

    $${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow {}^{7}_{4}Be + {}^{1}_{0}n$$

    Released kinetic energy (Q)
    $$= (7.016003 + 1.007825 - 7.016928 - 1.008665)uc^2 = -1.64\ MeV$$

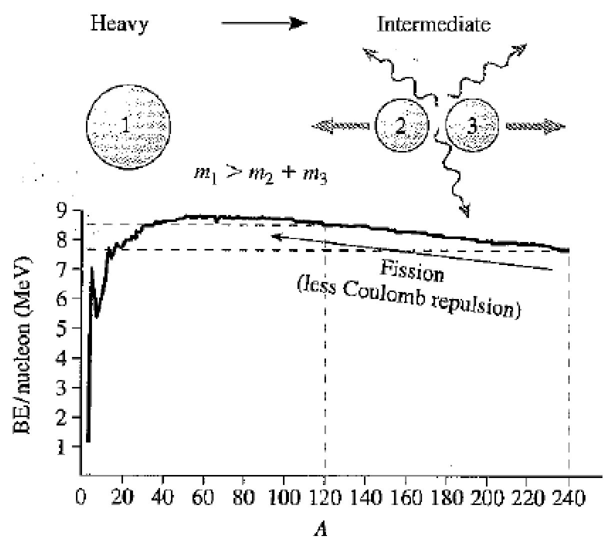
To release energy, mass must decrease after a nuclear reaction, meaning products should be more tightly bound. Considering that the binding energy per nucleon peaks at around A=60, there are two ways to achieve this:

- Nuclear fission: a heavy nucleus breaks into smaller nuclei
- Nuclear fusion: small nuclei fuse together

Nuclear Fission

- A heavy nucleus breaks into two smaller nuclei.
- When this occurs, two or more neutrons are released as soon as small nuclei are formed since large nuclei tend to include more neutrons. Then, subsequent beta decays bring their neutron/proton ratios to stable values.
- The driving force in the binding energy reduction is decrease in Coulomb repulsion.
- Figure 11.27 shows how much energy can be released per nucleon when a fission occurs from A=240 to A=120. The energy difference is about 0.9 MeV. So, this reaction can potentially generate over 200 MeV when all 240 nucleons are considered in the nucleus of A=240.

Figure 11.27 Decreasing BE/nucleon via fission.



- Compare energy release in fission with other energies generated: typical chemical reactions ~ a few eV; spontaneous radioactive decay ~ a few MeV.
- Using the liquid-drop model, the fission process is shown below. A typical excited nucleus can be regarded as an oscillating sphere where surface tension due to strong force and Coulomb repulsions take into play. An excited nucleus can come back to its original sphere shape over time by emitting gamma rays. Sometimes, however, an excited nucleus can be distorted too much (thus Coulomb repulsions win over strong force that provides surface tension) so that the nucleus does not come back to its sphere shape and breaks apart.

FIGURE 24.11 The oscillations of a liquid drop.

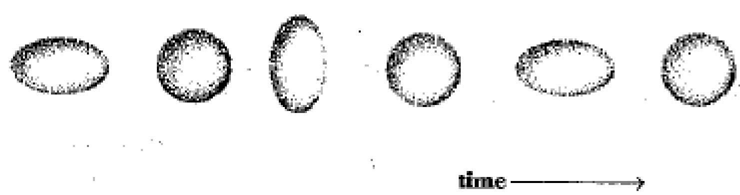
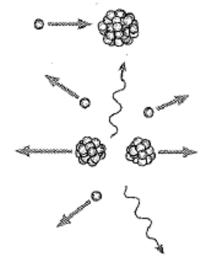


FIGURE 24.12 Nuclear fission according to the liquid-drop model.

- Examples of nuclear fission reactions:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n$$

Figure 11.28 Neutron-induced fission, freeing more neutrons.

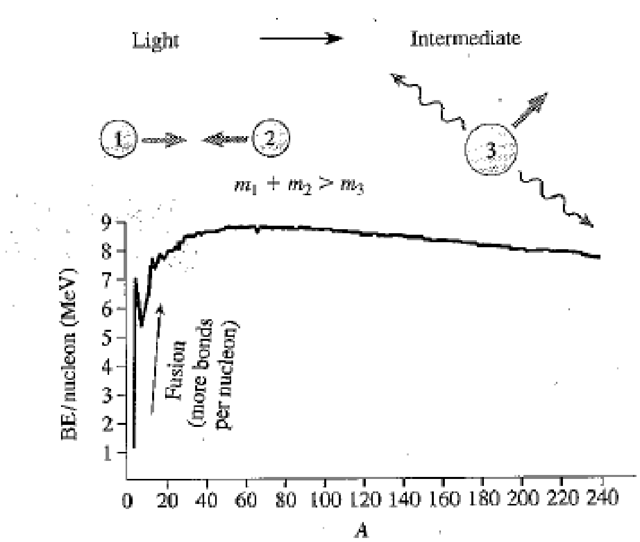


- Since nuclear fission reactions start with a highly energetic neutron striking a large nucleus and highly energetic neutrons are produced as a result of fission reactions, a chain reaction is possible when multiple nucleons are present. See Figure 11.28.

- Consider that a fission reaction generates E_0 and n neutrons. The first generation of fission would create n neutrons each of which can generate nE_0. The jth generation of fission reactions would create
$$E_j = E_0 n^j$$
- In nature, spontaneous chain reactions of fission do not occur because nuclei that participate in the process are not purified or near one another in mass. For a chain reaction to sustain, a critical assembly of a right size, a right geometry, and purification is needed.
- $^{235}_{92}U$ can be engaged in three fission processes. One process generates two neutrons, as compared to three in the other two. Thus the actual energy generated by the nuclear fission of $^{235}_{92}U$ can be written as
$$E_j = E_0 k^j$$
Where $k > 1$ exponential energy increase
$k = 1$ controlled energy release
$k < 1$ exponential energy decrease

Nuclear Fusion

Figure 11.30 Decreasing BE/nucleon via fusion.



- Light nuclei are less tightly bound than those of intermediate mass number. When light nuclei form heavier ones, the total mass decreases and therefore kinetic energy will be released.

  Example:

$$^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He + \gamma$$

$$Q = (2.0141 + 1.0078 - 3.0160)uc^2$$
$$= 5.48\ MeV$$

- A series of fusion occurs in stars.

Proton-Proton Cycle:

Step 1: ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{+1}\beta + \nu$ (Q=0.42 MeV)
Step 2: ${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He$ (Q=5.48 MeV)

Step 3: ${}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H$ (Q=12.9 MeV)

Step 1: ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{+1}\beta + \nu$ (Q=0.42 MeV)
Step 2: ${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He$ (Q=5.48 MeV)

Net Results: Four protons fuse to ${}^{4}_{2}He$ with Q=24.7 MeV

$${}^{1}_{1}H + {}^{1}_{1}H + {}^{1}_{1}H + {}^{1}_{1}H + {}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H + 2\,{}^{0}_{+1}\beta + 2\nu$$

Since there is no bound state between two protons, in Step 1, one proton changes into a neutron and emits a positron and a neutrino, which is relatively a slow process involving weak force.

Carbon Cycle: When the temperature and ${}^{4}_{2}He$ concentration is high enough
Two ${}^{4}_{2}He$ will fuse into ${}^{8}_{4}Be$ which is unstable and would naturally decay back to two ${}^{4}_{2}He$. However, if there is an enough number of fast moving ${}^{4}_{2}He$ is available, then a ${}^{8}_{4}Be$ and a ${}^{4}_{2}He$ will fuse to form a ${}^{12}_{6}C$. Once ${}^{12}_{6}C$ appears, more protons will fuse to ${}^{4}_{2}He$ at the faster pace than the Proton-Proton Cycle. The Carbon Cycle is shown below. At even higher temperatures, elements higher than carbon may form. Z higher than 60 may not form by fusion, but neutron capture and subsequent beta- decay may allow Z higher than 60 to form. Heavy elements in the universe are thought to be formed in supernovae.

Step 1: ${}^{1}_{1}H + {}^{12}_{6}C \rightarrow {}^{13}_{7}N$

Step 2: ${}^{13}_{7}N \rightarrow {}^{13}_{6}C + {}^{0}_{+1}\beta + \nu$

Step 3: ${}^{1}_{1}H + {}^{13}_{6}C \rightarrow {}^{14}_{7}N$

Step 4: ${}^{1}_{1}H + {}^{14}_{7}N \rightarrow {}^{15}_{8}O$

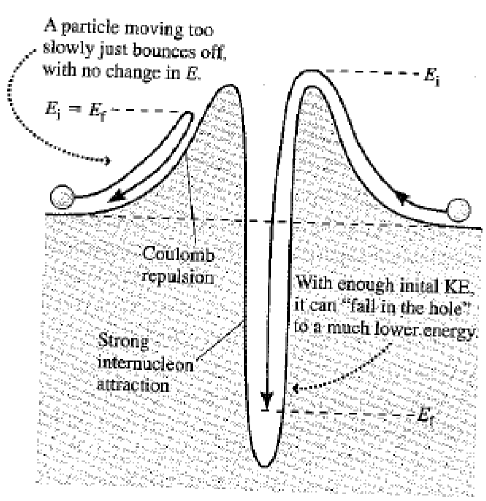
Step 5: ${}^{15}_{8}O \rightarrow {}^{15}_{7}N + {}^{0}_{+1}\beta + \nu$

Step 6: ${}^{1}_{1}H + {}^{15}_{7}N \rightarrow {}^{12}_{6}C + {}^{4}_{2}He$

Net effects: 4 protons fuse into a ${}^{4}_{2}He$.

Fission can occur spontaneously, fusion does not occur spontaneously because protons have charges and thus have Coulomb repulsion. Coulomb repulsion should be overcome. To do this, protons should have high initial kinetic energy to get to the lower, bound energy state.

Figure 11.32 Nuclear fusion: over the Coulomb hurdle, then into the strong force well.



High density and high temperature are necessary to allow more frequent collisions among particles and a greater number of particles have enough energy to surmount the potential barrier.

These conditions can be easily met in stars but not on Earth. Hydrogen bomb uses fusion, but the conditions are set by initially exploding an atomic bomb. Since the energy difference before and after reactions is much greater in fusion than in fission, much higher energy can be released in nuclear fusion reactions.

(Power use) Fission and fusion use much smaller amounts of fuel as compared to fossil fuel and provide energy sources that do not involve carbon emission, but produce radioactive byproducts.

	Fission	Fusion
Fuel	Uranium and Thorium: Not rare and should be mined	Deuterium: Abundant and non-toxic $\boldsymbol{{}^{2}_{1}D + {}^{3}_{1}T \rightarrow {}^{4}_{2}He + {}^{1}_{0}n}$ **Q=17.6 MeV**
Waste	Highly toxic Radioactive with long half-life Disposal is a problem	He isotopes (harmless) and Tritium (radioactive with short half life and not chemically hazardous)
Chain reaction	Chain reactions are possible, thus the fission process should be controlled.	Chain reactions are not possible.