

PH102, 2014W, Lecture Notes: March 4 , Tues, Class 17
Nuclear Physics: Nuclear Structure, Properties, and Models

Liquid Drop Model for Nucleus:

Binding energy= volume term + surface term + Coulomb term + Asymmetry term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

where

$$\begin{aligned} C_1 &= 15.8 \\ C_2 &= 17.8 \\ C_3 &= 0.71 \\ C_4 &= 23.7 \end{aligned}$$

The binding energy formula above is called the semiempirical binding energy formula because the formula is based on theoretical argument with empirical evidence.

Example: Fe-56

$$\begin{aligned} \text{Binding energy} &= 15.8 \times 56 - 17.8 \times 56^{2/3} - 0.71 \frac{26 \times 25}{56^{1/3}} - 23.7 \frac{(30-26)^2}{56} = 496.9 \text{ MeV} \\ \text{Binding energy/nucleon} &= 496.9 \text{ MeV}/56 = 8.87 \text{ MeV} \end{aligned}$$

Figure 11.16 shows the actual data with the semiempirical binding energy formula plot. Fit is good except very light nuclei such as He's nucleus.

Binding Energy

$$\text{Binding energy} = (\text{mass of individual nucleons} - \text{mass of nucleus})c^2$$

Example: Deuteron's binding energy

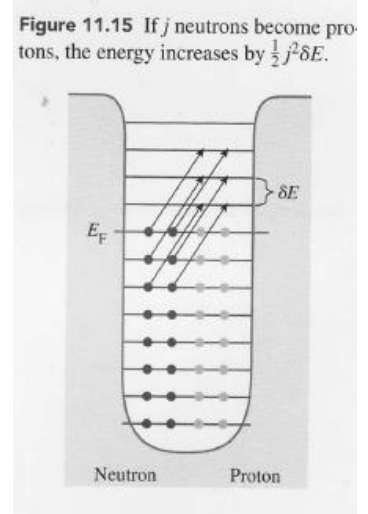
Deuteron = 1 proton + 1 neutron

$$\begin{aligned} \text{Binding Energy} &= (\text{proton mass} + \text{neutron mass} - \text{Deuteron mass})c^2 \\ &= (1.007276 \text{ u} + 1.008665 \text{ u} - 2.013553 \text{ u})c^2 = 0.002388 \text{ uc}^2 \\ &= 0.002388 \times 1.661 \times 10^{-27} \text{ kg} \times (3 \times 10^8 \text{ m/sec})^2 = 2.22 \text{ MeV} \end{aligned}$$

Compare the binding energy of electron in the hydrogen atom (=13.6 eV) and the binding energy of nucleons in the deuterium $\sim 10^{-5}$

$$\text{Binding Energy} = (Zm_H + Nm_n - M_{A_X})c^2$$

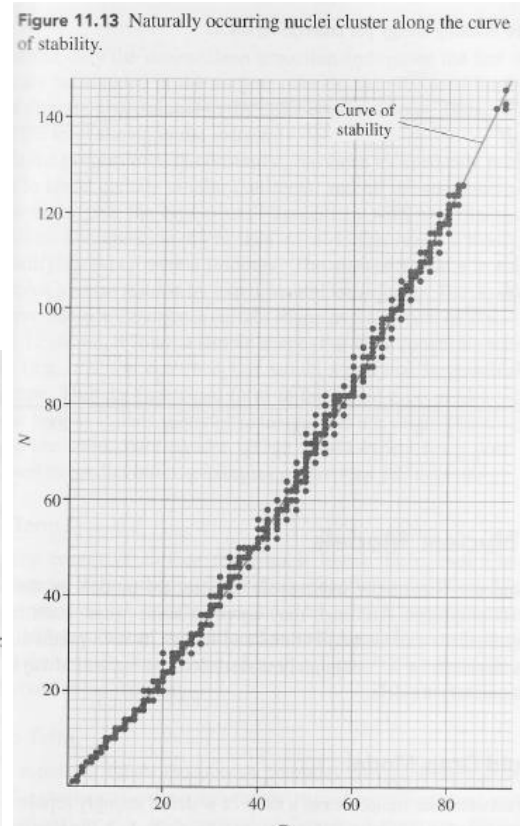
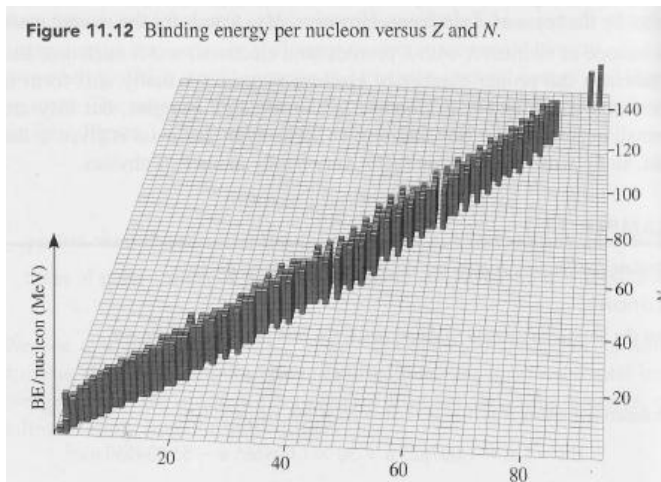
Where m_H = atomic mass of hydrogen



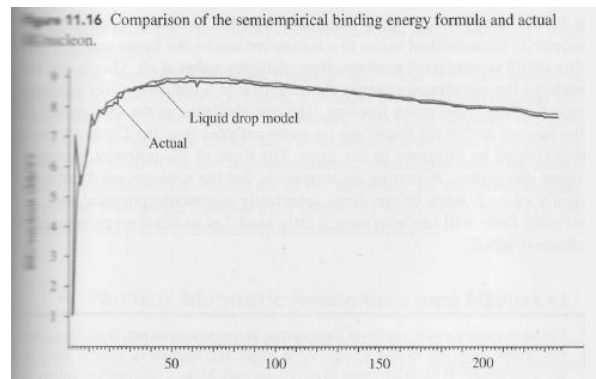
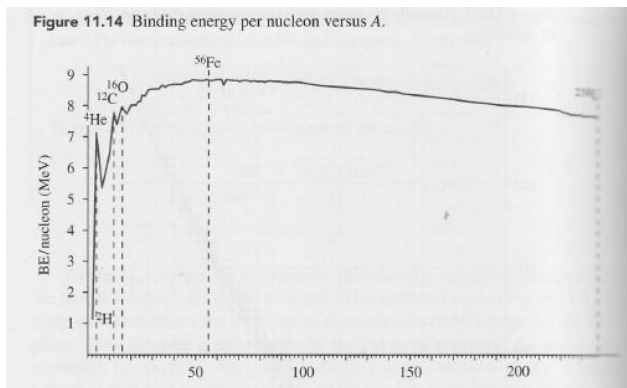
m_n = neutron mass
 $M_{Z^A X}$ = atomic mass of the nucleus

For example: ${}^{56}_{26}\text{Fe} = 55.934939 u$
 Binding energy = $(26 \times 1.007825 u + 30 \times 1.008665 u - 55.934939 u) c^2 = 0.528461 uc^2$
 $= 492.3 \text{ MeV}$ (consider $c^2 = 931.5 \text{ MeV}/u$).

The curve of stability: Binding energy/nucleon for naturally occurring isotopes
 Figure 11.12 and Figure 11.13



Binding energy per nucleon over A: Figure 11.14

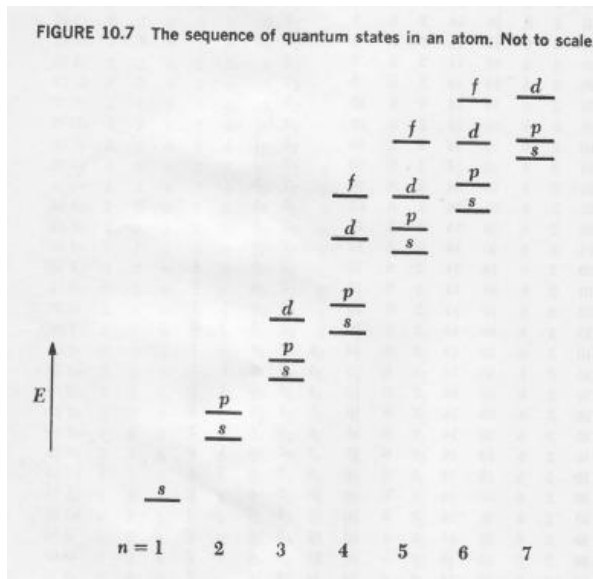


Nuclear Physics: Nuclear Shell Model

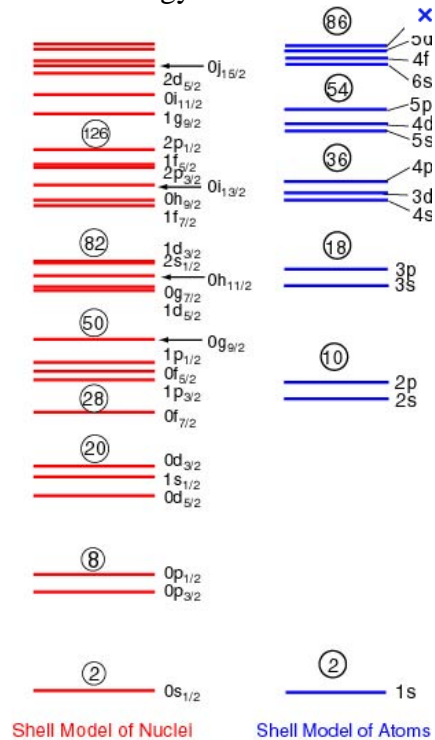
The Liquid Drop model can predict the nuclear binding energy reasonably well. The Liquid Drop model is based on the Strong Force between two adjacent nucleons in a nucleus. However, the Liquid Drop model cannot explain magic numbers: 2, 8, 20, 28, 50, 82, 126 where nuclei with magic numbers of nucleons are particularly stable. The Shell Model explains the magic numbers, the tendency for Z and N to be simply even numbers, and the model can explain angular momenta of nuclei.

	Atomic Shell Model	Nuclear Shell Model
Potential	Electrostatic between nucleus and electrons	Net effect of all the forces nucleons experience in a nucleus
Magic numbers	Atoms with closed electronic shells are stable such as He (2 electrons), Ne (10), Ar (18), Kr (36), Xe (54), Rn (86).	Nuclei are particularly stable when the number of nucleons is 2, 8, 20, 28, 50, 82, and 126.
Exclusion principle	Electrons follow the Exclusion principle	Protons and neutrons separately follow the Exclusion Principle
Movement	Electrons move in orbitals	Nucleons do not move like electrons because most nucleons fill states to a maximum level, preventing them from changing momentum.

Atomic Energy States:



Nuclear Energy States



The nuclear shell model is based on three dimensional harmonic oscillator solutions.

$$\frac{-\hbar^2}{2m} \nabla^2 \psi(x) + \frac{1}{2} \mu \omega^2 x^2 \psi(x) = E \psi(x)$$

$$\psi_{klm}(r, \theta, \phi) = N_{kl} r^l e^{-\nu r^2} L_k^{(l+\frac{1}{2})}(2\nu r^2) Y_{lm}(\theta, \phi)$$

where

$$N_{kl} = \sqrt{\frac{2\nu^3}{\pi} \frac{2^{k+2l+3} k! \nu^l}{(2k+2l+1)!!}}$$

is a normalization constant.

$$\nu \equiv \frac{\mu\omega}{2\hbar}$$

$L_k^{(l+\frac{1}{2})}(2\nu r^2)$ are [generalized Laguerre polynomials](#). The order k of the polynomial is a non-negative integer.

$Y_{lm}(\theta, \phi)$ is a [spherical harmonic function](#).

The energy eigenvalue is

$$E = \hbar\omega \left(2k + l + \frac{3}{2} \right)$$

The energy is usually described by the single [quantum number](#)

$$n \equiv 2k + l$$

- For every even $n, l = 0, 2, \dots, n - 2, n$
- For every odd $n, l = 1, 3, \dots, n - 2, n$
- $-l \leq m \leq l$
- Every n and l , there are $2l + 1$ energy degeneracies, which can accommodate $2(2l + 1)$ nucleons

Using these rules, we can obtain the following table:

n	k	l	No. of nucleons in (n, k, l)	No. of nucleons in n	Total nucleons	Energy
0	0	0	2	2	2	$\frac{3}{2} \hbar\omega$
1	0	1	6	6	8	$\frac{5}{2} \hbar\omega$
2	0	2	10	12	20	$\frac{7}{2} \hbar\omega$
	1	0	2			
3	0	3	14	20	40	$\frac{9}{2} \hbar\omega$
	1	1	6			
4	0	4	18	30	70	$\frac{11}{2} \hbar\omega$
	1	2	10			
	2	0	2			
5	0	5	22	42	112	$\frac{13}{2} \hbar\omega$
	1	3	14			
	2	1	6			

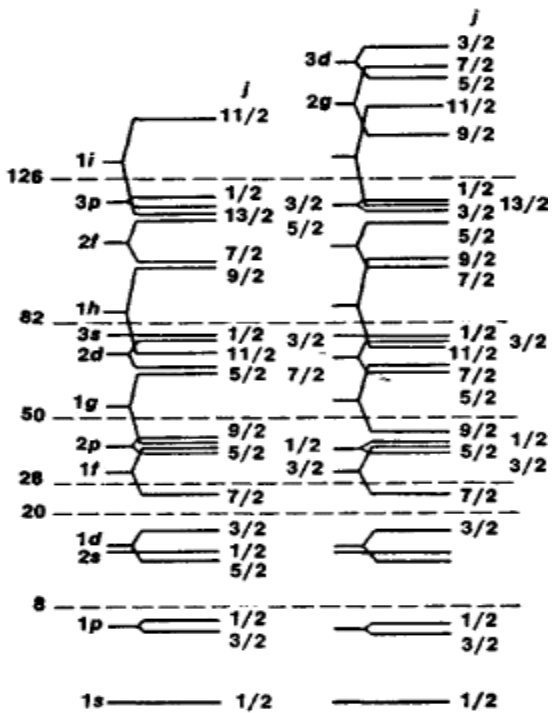
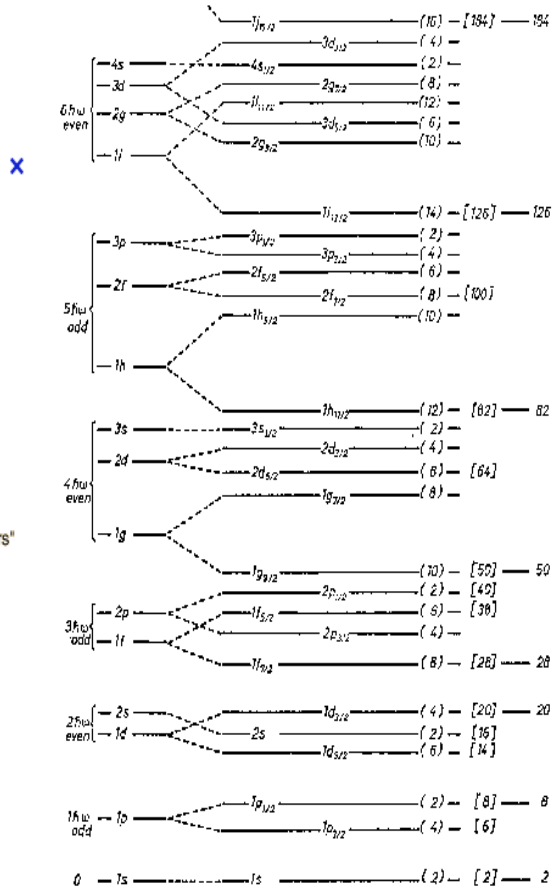
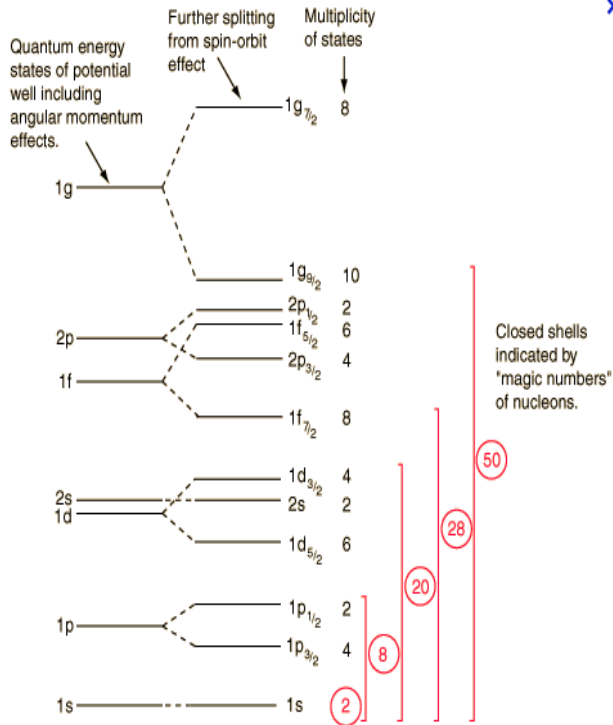
The above table predicts the magic numbers of 2, 8, and 20, but not higher than these numbers. To account for this, we need to consider the L-S coupling, which is similar to the L-S coupling of electrons.

$$\vec{j} = \vec{L} + \vec{S}$$

This means that, instead of l and m_l and spin (as two times of each state), we need to consider j and m_j . This will result in each (n, k, l) energy state to split into two more states.

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in (n, j)	Energy	
0	0	0	1/2	2	2	$\frac{3}{2} \hbar \omega$	
1	0	1	3/2	4	6	$\frac{5}{2} \hbar \omega$	
			1/2	2			
2	0	2	5/2	6	10	$\frac{7}{2} \hbar \omega$	
			3/2	4			
	1	0	1/2	2	2		
3	0	3	7/2	8	14	$\frac{9}{2} \hbar \omega$	
			5/2	6			
	1	1	3/2	4	6		
			1/2	2			
4	0	4	9/2	10	18	$\frac{11}{2} \hbar \omega$	
			7/2	8			
	1	2	5/2	6	10		
				3/2	4		
	2	0	1/2	2	2		
5	0	5	11/2	12	22	$\frac{13}{2} \hbar \omega$	
			9/2	10			
	1	3	7/2	8	14		
				5/2	6		
		2	1	3/2	4	6	
			1/2	2			

Then, each (n, k, l) state splits into two j states, except $j = 0$. When it does, the split becomes greater as l gets greater for the same k . In each split, the higher j state is lower than the lower j state.



Proton energy levels Neutron energy levels