PHY 102 Modern Physics Mid-Term Exam I
Date: January 28, 2014 **Name: ________________________________**

1. (**12 points**) Consider the following four particles.

Particle A with energy E moves freely in a 3 dimensional space. $U(\vec{r}) = 0$ everywhere

Particle B with energy E moves inside a spherical hollow where $U(\vec{r}) = \begin{cases} 0, & |\vec{r}| \le L \\ \infty, & \text{otherwise} \end{cases}$

Particle C with energy E moves inside a cubic box where $U(\vec{r}) = \begin{cases} 0, & 0 \le x, y, z \le L \\ \infty, & \text{otherwise} \end{cases}$

An electron D moving around a proton (hydrogen atom) where $U(\vec{r}) = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$

Hint: Determine whether the Hamiltonian of the particles listed above are translationally and/or rotationally invariant. Based on this information, determine whether linear or angular momentum is conserved.

(1) (1 point) Which of the four particles listed above will have its linear momentum conserved?
Check all that apply. __ A __ B __ C __ D

(2 points) Explain why.

(2) (1 point) Which of the four particles listed above will have its angular momentum conserved?
Check all that apply. __ A __ B __ C __ D

(2 points) Explain why.

(3) (1 point) Which of the following particles' angular motions might be described using spherical harmonics ($Y_l^{m_l}$)?
Check all that apply: __ A __ B __ C __ D

(2 points) Explain why.

(4) (1 point) Which of the following particles' wave function for the linear momentum operator (p) might be the same as their wave function for the Hamiltonian operator?
Check all that apply: __ A __ B __ C __ D

(2 points) Explain why.

2. (**10 points**) Identify different quantum states that are possible for an electron in the 3d orbital. Consider both spin and orbital angular momenta for (2) and (3), but ignore spin-related degeneracies for (1).

(1) (2 points) Write all possible degenerate orbital wave functions associated with the 3d orbital. Use the table 7.3 in the textbook p.257 to construct wave functions. Ignore spin-related degeneracies.

(2) (4 points) When a strong magnetic field is present (Hint: Use (n, l, m_l, m_s))

(3) (4 points) When a weak magnetic field is present (Hint: Use (n, l, j, m_j))

3. (**15 points**) Draw a diagram for each of the three systems described below. Label the diagram for proton, electron, nucleus and their charges and distances among them. Then, write a Schrodinger Equation for each system as completely as possible.

(a) (2 points) Hydrogen atom (H) consisting of one proton (+e) and one electron (-e) in a 3 dimensional space

(b) (3 points) Helium ion (He^+) consisting of He nucleus (+2e) and one electron (-e) in a 3 dimensional space

(c) (3 points) He atom (He) consisting of He nucleus (+2e) and two electrons in a 3 dimensional space

(d) (1 point) Rank these three systems in terms of the amount of energy necessary to remove one electron from the system: (1: largest, 3: smallest)

__ Hydrogen atom
__ Helium ion
__ Helium atom

(3 points) Explain your ranking. In your explanation, use theory as well as actual estimates.

(e) (1 point) Which of the following would be true for the first and the second ionization energies for the Helium atom? (Circle one)

_____ $E_{first\ ionization} > E_{second\ ionization}$
____ $E_{first\ ionization} = E_{second\ ionization}$
____ $E_{first\ ionization} < E_{second\ ionization}$

Note that the first ionization energy refers to the amount of energy needed to free the first electron from a neutral atom. Second ionization energy refers to the amount of energy needed to free the second electron after the first electron.

(2 points) Explain your answer.

$$l = 1, m_l = 0, \qquad \Theta_{10} = \frac{\sqrt{6}}{2}\cos\theta\,, \Phi_0 = \frac{1}{\sqrt{2\pi}}$$

$$l = 1, m_l = \pm 1, \qquad \Theta_{1\pm1} = \frac{\sqrt{3}}{2}\sin\theta\,, \Phi_{\pm1} = \frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$$

4. **(9 points)** Let us consider angular wave functions for $l = 1$. We leave n as unspecified and general so that this problem can be applied to all p orbitals such as $2p$ orbitals, $3p$ orbitals, or $4p$ orbitals, etc. (Note: $\Theta_{l,m_l}\Phi_{m_l}$= $Y_l^{m_l}$, spherical harmonics).

(1) (3 points) Write three angular wave functions for an electron in a hydrogen atom when it is in the following states:

Y_{10}

Y_{11}

Y_{1-1}

(2) (6 points) Prove that these three angular wave functions are orthogonal to each other. That is,

$$\int_0^{\pi}\int_0^{2\pi} Y_{1m_l}^{*} Y_{1m_{l'}}^{*} sin\theta d\theta d\phi = \begin{cases} 1 & if\ m_l = m_{l'} \\ 0 & if\ m_l \neq m_{l'} \end{cases}$$

(Hint: Use $sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$)

5**. True/False Questions (18 points)** Determine whether each of the following statements is true or false. Then, explain your answer.

(a) (3 points) The magnitude of the angular momentum $|\vec{L}|$ of an electron in the Hydrogen atom is always greater than its largest allowed m_l value.

___ True ___ False

Explain your answer.

(b) (3 points) Anti-parallel spin arrangements in the same orbital are more stable than parallel spin arrangements when the orbital shell is less than half full.

___ True ___ False

Explain your answer.

(c) (3 points) The ground state energy level for an electron in the Hydrogen atom (H) is the same as the ground state energy level for an electron in the Helium ion (He^+)

___ True ___ False

Explain your answer.

(d) (3 points) When n (the primary quantum number) equals 2, electrons occupying 2s and 2p orbitals have the same energy. ___ True ___ False

Explain your answer.

(e) (3 points) A wave function for a system of electrons is symmetric under the particle exchange operation. ___ True ___ False

Explain your answer.

(f) (3 points) The most stable electron arrangement for the Helium atom is found when both electrons occupy 1s states with identical spin orientations (both "up" for example).
___ True ___ False

Explain your answer.