PHY 102 Modern Physics Final Exam
Date: March 19, 2014 **Name: ___________________________________**

1. Schrodinger Equation (7 points)
(a) (1 point) Write a one dimensional time-dependent Schrodinger Equation for a free particle with mass m.

(b) (2 points) Show that $\Psi = e^{ikx-i\omega t}$ can be a solution to the time-dependent Schrodinger Equation you wrote in (a) where $k = \sqrt{\frac{2mE}{\hbar^2}}$.

(c) (2 points) What relationship between k and ω exists for $\Psi = e^{ikx-i\omega t}$ to become a solution to the Schrodinger Equation.

(d) (2 points) Prove that $\Psi^*\Psi$ is time independent (that is, conserved).

2. Klein-Gordon Equation (9 points)

(a) (1 point) Write a one dimensional Klein-Gordon equation for a particle with mass *m*.

(b) (2 points) Prove that the following wave functions are solutions to the Klein-Gordon Equation you wrote.

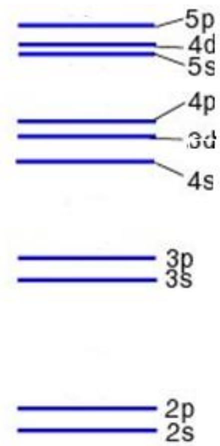
$$\Psi_{-}(x,t) = Ae^{\frac{ipx}{\hbar}+\frac{iEt}{\hbar}}$$
$$\Psi_{+}(x,t) = Ae^{-\frac{ipx}{\hbar}-\frac{iEt}{\hbar}}$$

(c) (3 points) Which wave function, $\Psi_{-}\ or\ \Psi_{+}$, has a positive energy solution (eigenvalue) and which wave function has a negative energy solution?

Explain.

(d) (3 points) Show that the charge density defined as $i\Psi^{*}\frac{\partial}{\partial t}\Psi - i\Psi\frac{\partial}{\partial t}\Psi^{*}$ is not time dependent (that is conserved). Use $\Psi_{-}\ or\ \Psi_{+}$ to create the charge density and examine whether positive or negative charge is associated the charge density you are constructing.

3. Electron Shell Model and Nuclear Shell Model (8 points)

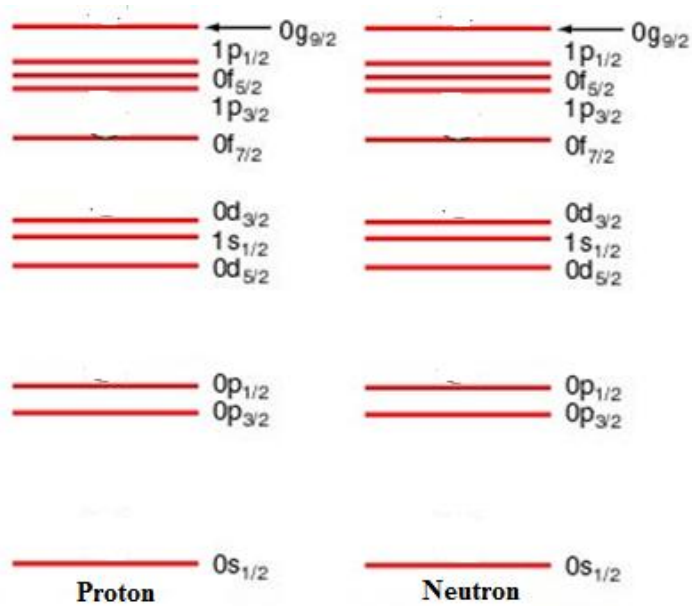


(a) (3 points) The atomic energy diagram for ${}^{40}_{20}Ca$ is shown on the right. Show how electrons in the ground state of the Ca atom fill up the electron energy states. Use ↑↓ for electrons.

Using your energy diagram, is Ca chemically inert or reactive?

Explain your answer.

(b) (2 points) Using energy diagrams on the right show how protons and neutrons in the ground state of the ${}^{40}_{20}Ca$ nucleus fill up the energy states. Use • for a nucleon.



(c) (3 points) Ca (Z=20) has 8 isotopes with A being 35, 40, 41, 42, 43, 44, 46, and 48. Which one of these eight isotopes is mostly likely to be found in nature?

Explain why this is the case. Base your reasoning on your energy diagrams.

4. Nuclear Binding Energy, Decay, and Reactions (14 points)

(a) (1 point) Find the binding energy per nucleon for ${}^{35}_{20}Ca$ in MeV.
Note that m_H= 1.007825u; m_n=1.008665u; $m_{{}^{35}_{20}Ca}$ =34.99523u; uc^2 =931.5 MeV.

(b) (1 point) ${}^{35}_{20}Ca$ is naturally unstable and goes through decays. Write the most likely decay process involving ${}^{35}_{20}Ca$.

(c) (2 points) Using the liquid drop model semiempirical formula, discuss why the decay process you chose most likely to occur. Include as many relevant terms as possible.

(d) (6 points) The isotope ${}^{11}_{6}C$ can decay into ${}^{11}_{5}B$ in both β^+ and electron capture processes. Express each decay process and calculate released energy. Note that mass of ${}^{11}_{6}C$ is 11.011430u and that of ${}^{11}_{5}B$ is 11.009305u and electron mass is 0.0005486u.

β^+ decay process:

Released energy during the β^+ decay process =

Electron capture process:

Released energy during the β^+ decay process =

Which process would ${}^{11}_{6}C$ more frequently go through? ___ electron capture ___ β^+ decay

Explain your answer.

(e) (2 points) Explain why nuclear fusion cannot happen in everyday settings. Draw a diagram to support your explanation.

(f) (2 points) Write a nuclear reaction process related to fusion of three alpha particles to form a ${}^{12}_{6}C$ nucleus.

How much energy does each such reaction produce?

5. Particle decays and conservation rules (2 points; 8 points each)
Indicate whether or not the proposed decay is possible and explain why. Consider mass, spin angular momentum, strangeness, charge, Baryon number and Lepton number before and after the decay. If the decay process is possible, draw a Feynman Diagram for the process.

(a) $\Xi^- \rightarrow \Lambda^0 + \pi^-$ __ possible __ impossible

(b) $\Lambda^0 \rightarrow n + \pi^-$ __ possible __ impossible

(c) $\Xi^{*-} \rightarrow \Sigma^0 + e^- + \nu_e$ __ possible __ impossible

(d) $\bar{n} \rightarrow p + e^- + \overline{\nu_e}$ __ possible __ impossible

6. Uncertainty principle vs. Exclusion principle (18 points)
We learned uncertainty and exclusion principles and their applications throughout this course. Determine which principle is applied to each statement and explain how.

(a) (3 points) L^2 and L_z related to the angular momentum of an electron in a hydrogen atom are quantized. __ Uncertainty __ Exclusion
Explain.

(b) (3 points) Hund's rule: If two or more orbitals of equal energy are available, electrons will occupy them singly before filling them in pairs. __ Uncertainty __ Exclusion
Explain.

(c) (3 points) At T=0, a total energy of fermion gas will be higher than that of boson gas.
Explain. __ Uncertainty __ Exclusion

(d) (3 points) Pions (π^+, π^-, π^0) are mediating bosons (field quanta) for the residual strong force between two nucleons. Pions should have mass to mediate such force.
Explain. __ Uncertainty __ Exclusion

(e) (3 points) There are three ways two nucleons can combine: proton-proton, proton-neutron, and neutron-neutron. In nature, only proton-neutron arrangement produces a stable nucleus.
Explain. __ Uncertainty __ Exclusion

(f) (3 points) A wave function that represents a system of fermions should be antisymmetric under exchange operation.
Explain. __ Uncertainty __ Exclusion

Optional

7. Slater Determinant (5 points)

To describe a many-electron state, an elementary way is to use the Slater Determinant. Consider putting two electrons in 1s shell in He. The Slater determinant wave function is given by

$$\psi(x_1, x_2, u_1, u_2) = \frac{1}{\sqrt{2}} \begin{vmatrix} \psi_{1s}(x_1) \uparrow_1 & \psi_{1s}(x_1) \downarrow_1 \\ \psi_{1s}(x_2) \uparrow_2 & \psi_{1s}(x_2) \downarrow_2 \end{vmatrix}$$

where

- $\uparrow_1$ means the spin wave function where the "spin coordinate" $u_1 \equiv m_{s,1} = \frac{1}{2}$
- $\downarrow_1$ means the spin wave function $u_1 \equiv m_{s,1} = -\frac{1}{2}$.
- x_1, u_1 are the spatial and spin coordinates of particle 1 and x_2, u_2 are those of particle 2.
- The spin coordinate for an electron is binary by nature: $u_1 = \frac{1}{2}$ or $-\frac{1}{2}$ and $u_2 = \frac{1}{2}$ or $-\frac{1}{2}$.
- Note that the determinant of a 2 x 2 matrix is $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

(a) (2 points) Determine whether $\psi(x_1, x_2, u_1, u_2)$ is symmetric or anti-symmetric under the spin and spatial state exchange.

(b) (3 points) If both electrons are in the 1s shell, then which of the following spin arrangements are possible (choose one)

___ both spins up
___ one spin up and the other down
___ both spins down

Explain your answer using the Slater Determinant.