Announcements

- Mid-term exam: 01/29, Tues, in class
  - Chapters 7 and 8
  - Open book and open lecture notes but no personal notes or electronics
  - 6 questions
    - 1 question from each homework set
    - 1 true/false question (multiple sub questions)
    - Learning experience while taking exams
  - More conceptual, simple calculations if any.
  - Review sessions voting

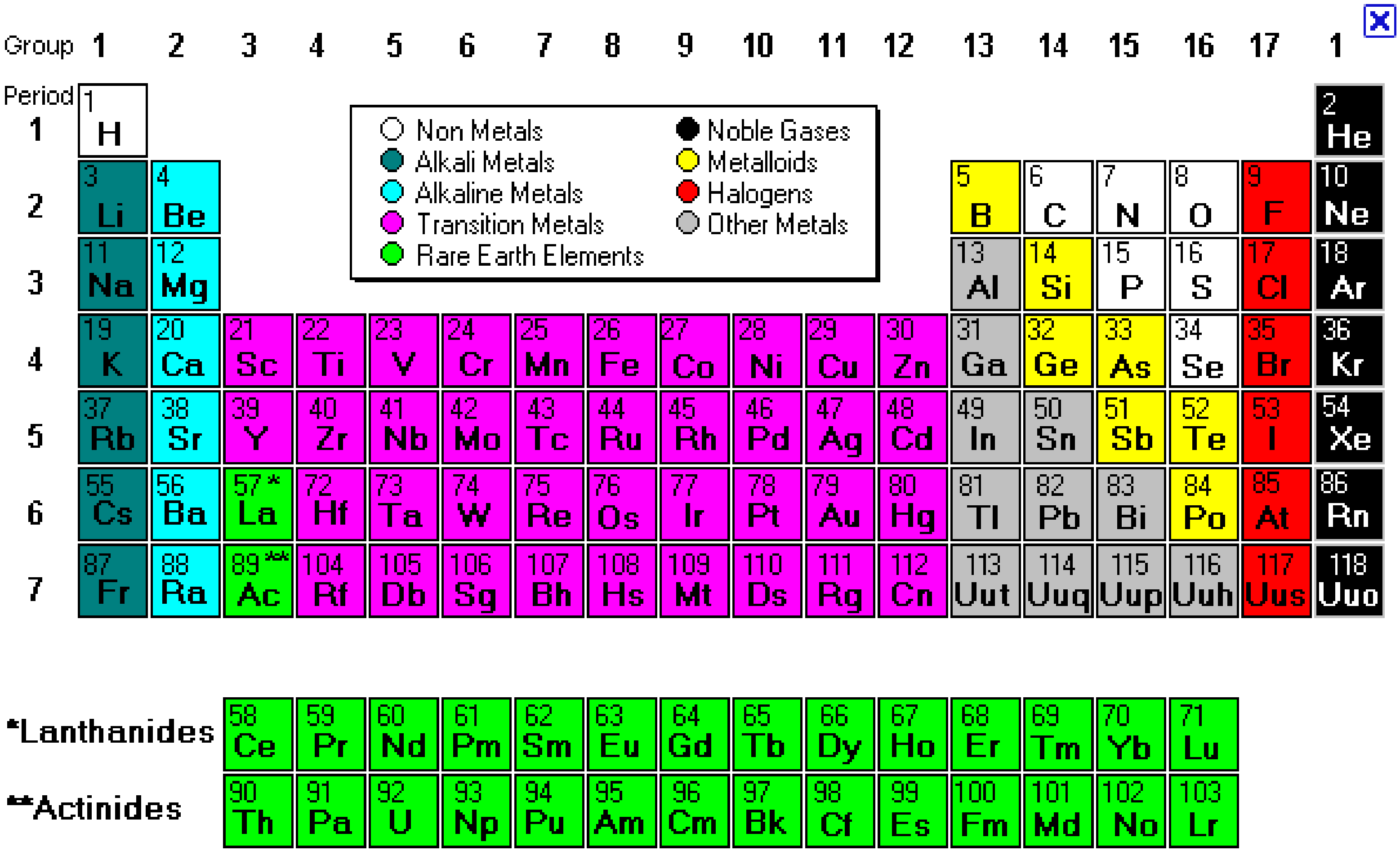
  Fri, 3-5 pm, 4-6 pm, 5-7 pm, 6-8 pm

  Mon, 10-12pm, 11-1 pm, 12-2 pm, 5-7pm, 6-8pm, 7-9pm

Lecture 6 Topics:

- Periodic table trends
- Paschen-Back Effect
- Zeeman Effect
- LS coupling
- Energy split under magnetic field
- Spectral line split

Periodic Table



Periodic Table: Two basic rules

- A system of particles is stable when its total energy is a minimum
- Only one electron can exist in any particular quantum state in an atom (exclusion principle).
- Extending from energy levels conceived from the hydrogen atom solutions

$$\mathrm{H}\psi_{n,l,m_l} = E_n\psi_{n,l,m_l}$$

$$\mathrm{L}^2\psi_{n,l,m_l} = l(l+1)\hbar^2\psi_{n,l,m_l}$$

$$\mathrm{L_z}\psi_{n,l,m_l} = m_l\hbar\psi_{n,l,m_l}$$

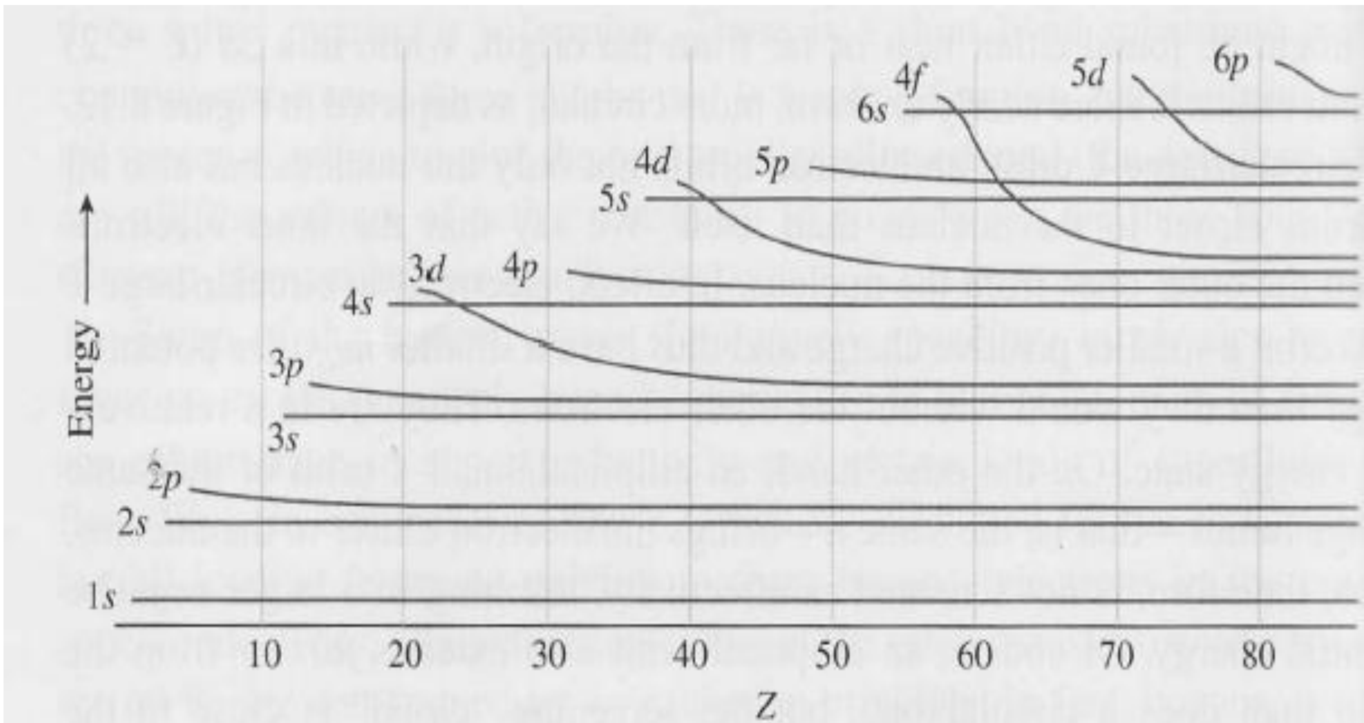
- Lower Energy can be obtained
  - When n is lower.
  - With a given , when *l* is lower.
  - With a given *l*, parallel spin arrangements lower energy.

What we know so far:

- Number of electrons in each atom
- Electrons should be in one of the orbitals determined by *n*, *l*, and m_l
  - n limits what types of l orbitals an electron can occupy.
  - Each l orbital has 2l+1 possible m_l states:
    - *s* orbital (l=0) = 1
    - *p* orbital (l=1) = 2 x 1 + 1 = 3
    - *d* orbital (l=2) = 2 x 2 + 1 = 5
    - *f* orbital (l=3) = 2 x 3 + 1 = 7
- Due to electron's spin where ½ and – ½ are possible, each m_l orbital can have two additional possible states.
- As a result, in each n, there are $2n^2$ possible energy states.

TABLE 8.2 Subshell ordering and capacity

Subshell $n\ell$	1s	2s	2p	3s	3p	4s	3d	4p	5s	4d	5p	6s	4f	5d	6p	7s	5f	6d
$n + \ell$	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8
Number of electrons $2(2\ell + 1)$	2	2	6	2	6	2	10	6	2	10	6	2	14	10	6	2	14	10



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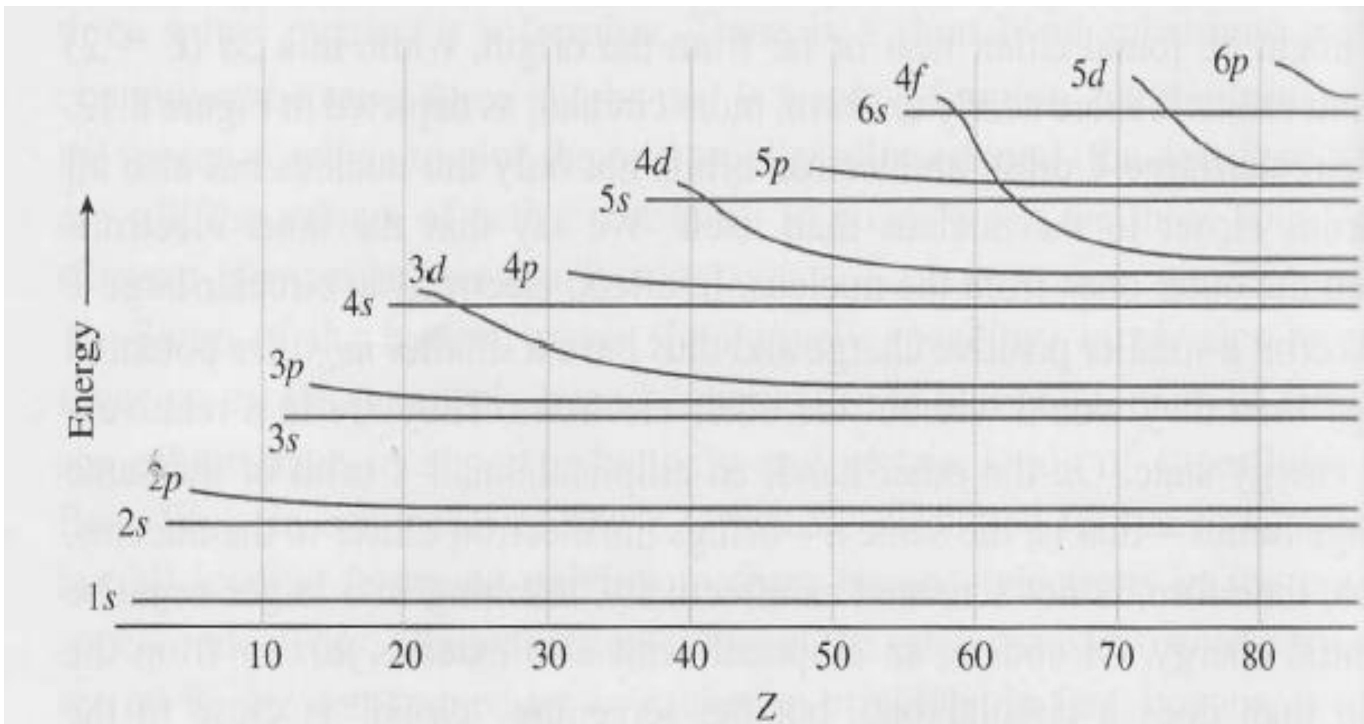
Subshell $n\ell$	1s	2s	2p	3s	3p	4s	3d	4p	5s	4d	5p	6s	4f	5d	6p	7s	5f	6d
$n+\ell$	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8
Number of electrons $2(2\ell+1)$	2	2	6	2	6	2	10	6	2	10	6	2	14	10	6	2	14	10

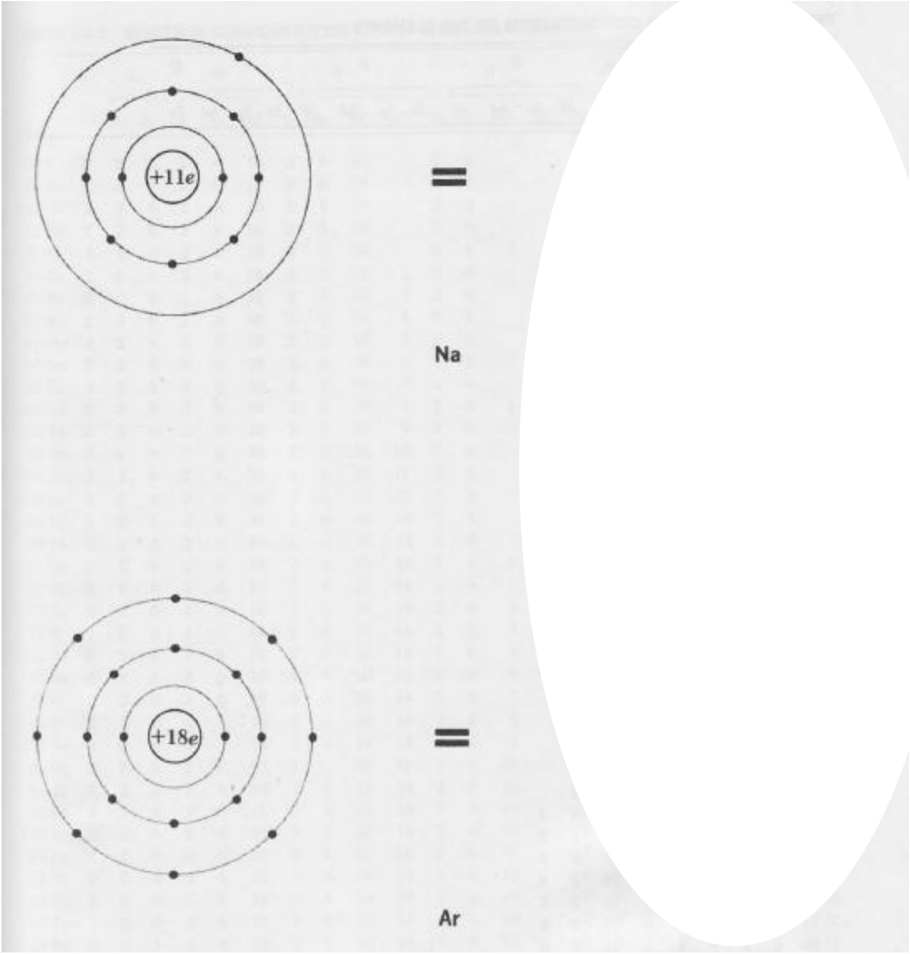
	1s	2s	2p	3s	3p	3d	4s	4p	4d	4f	5s	5p	5d	5f	6s	6p	6d	7s
1 H	1																	
2 He	2																	
3 Li	2	1																
4 Be	2	2																
5 B	2	2	1															
6 C	2	2	2															
7 N	2	2	3															
8 O	2	2	4															
9 F	2	2	5															
10 Ne	2	2	6															
11 Na	2	2	6	1														
12 Mg	2	2	6	2														
13 Al	2	2	6	2	1													
14 Si	2	2	6	2	2													
15 P	2	2	6	2	3													
16 S	2	2	6	2	4													
17 Cl	2	2	6	2	5													
18 A	2	2	6	2	6													
19 K	2	2	6	2	6		1											
20 Ca	2	2	6	2	6		2											
21 Sc	2	2	6	2	6	1	2											
22 Ti	2	2	6	2	6	2	2											
23 V	2	2	6	2	6	3	2											
24 Cr	2	2	6	2	6	5	1											
25 Mn	2	2	6	2	6	5	2											
26 Fe	2	2	6	2	6	6	2											
27 Co	2	2	6	2	6	7	2											
28 Ni	2	2	6	2	6	8	2											
29 Cu	2	2	6	2	6	10	1											
30 Zn	2	2	6	2	6	10	2											
31 Ga	2	2	6	2	6	10	2	1										
32 Ge	2	2	6	2	6	10	2	2										
33 As	2	2	6	2	6	10	2	3										
34 Se	2	2	6	2	6	10	2	4										
35 Br	2	2	6	2	6	10	2	5										
36 Kr	2	2	6	2	6	10	2	6										
37 Rb	2	2	6	2	6	10	2	6			1							
38 Sr	2	2	6	2	6	10	2	6			2							
39 Y	2	2	6	2	6	10	2	6	1		2							
40 Zr	2	2	6	2	6	10	2	6	2		2							
41 Nb	2	2	6	2	6	10	2	6	4		1							
42 Mo	2	2	6	2	6	10	2	6	5		1							
43 Tc	2	2	6	2	6	10	2	6	5		2							
44 Ru	2	2	6	2	6	10	2	6	7		1							
45 Rh	2	2	6	2	6	10	2	6	8		1							
46 Pd	2	2	6	2	6	10	2	6	10		1							
47 Ag	2	2	6	2	6	10	2	6	10		1							
48 Cd	2	2	6	2	6	10	2	6	10		2							
49 In	2	2	6	2	6	10	2	6	10		2	1						
50 Sn	2	2	6	2	6	10	2	6	10		2	2						
51 Sb	2	2	6	2	6	10	2	6	10		2	3						
52 Te	2	2	6	2	6	10	2	6	10		2	4						
53 I	2	2	6	2	6	10	2	6	10		2	5						
54 Xe	2	2	6	2	6	10	2	6	10		2	6						
55 Cs	2	2	6	2	6	10	2	6	10		2	6			1			
56 Ba	2	2	6	2	6	10	2	6	10		2	6			2			
57 La	2	2	6	2	6	10	2	6	10		2	6	1		2			
58 Ce	2	2	6	2	6	10	2	6	10	2	2	6			2			
59 Pr	2	2	6	2	6	10	2	6	10	3	2	6			2			
60 Nd	2	2	6	2	6	10	2	6	10	4	2	6			2			
61 Pm	2	2	6	2	6	10	2	6	10	5	2	6			2			
62 Sm	2	2	6	2	6	10	2	6	10	6	2	6			2			
63 Eu	2	2	6	2	6	10	2	6	10	7	2	6			2			
64 Gd	2	2	6	2	6	10	2	6	10	7	2	6	1		2			
65 Tb	2	2	6	2	6	10	2	6	10	9	2	6			2			
66 Dy	2	2	6	2	6	10	2	6	10	10	2	6			2			
67 Ho	2	2	6	2	6	10	2	6	10	11	2	6			2			
68 Er	2	2	6	2	6	10	2	6	10	12	2	6			2			
69 Tm	2	2	6	2	6	10	2	6	10	13	2	6			2			
70 Yb	2	2	6	2	6	10	2	6	10	14	2	6			2			
71 Lu	2	2	6	2	6	10	2	6	10	14	2	6	1		2			
72 Hf	2	2	6	2	6	10	2	6	10	14	2	6	2		2			
73 Ta	2	2	6	2	6	10	2	6	10	14	2	6	3		2			
74 W	2	2	6	2	6	10	2	6	10	14	2	6	4		2			
75 Re	2	2	6	2	6	10	2	6	10	14	2	6	5		2			
76 Os	2	2	6	2	6	10	2	6	10	14	2	6	6		2			
77 Ir	2	2	6	2	6	10	2	6	10	14	2	6	7		2			
78 Pt	2	2	6	2	6	10	2	6	10	14	2	6	9		1			
79 Au	2	2	6	2	6	10	2	6	10	14	2	6	10		1			
80 Hg	2	2	6	2	6	10	2	6	10	14	2	6	10		2			
81 Tl	2	2	6	2	6	10	2	6	10	14	2	6	10		2	1		
82 Pb	2	2	6	2	6	10	2	6	10	14	2	6	10		2	2		
83 Bi	2	2	6	2	6	10	2	6	10	14	2	6	10		2	3		
84 Po	2	2	6	2	6	10	2	6	10	14	2	6	10		2	4		
85 At	2	2	6	2	6	10	2	6	10	14	2	6	10		2	5		
86 Rn	2	2	6	2	6	10	2	6	10	14	2	6	10		2	6		
87 Fr	2	2	6	2	6	10	2	6	10	14	2	6	10		2	6		1
88 Ra	2	2	6	2	6	10	2	6	10	14	2	6	10		2	6		2
89 Ac	2	2	6	2	6	10	2	6	10	14	2	6	10		2	6	1	2
90 Th	2	2	6	2	6	10	2	6	10	14	2	6	10		2	6	2	2
91 Pa	2	2	6	2	6	10	2	6	10	14	2	6	10	2	2	6	1	2
92 U	2	2	6	2	6	10	2	6	10	14	2	6	10	3	2	6	1	2
93 Np	2	2	6	2	6	10	2	6	10	14	2	6	10	4	2	6	1	2
94 Pu	2	2	6	2	6	10	2	6	10	14	2	6	10	5	2	6	1	2
95 Am	2	2	6	2	6	10	2	6	10	14	2	6	10	6	2	6	1	2
96 Cm	2	2	6	2	6	10	2	6	10	14	2	6	10	7	2	6	1	2
97 Bk	2	2	6	2	6	10	2	6	10	14	2	6	10	8	2	6	1	2
98 Cf	2	2	6	2	6	10	2	6	10	14	2	6	10	10	2	6		2
99 E	2	2	6	2	6	10	2	6	10	14	2	6	10	11	2	6		2
100 Fm	2	2	6	2	6	10	2	6	10	14	2	6	10	12	2	6		2
101 Md	2	2	6	2	6	10	2	6	10	14	2	6	10	13	2	6		2
102 No	2	2	6	2	6	10	2	6	10	14	2	6	10	14	2	6		2
103 Lw	2	2	6	2	6	10	2	6	10	14	2	6	10	14	2	6	1	2

TABLE 8.2 Subshell ordering and capacity

Subshell $n\ell$	1*s*	2*s*	2*p*	3*s*	3*p*	4*s*	3*d*	4*p*	5*s*	4*d*	5*p*	6*s*	4*f*	5*d*	6*p*	7*s*	5*f*	6*d*
$n + \ell$	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8
Number of electrons $2(2\ell + 1)$	2	2	6	2	6	2	10	6	2	10	6	2	14	10	6	2	14	10



Effective charge of the nucleus



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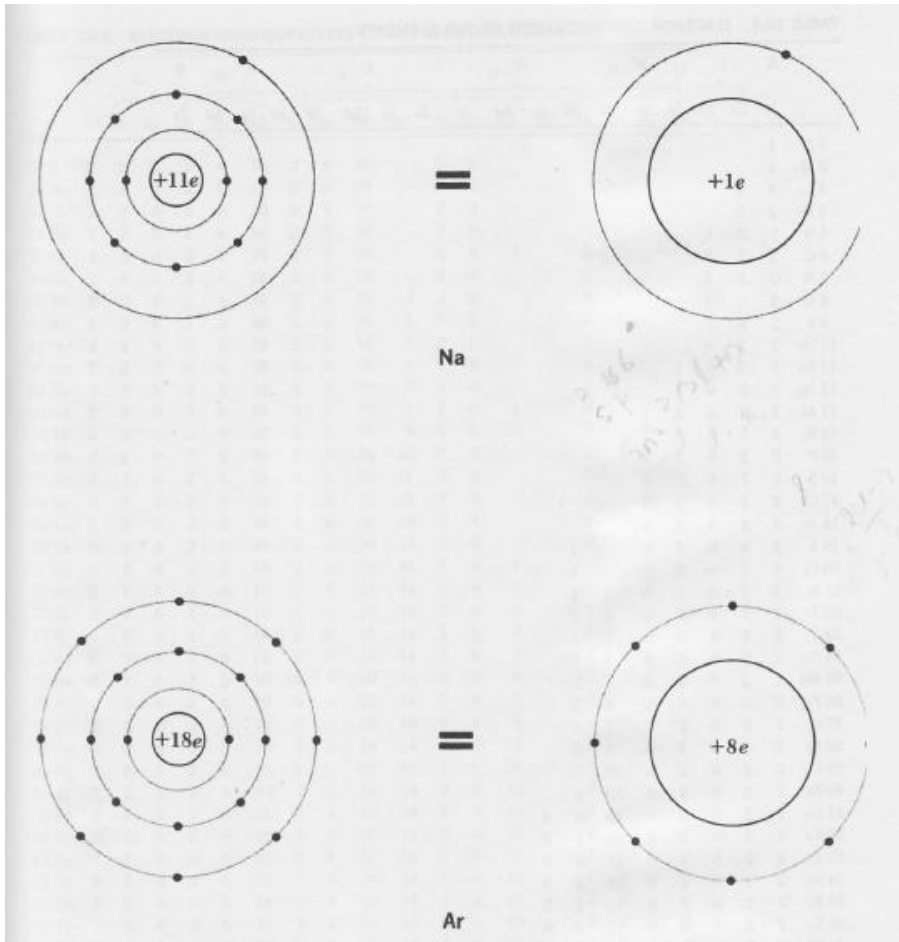
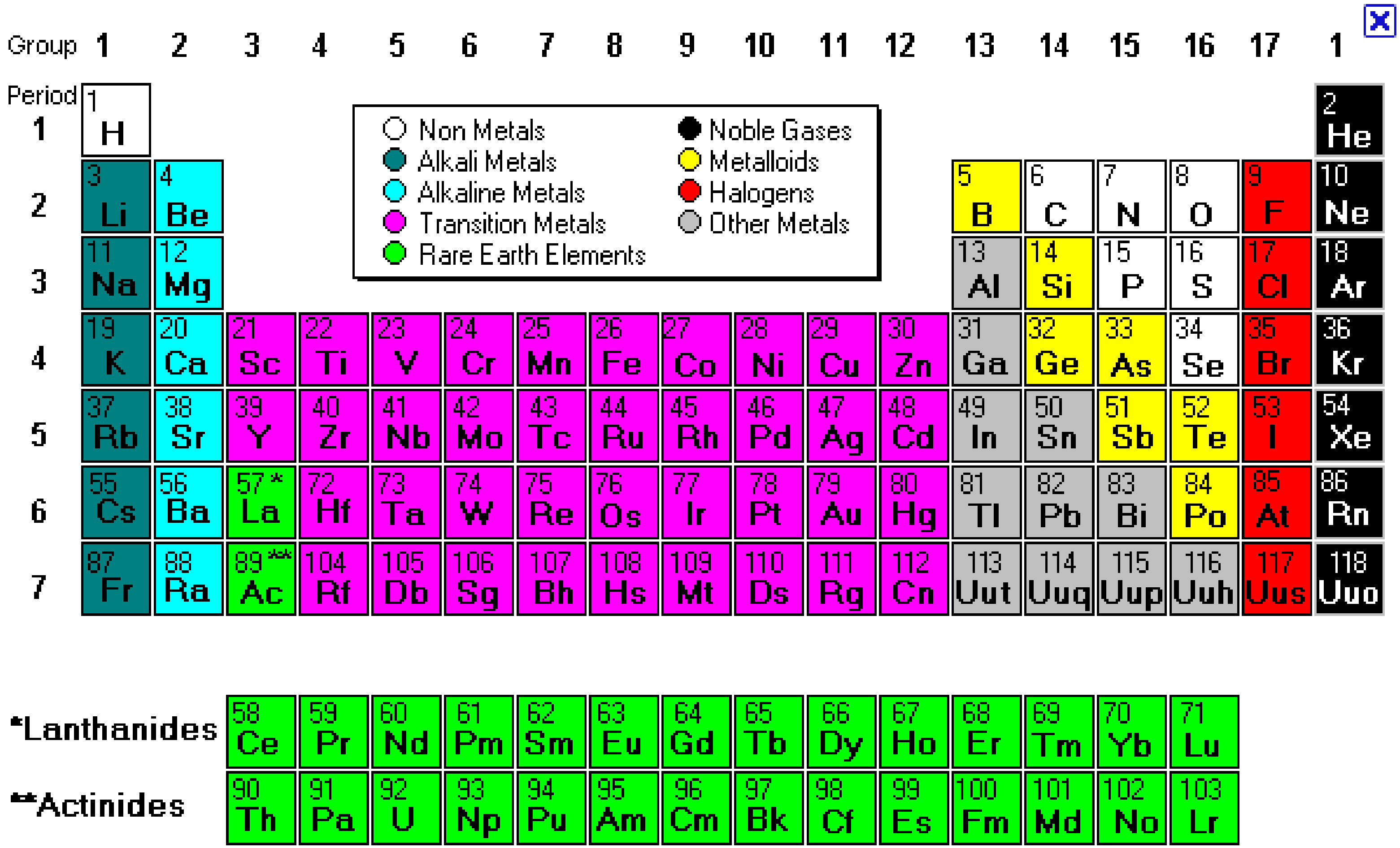


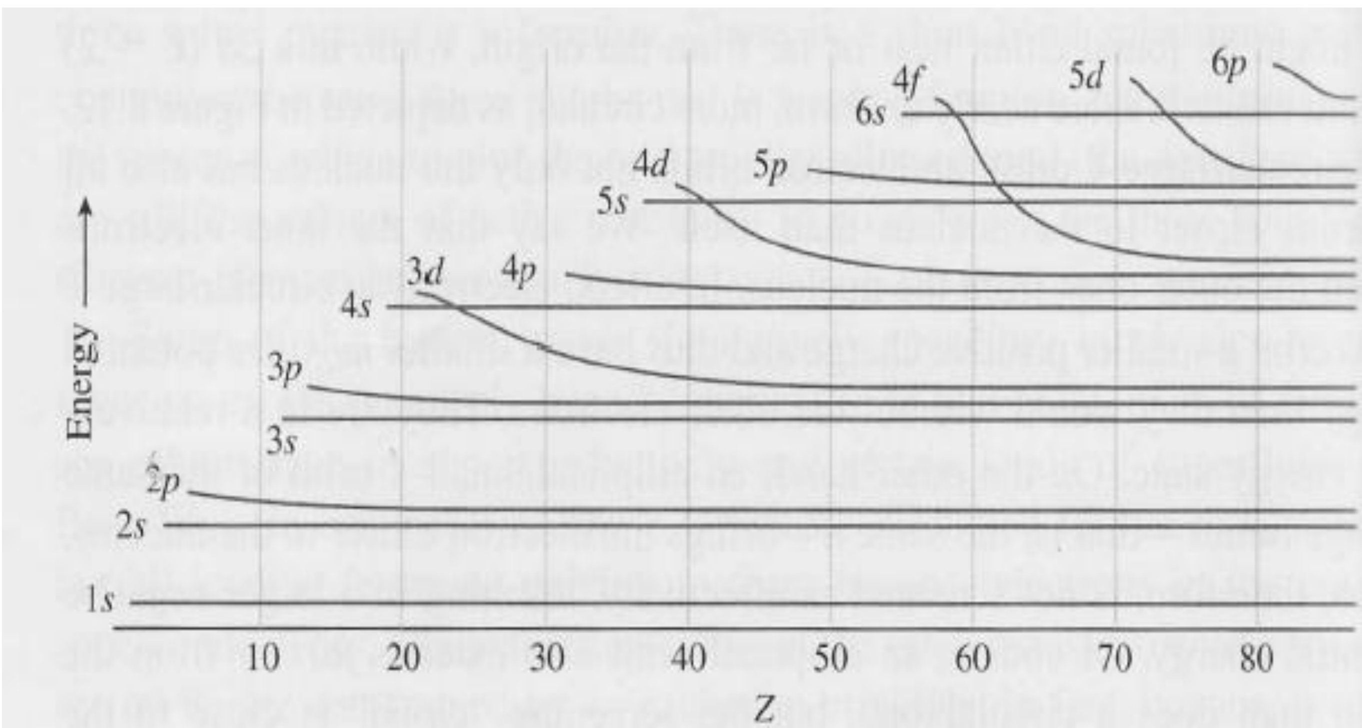
Noble Gases



Group 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 1
Period
Non Metals
Alkali Metals
Alkaline Metals
Transition Metals
Rare Earth Elements
Noble Gases
Metalloids
Halogens
Other Metals
1 1 H 2 He
2 3 Li 4 Be 5 B 6 C 7 N 8 O 9 F 10 Ne
3 11 Na 12 Mg 13 Al 14 Si 15 P 16 S 17 Cl 18 Ar
4 19 K 20 Ca 21 Sc 22 Ti 23 V 24 Cr 25 Mn 26 Fe 27 Co 28 Ni 29 Cu 30 Zn 31 Ga 32 Ge 33 As 34 Se 35 Br 36 Kr
5 37 Rb 38 Sr 39 Y 40 Zr 41 Nb 42 Mo 43 Tc 44 Ru 45 Rh 46 Pd 47 Ag 48 Cd 49 In 50 Sn 51 Sb 52 Te 53 I 54 Xe
6 55 Cs 56 Ba 57 * La 72 Hf 73 Ta 74 W 75 Re 76 Os 77 Ir 78 Pt 79 Au 80 Hg 81 Tl 82 Pb 83 Bi 84 Po 85 At 86 Rn
7 87 Fr 88 Ra 89 ** Ac 104 Rf 105 Db 106 Sg 107 Bh 108 Hs 109 Mt 110 Ds 111 Rg 112 Cn 113 Uut 114 Uuq 115 Uup 116 Uuh 117 Uus 118 Uuo
*Lanthanides 58 Ce 59 Pr 60 Nd 61 Pm 62 Sm 63 Eu 64 Gd 65 Tb 66 Dy 67 Ho 68 Er 69 Tm 70 Yb 71 Lu
**Actinides 90 Th 91 Pa 92 U 93 Np 94 Pu 95 Am 96 Cm 97 Bk 98 Cf 99 Es 100 Fm 101 Md 102 No 103 Lr

Noble Gases

He (2) = $1s^2$
Ne (10) = $1s^2\ 2s^2\ 2p^6$
Ar (18) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$
Kr (36) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6$
Xe (54) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 5s^2\ 5p^6$
Rn (86) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 4f^{14}\ 5s^2\ 5p^6\ 5d^{10}\ 6s^2\ 6p^6$



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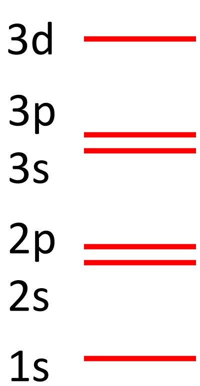

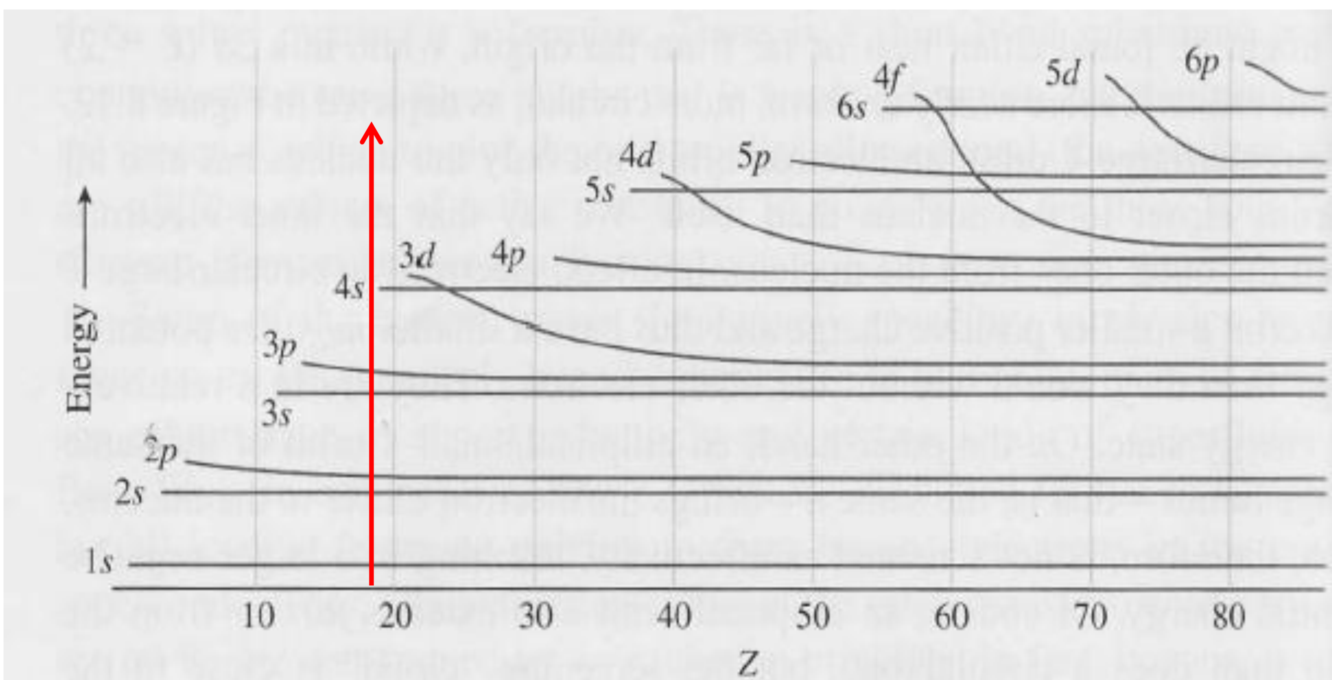

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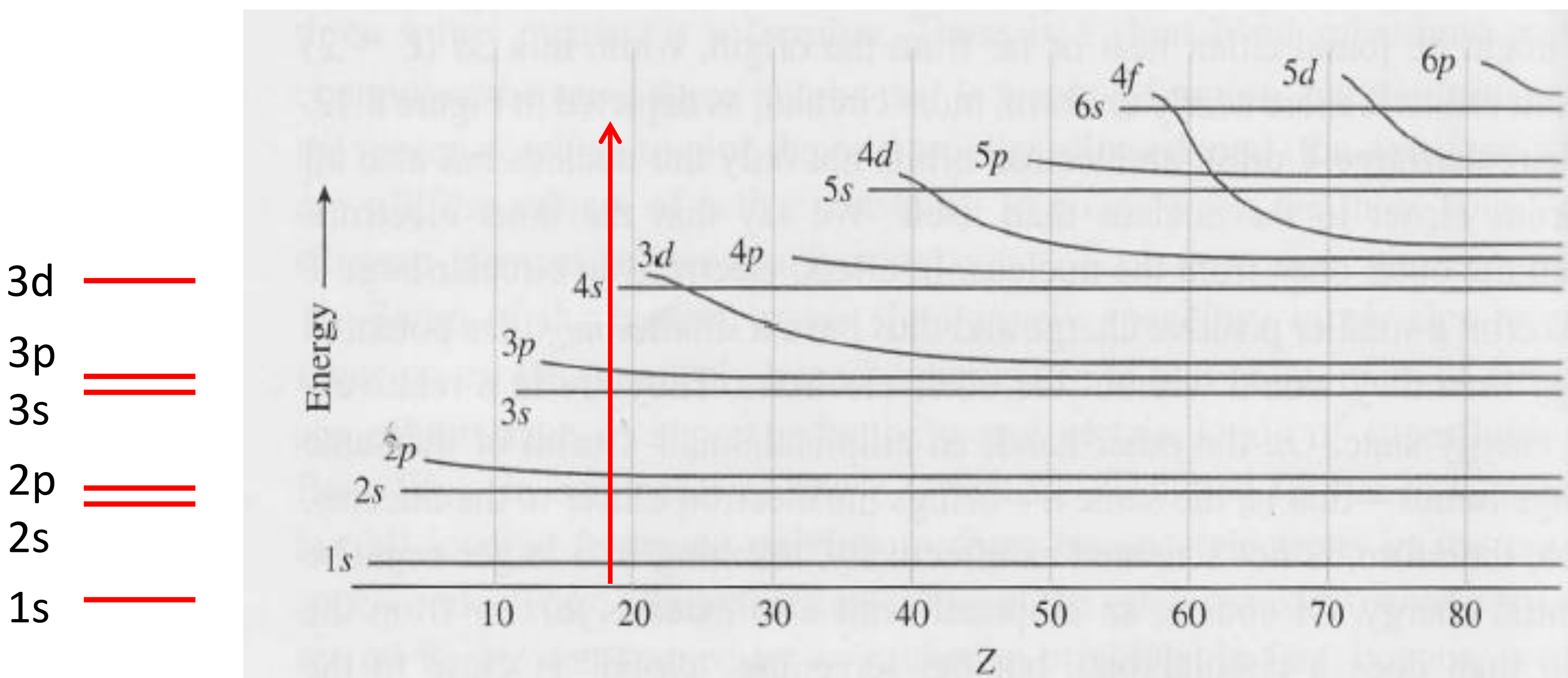

Noble Gases

He (2) = $1s_2$

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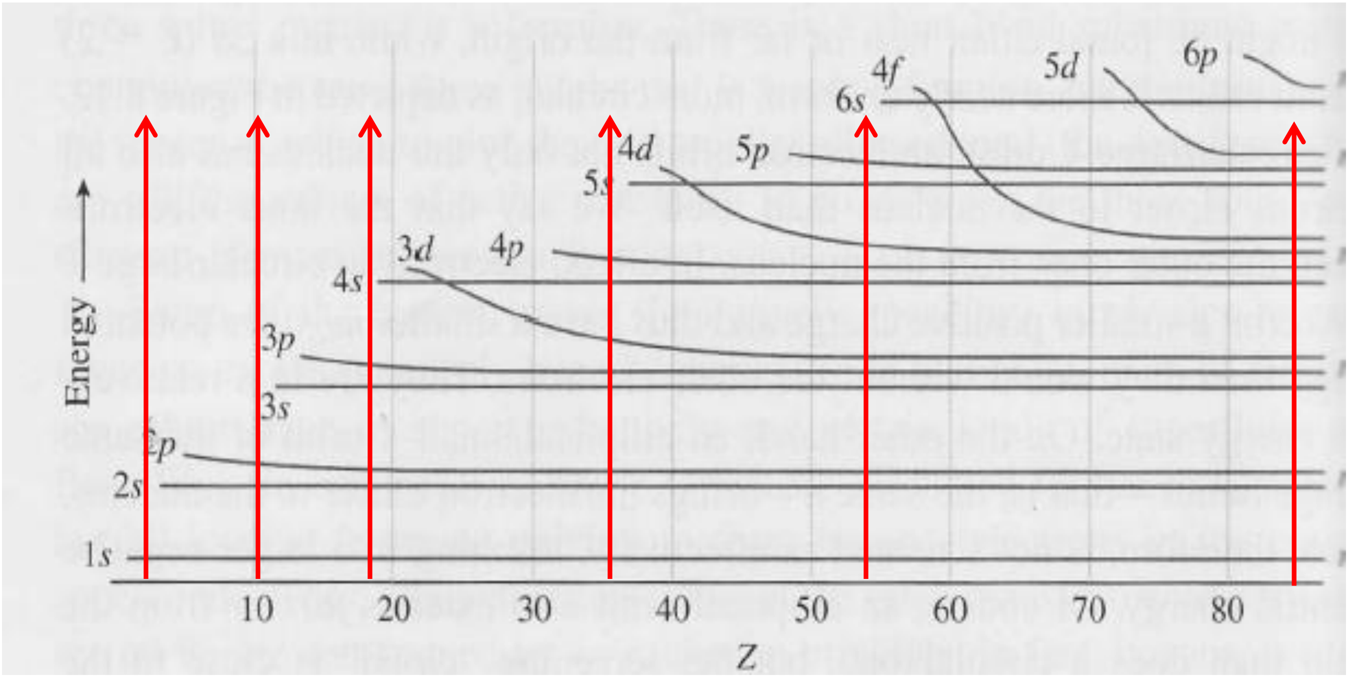
Rn (86) = $1s_2$ $2s_2$ $2p_6$ $3s_2$ $3p_6$ $3d_{10}$ $4s_2$ $4p_6$ $4d_{10}$ $4f_{14}$ $5s_2$ $5p_6$ $5d_{10}$ $6s_2$ $6p_6$



Angular Probability Density

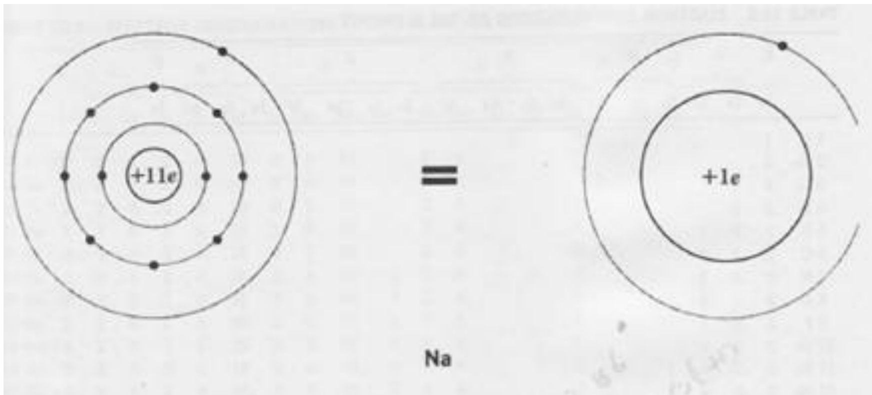
$$\Theta(\theta)^2\Phi(\phi)^2 \equiv \mathbf{Y}_l^{m_l *}\mathbf{Y}_l^{m_l}$$

n	l	m_l	$\Phi(\phi)$	$\Theta(\theta)$	$R(r)$	$\psi(r,\theta,\phi)$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0^{3/2}}e^{-r/a_0}$	$\frac{1}{\sqrt{\pi}\,a_0^{3/2}}e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}\,a_0^{3/2}}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\,a_0^{3/2}}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{1}{2\sqrt{6}\,a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\,a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\cos\theta$
2	1	±1	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$	$\frac{1}{2\sqrt{6}\,a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi}\,a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\sin\theta\, e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3}\,a_0^{3/2}}\left(27-18\frac{r}{a_0}+2\frac{r^2}{a_0^2}\right)e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi}\,a_0^{3/2}}\left(27-18\frac{r}{a_0}+2\frac{r^2}{a_0^2}\right)e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{4}{81\sqrt{6}\,a_0^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi}\,a_0^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}\cos\theta$
3	1	±1	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$	$\frac{4}{81\sqrt{6}\,a_0^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\,a_0^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}\sin\theta\, e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4}(3\cos^2\theta-1)$	$\frac{4}{81\sqrt{30}\,a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi}\,a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}(3\cos^2\theta-1)$
3	2	±1	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$	$\frac{\sqrt{15}}{2}\sin\theta\cos\theta$	$\frac{4}{81\sqrt{30}\,a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\,a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}\sin\theta\cos\theta\, e^{\pm i\phi}$
3	2	±2	$\frac{1}{\sqrt{2\pi}}e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4}\sin^2\theta$	$\frac{4}{81\sqrt{30}\,a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi}\,a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}\sin^2\theta\, e^{\pm 2i\phi}$

Unsöld's theorem

$$\sum_{m_l=-l}^{l}|\Theta_{l,m_l}|^2|\Phi_{m_l}|^2 = Constant$$

Hydrogenlike atoms

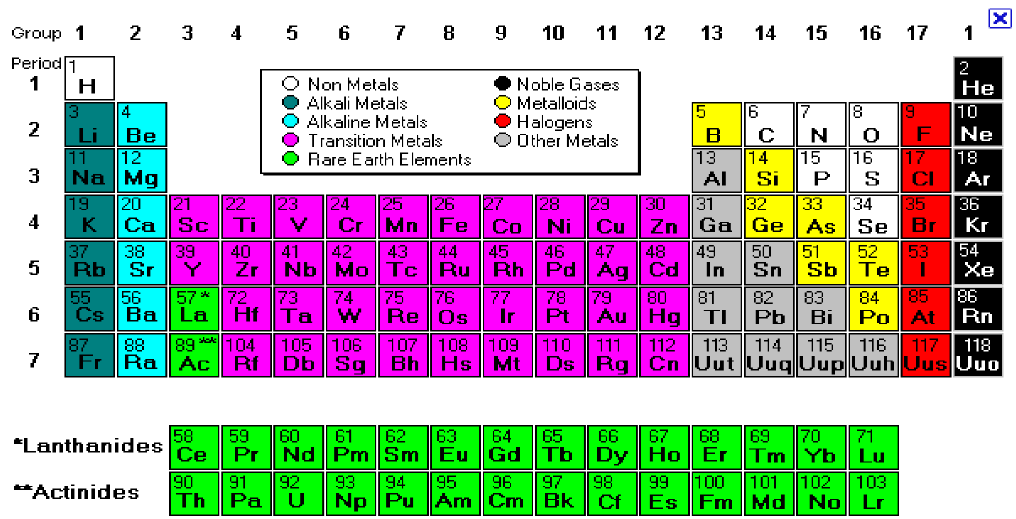


Hydrogen atom

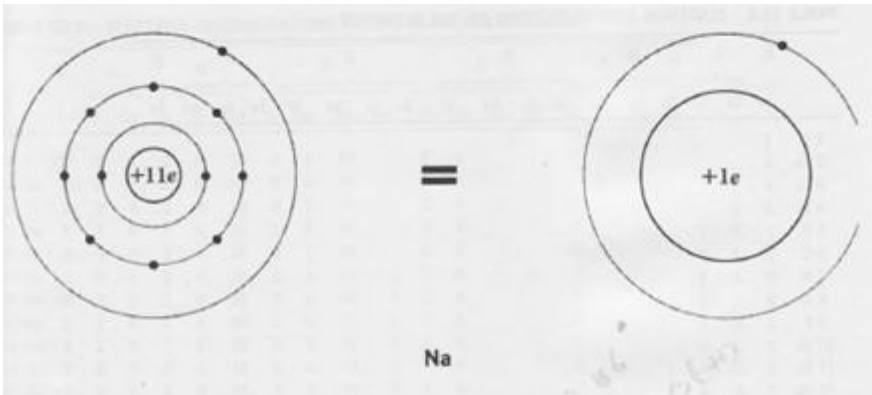
$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{-e^2}{r}$$

Hydrogen-like atom

$$U(r) = -\frac{1}{4\pi\varepsilon_0}\frac{e \cdot Ze}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$



Hydrogenlike atoms



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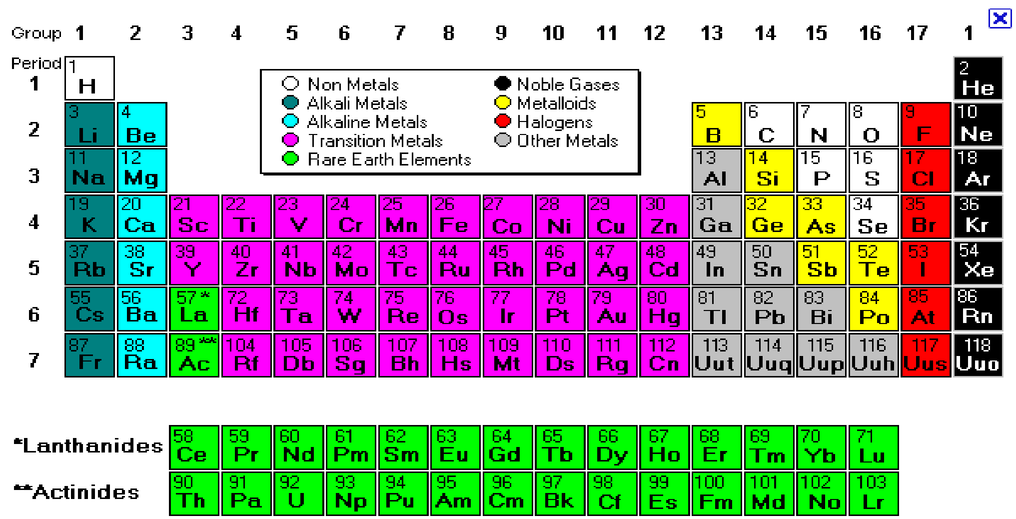
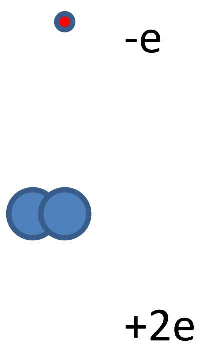
Hydrogen atom solutions can be applied with charge modifications



Iodized Helium



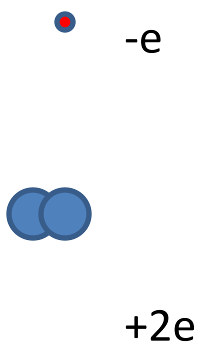
Iodized Helium

Example: ionized Helium where the nucleus has a charge of $+2e$ and the orbiting electron has a charge of $-e$.

In that case,

$$E_{n\ hydrogen} = -\frac{m(e^2)e^2}{32\pi^2\varepsilon_0^2\hbar^2}\left(\frac{1}{n^2}\right) = -\left(\frac{e^2}{8\pi\varepsilon_0}\right)\left(\frac{me^2}{4\pi\varepsilon_0^{\square}\hbar^2}\right)\left(\frac{1}{n^2}\right) = -\left(\frac{e^2}{8\pi\varepsilon_0 a_0}\right)\left(\frac{1}{n^2}\right) = -13.6\ eV\left(\frac{1}{n^2}\right)$$

$$E_{n\ hydrogenlike} = -\frac{m(Z^2e^2)e^2}{32\pi^2\varepsilon_0^2\hbar^2}\left(\frac{1}{n^2}\right) = -\left(\frac{Z^2e^2}{8\pi\varepsilon_0}\right)\left(\frac{me^2}{4\pi\varepsilon_0^{\square}\hbar^2}\right)\left(\frac{1}{n^2}\right) = -\left(\frac{Z^2e^2}{8\pi\varepsilon_0 a_0}\right)\left(\frac{1}{n^2}\right) = -13.6\ eV\left(\frac{Z^2}{n^2}\right)$$



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Example: ionized Helium where the nucleus has a charge of $+2e$ and the orbiting electron has a charge of $-e$.

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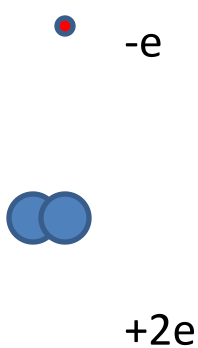
$$E_{n\ hydrogenlike} = -\frac{m(Z^2e^2)e^2}{32\pi^2\varepsilon_0^2\hbar^2}\left(\frac{1}{n^2}\right) = -\left(\frac{Z^2e^2}{8\pi\varepsilon_0}\right)\left(\frac{me^2}{4\pi\varepsilon_0\hbar^2}\right)\left(\frac{1}{n^2}\right) = -\left(\frac{Z^2e^2}{8\pi\varepsilon_0 a_0}\right)\left(\frac{1}{n^2}\right) = -13.6\ eV\left(\frac{Z^2}{n^2}\right)$$



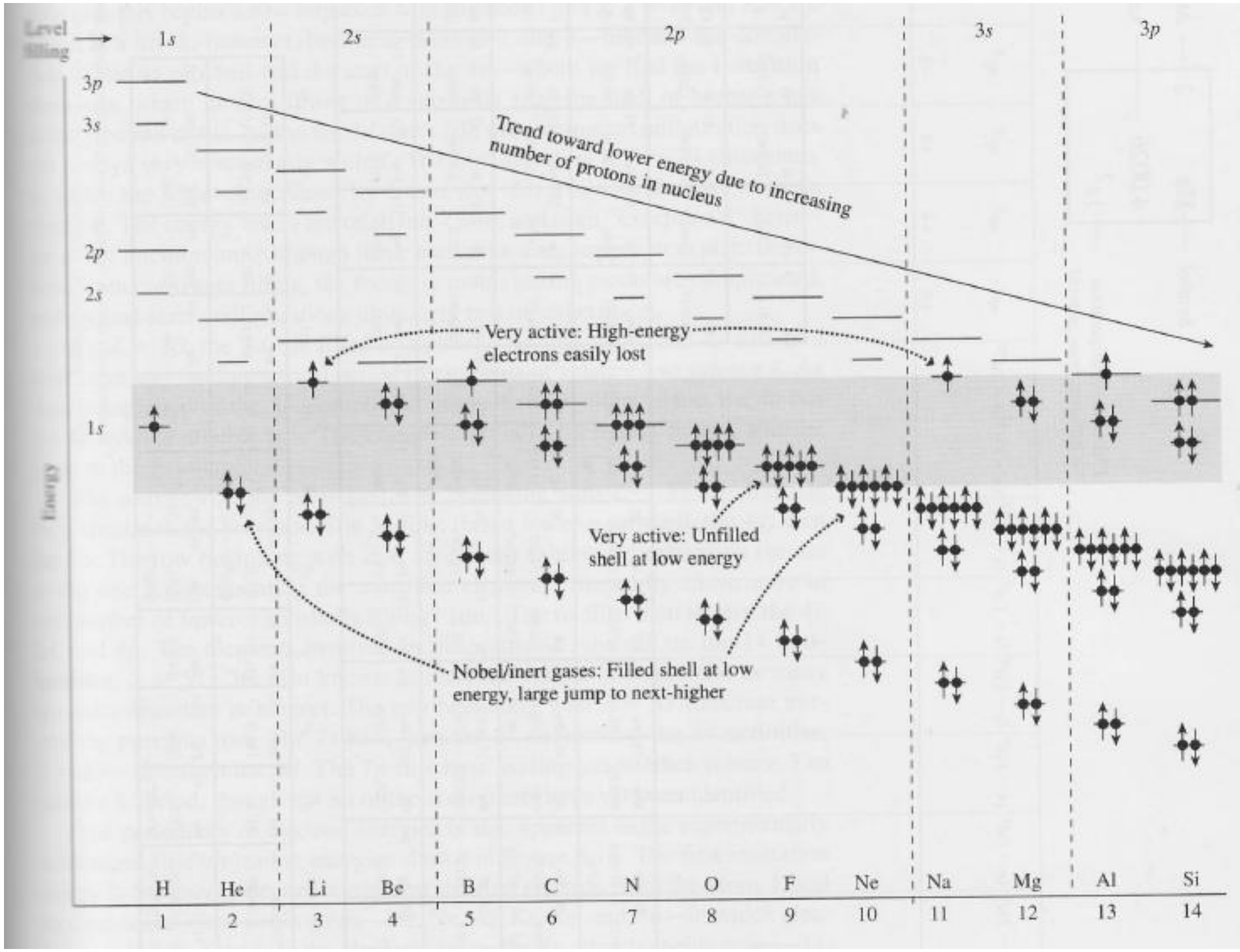
The most probable r vale for the orbital $l = n - 1$

$$r_{n,\ l=n-1\ (most\ probable)\ hydrogen} = n^2 a_0$$

$$r_{n,\ l=n-1\ (most\ probable)\ hydrogenlike} = \frac{1}{Z} n^2 a_0$$

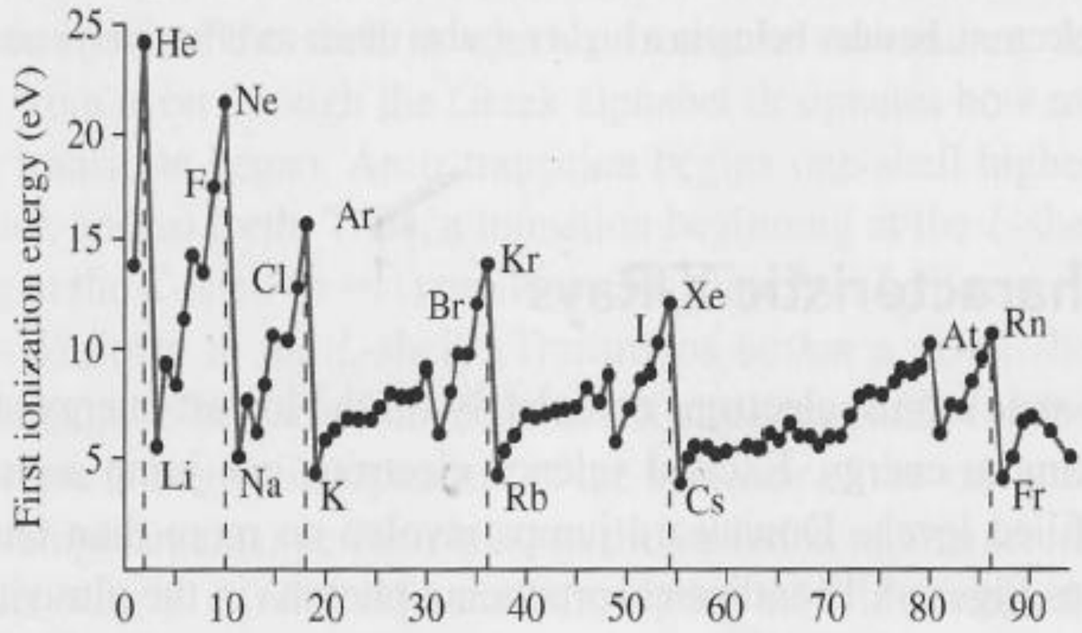
Bound energies are deeper by Z^2, the orbital most probably radii smaller by $\frac{1}{Z}$

Orbital Energy levels

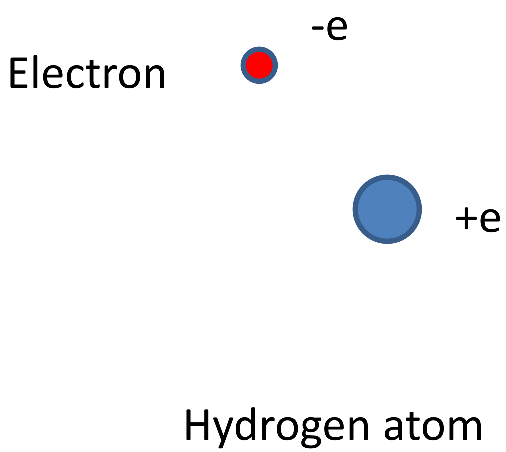


Ionization Energy

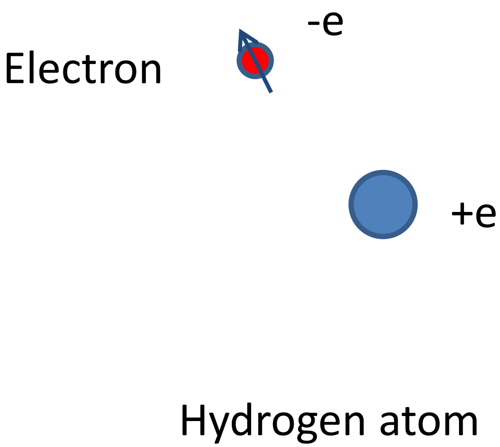
Figure 8.16 First ionization energies of the elements.



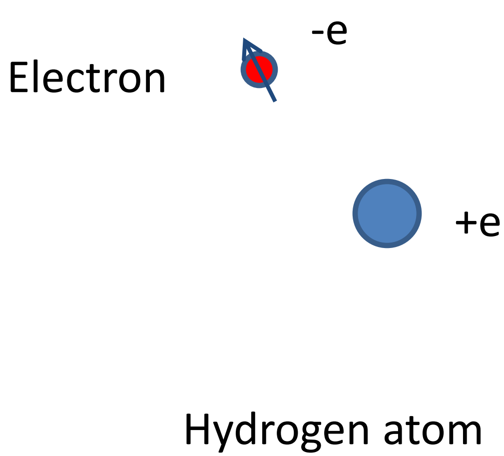
Hydrogen atom

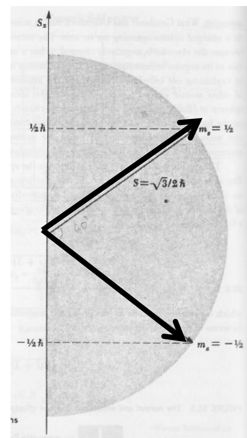


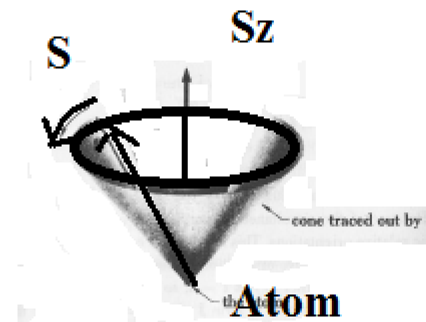
Spin $\vec{S}$



Spin $\vec{S}$

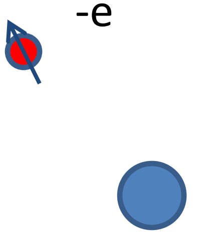




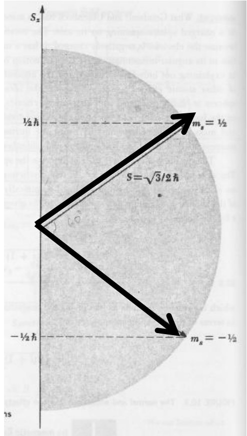


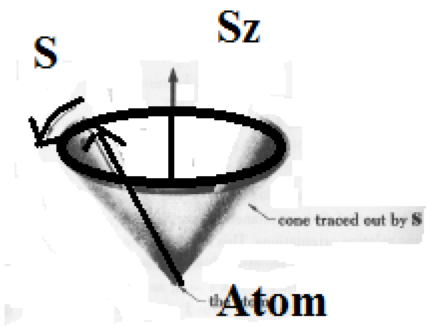
Spin $\vec{S}$

Electron



Hydrogen atom





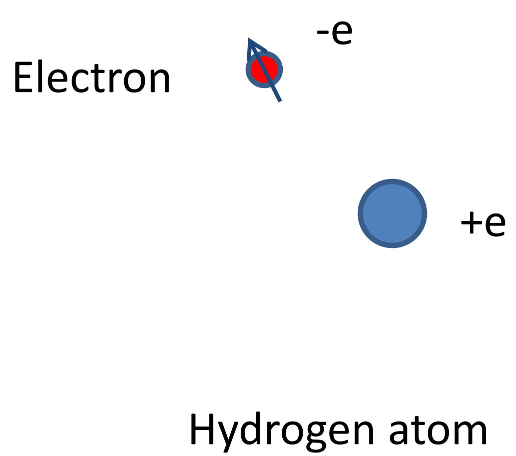
$$|\vec{S}| = \sqrt{s(s+1)}\hbar$$

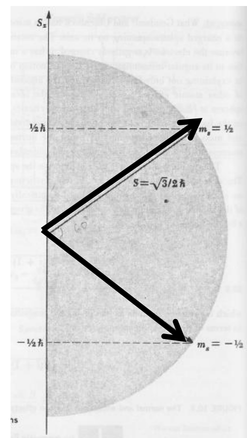
where s is a number intrinsic to a given particle

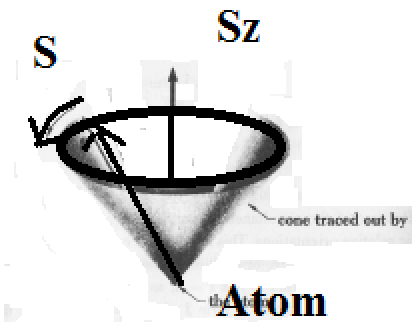
$$S_z = m_s\hbar$$

$$\text{where } m_s = -s, -s+1, \ldots s-1, s$$

Spin $\vec{S}$







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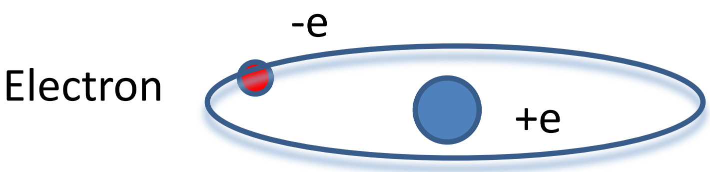
where $m_s = -s, -s+1, \ldots s-1, s$

$$\vec{\mu}_S = -\left(\frac{e}{m}\right)\vec{S}$$

Magnetic moment under an external magnetic field

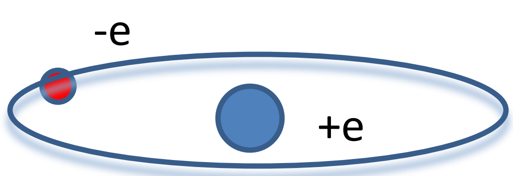
Orbital angular momentum $\vec{L}$

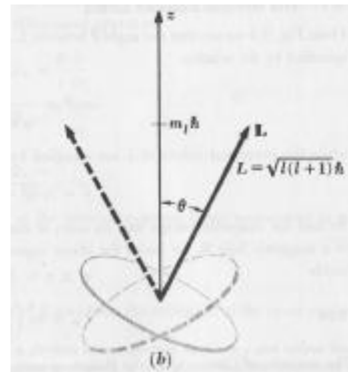


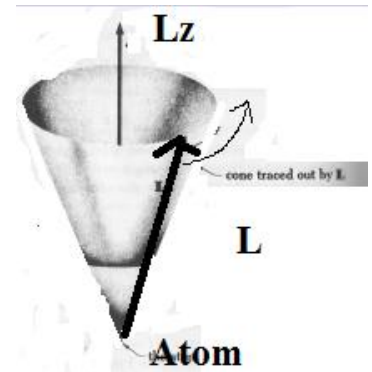
Hydrogen atom

Orbital angular momentum $\vec{L}$

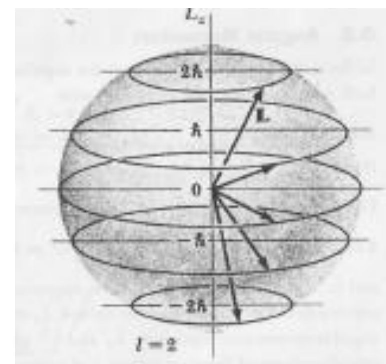
Electron



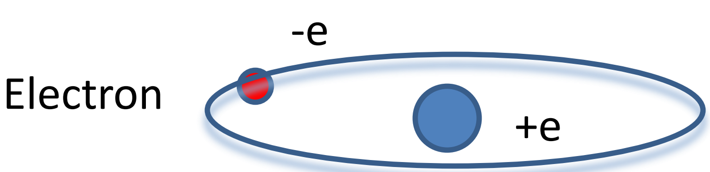




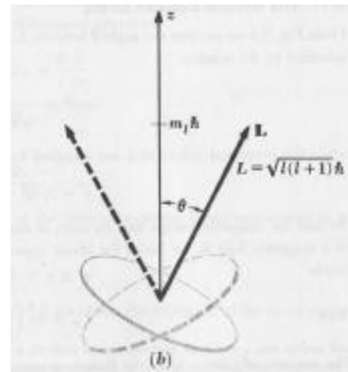
Hydrogen atom

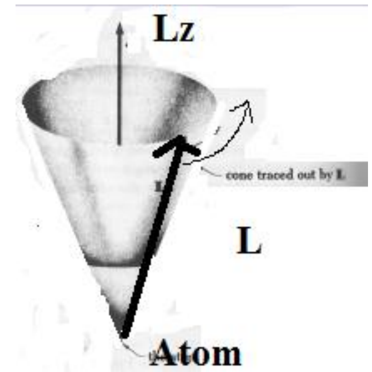


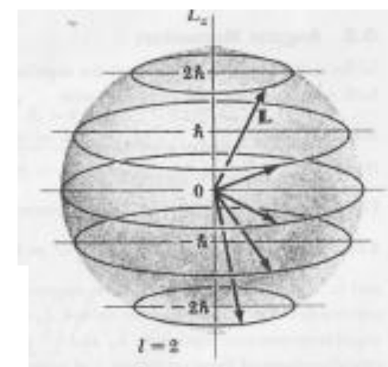
Orbital angular momentum $\vec{L}$



Hydrogen atom



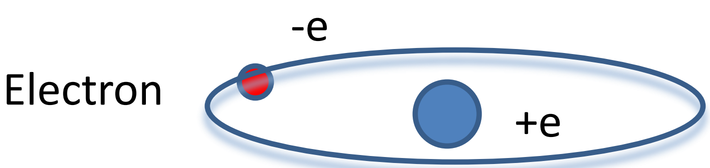




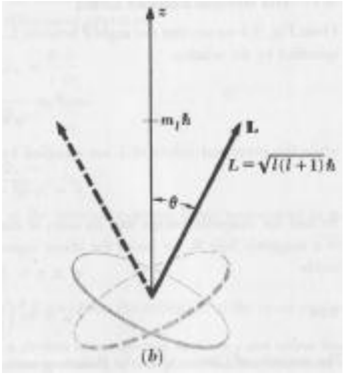
$$|\vec{L}| = \sqrt{l(l+1)}\hbar \qquad where\ l = 0, 1, 2, \ldots n-1$$

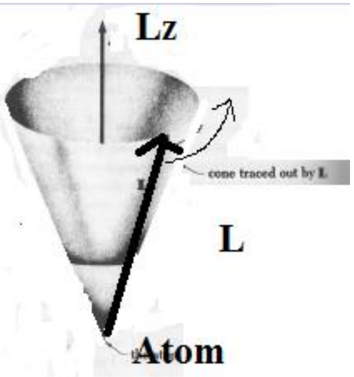
$$L_z = m_l\hbar \qquad where\ m_l = -l, -l+1, \ldots l-1, l$$

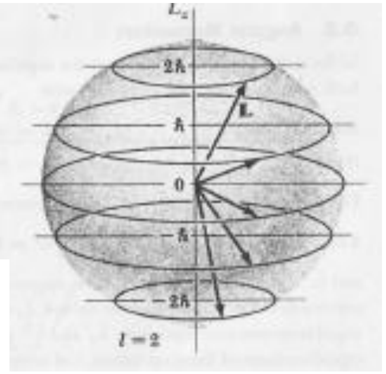
Orbital angular momentum $\vec{L}$



Hydrogen atom







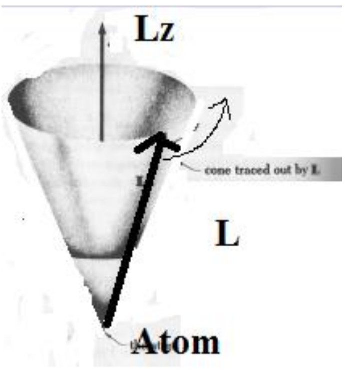
$$|\vec{L}| = \sqrt{l(l+1)}\hbar \qquad where\ l = 0, 1, 2, \dots n-1$$

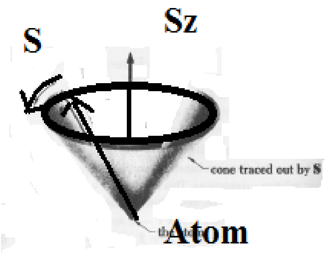
$$L_z = m_l \hbar \qquad where\ m_l = -l, -l+1, \dots l-1, l$$

$$\vec{\mu}_L = -\left(\frac{e}{2m}\right)\vec{L}$$

Magnetic moment under an external magnetic field

Strong $\vec{B}$: Paschen-Back Effect





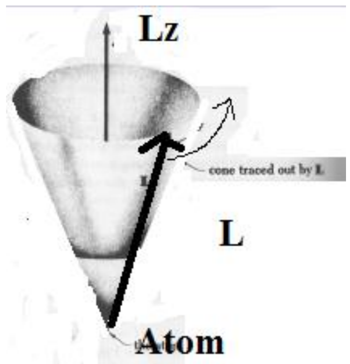
$$\vec{\mu}_L = -\left(\frac{e}{2m}\right)\vec{L}$$
$$\vec{\mu}_S = -\left(\frac{e}{m}\right)\vec{S}$$

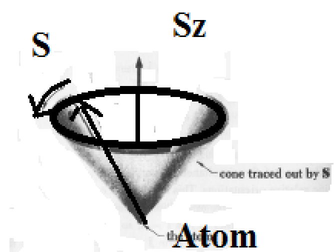
$$U_L = -\vec{\mu}_L \cdot \vec{B} = -\mu_{L_z} B_z = -\left(-\frac{e}{2m} L_z\right) B_z = \frac{e}{2m} m_l \hbar B_z$$

$$U_S = -\vec{\mu}_S \cdot \vec{B} = -\mu_{S_z} B_z = -\left(-\frac{e}{m} S_z\right) B_z = \frac{e}{m} m_s \hbar B_z$$

$$E_{strong\ magnetic\ field} = E_n + \frac{e}{2m}(m_l + 2m_s)\hbar B_z$$

Strong $\vec{B}$: Paschen-Back Effect





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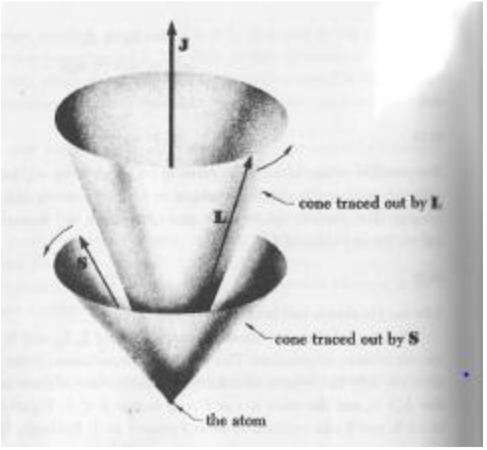
$$E_{strong\ magnetic\ field} = E_n + \frac{e}{2m}(m_l + 2m_s)\hbar B_z$$

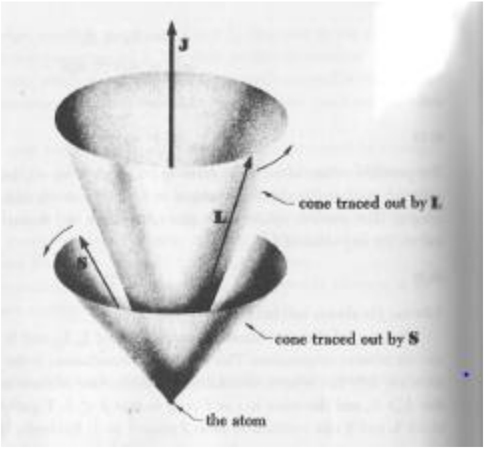
For a 2p electron, how many states are available and how much energy is deviated from the E_n value for each state?

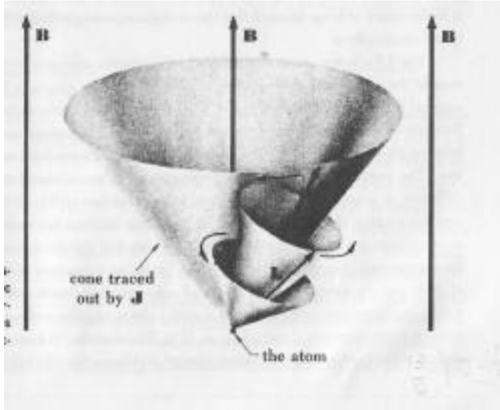
Weak $\vec{B}$: Zeeman Effect



$$\vec{J} = \vec{L} + \vec{S}$$

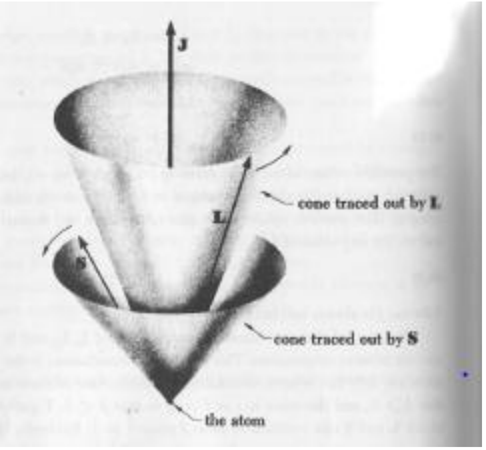
Weak $\vec{B}$: Zeeman Effect

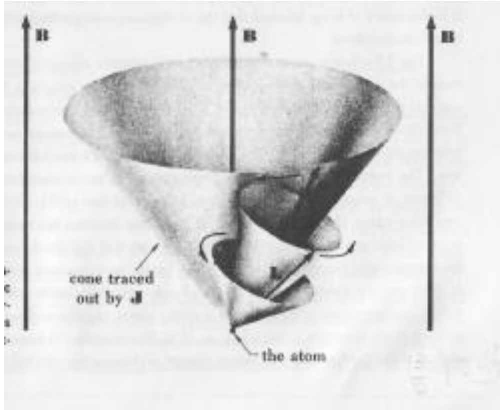




$$\vec{J} = \vec{L} + \vec{S}$$

Weak $\vec{B}$: Zeeman Effect





$$\vec{J} = \vec{L} + \vec{S}$$

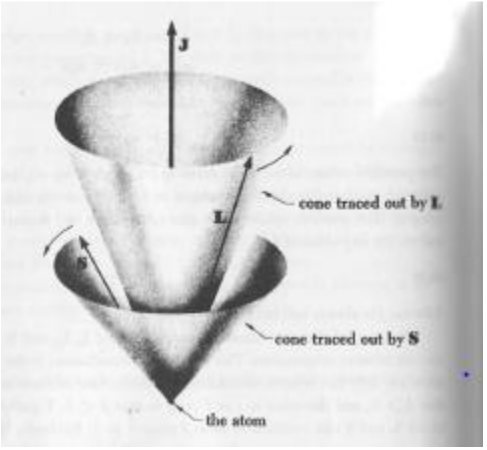
$$|\vec{J}| = \sqrt{j(j+1)}\hbar \qquad \textit{where } j = |l-s|, |l-s|+1, \ldots |l+s|-1, |l+s|$$

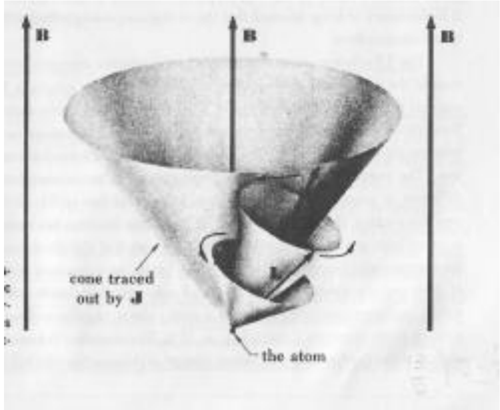
$$J_z = m_j\hbar \qquad \text{where } m_j = -j, -j+1, \ldots, j-1, j$$

$$J_z = L_z \pm S_z$$

$$m_j\hbar = m_l\hbar + m_s\hbar$$

Weak $\vec{B}$: Zeeman Effect





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$$J_z = L_z \pm S_z$$

$$m_j\hbar = m_l\hbar + m_s\hbar$$

where $j = |l-s|, |l-s|+1, \ldots |l+s|-1, |l+s|$

where $m_j = -j, -j+1, \ldots, j-1, j$

For a 2p electron, how many states are available?

$$\vec{\mu}_L = -\left(\frac{e}{2m}\right)\vec{L}$$
$$\vec{\mu}_S = -\left(\frac{e}{m}\right)\vec{S}$$

$$\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S = -\left(\frac{e}{2m}\right)(\vec{L} + 2\vec{S})$$

$$|\overrightarrow{\mu_J}||\vec{J}|\cos(\pi + \delta) = \vec{\mu}_J \cdot \vec{J} \qquad \cos\delta = -\frac{\vec{\mu}_J \cdot \vec{J}}{|\overrightarrow{\mu_J}||\vec{J}|}$$

$$|\overrightarrow{\mu_J}|_{\text{avg}} = |\overrightarrow{\mu_J}|\cos\delta = |\overrightarrow{\mu_J}|\left(-\frac{\vec{\mu}_J \cdot \vec{J}}{|\overrightarrow{\mu_J}||\vec{J}|}\right) = -\frac{\vec{\mu}_J \cdot \vec{J}}{|\vec{J}|} = \frac{e}{2m}\frac{(\vec{L} + 2\vec{S}) \cdot (\vec{L} + \vec{S})}{|\vec{J}|}$$

$$= \frac{e}{2m}\frac{|\vec{L}|^2 + 2|\vec{S}|^2 + 3\vec{L} \cdot \vec{S}}{|\vec{J}|} \qquad \textit{since } \vec{L} \cdot \vec{S} = \frac{1}{2}(|\vec{J}|^2 - |\vec{L}|^2 - |\vec{S}|^2)$$

$$|\overrightarrow{\mu_J}|_{\text{avg}} = \frac{e}{2m}\frac{3|\vec{J}|^2 - |\vec{L}|^2 + |\vec{S}|^2}{2|\vec{J}|} = \frac{e}{2m}\frac{3j(j+1) - l(l+1) + s(s+1)}{2\sqrt{j(j+1)}}\hbar$$

$\vec{B} = B_z \hat{z}$, then potential energy since $cos\theta = \frac{J_z}{|\vec{J}|} = \frac{J_z}{\sqrt{j(j+1)}\hbar}$

$$U = -\vec{\mu}_{J.} \cdot \vec{B} = |\vec{\mu}_{J.}| B_z cos\theta = |\vec{\mu}_{J.}| B_z \frac{J_z}{\sqrt{j(j+1)}\hbar} = \frac{e J_z B_z}{2m} \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

$$= g_{Laude} \frac{e J_z B_z}{2m} \qquad = g_{Lande} \frac{e B_z}{2m} (m_j \hbar)$$

Where $g_{Lande} = \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$

$$E_{weak\ external\ magnetic\ field} = E_n + g_{Lande} \frac{e B_z}{2m} (m_j \hbar)$$

How strong is strong?

- What is the amount of energy comparable to LS coupling?
- Consider: a 2p electron
  - n=2, l=1, and $r \approx n^2 a_0 \approx 4a_0$

$$U = -\vec{\mu}_{S.} \cdot \vec{B}_{due\ to\ \vec{L}}$$

$$\vec{B} = \frac{\mu_0 e}{4\pi m r^3} \vec{L}$$

$$B = \frac{\mu_0 e}{4\pi m (4a_0)^3} \sqrt{1 \cdot 2}\hbar = 0.28\ \mathrm{T}$$

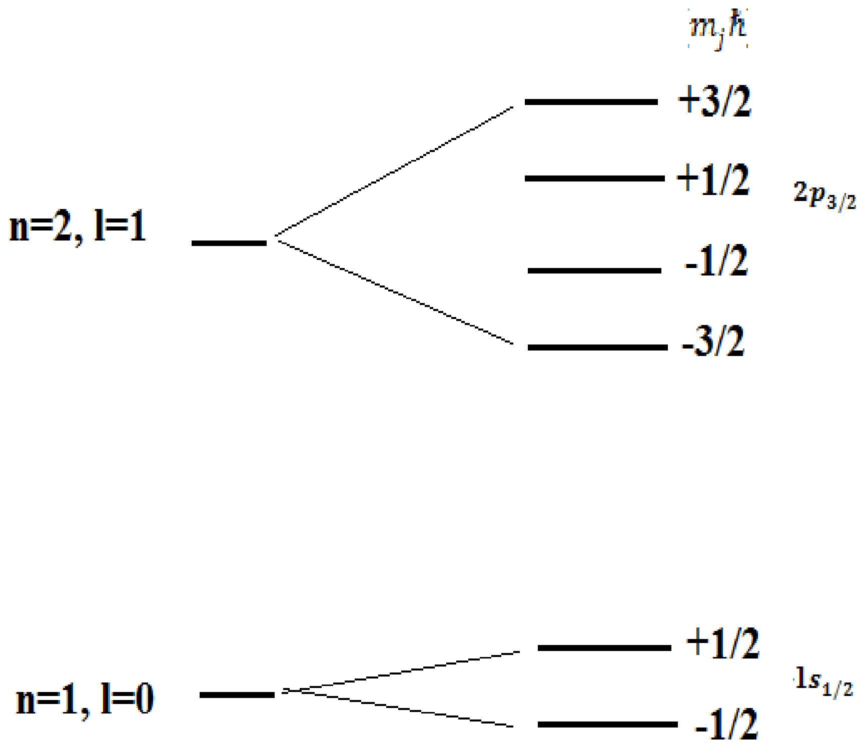
$$U = \mu_S B = 2 \times 10^{-5}\ eV$$

: $2p_{3/2} \rightarrow 1s_{1/2}$ **in a weak external magnetic field of .05T**

Obtain energy split for each.

Draw an energy diagram without B and with B. Indicate how the energy without B is split when B is present.

: $2p_{3/2} \rightarrow 1s_{1/2}$ in a weak external magnetic field of .05T



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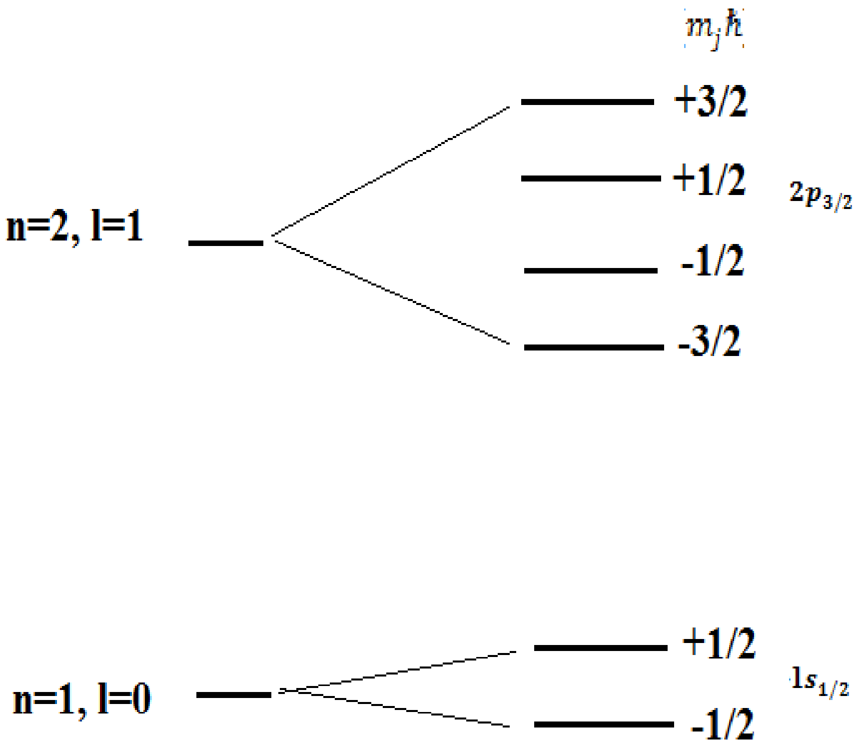

Calculate

$$g_{Lande} = \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

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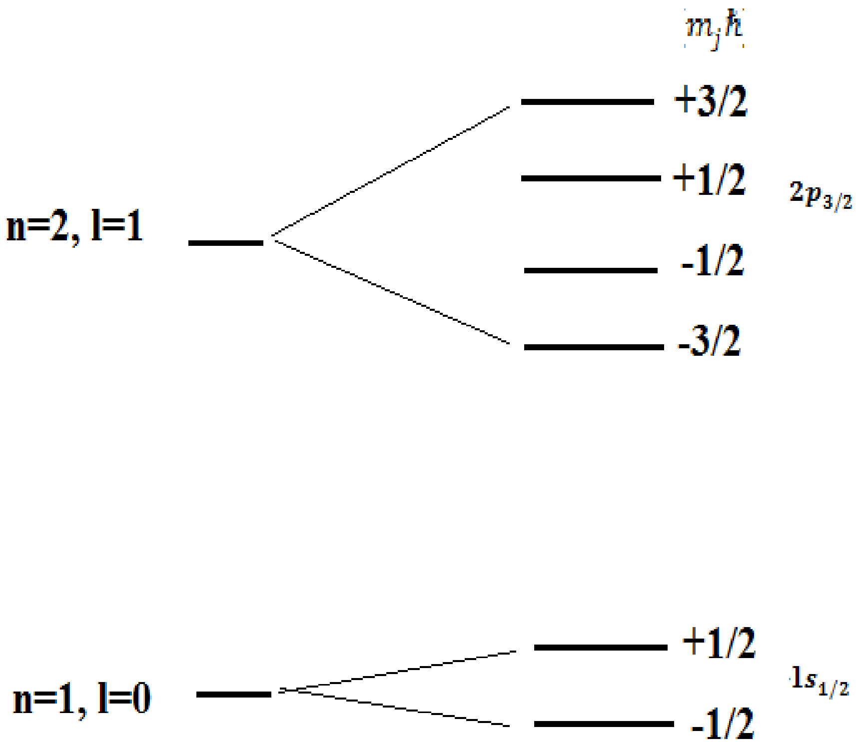


Calculate

$$g_{Lande} = \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

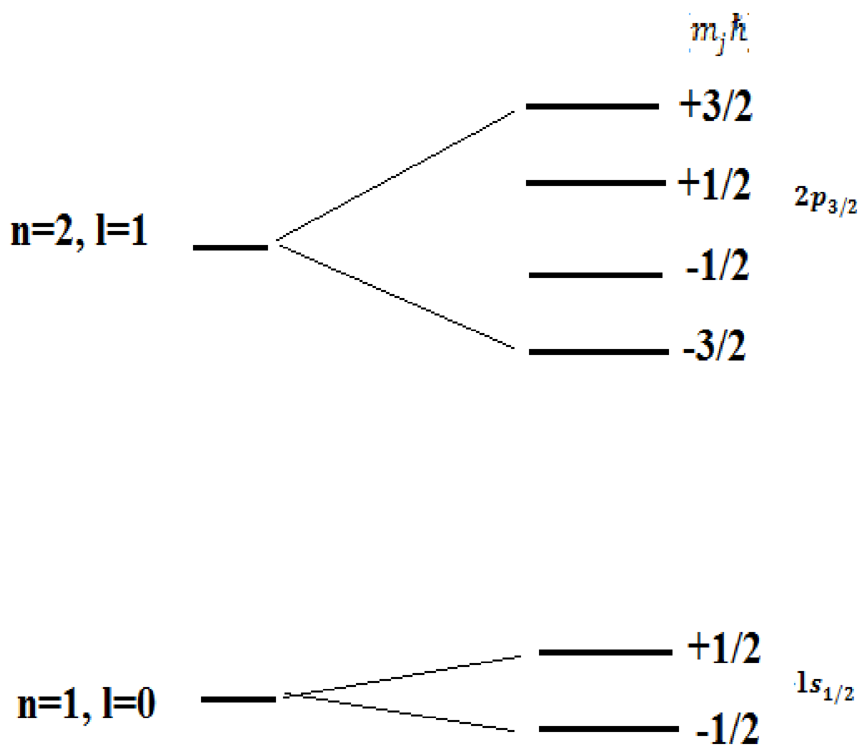
For $1s_{\frac{1}{2}}$, $l = 0, s = \frac{1}{2}, j = \frac{1}{2}$; $g_{Laude} = 2$

For $2p_{3/2}$, $l = 1, s = \frac{1}{2}, j = \frac{3}{2}$; $g_{Lande} = \frac{4}{3}$

Calculate

$$\Delta E = g_{Lande} \frac{eB_z}{2m} (m_j \hbar)$$

: $2p_{3/2} \rightarrow 1s_{1/2}$ in a weak external magnetic field of .05T



Calculate

$$g_{Lande} = \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

For $1s_{\frac{1}{2}}, l = 0, s = \frac{1}{2}, j = \frac{1}{2};\ g_{Laude} = 2$

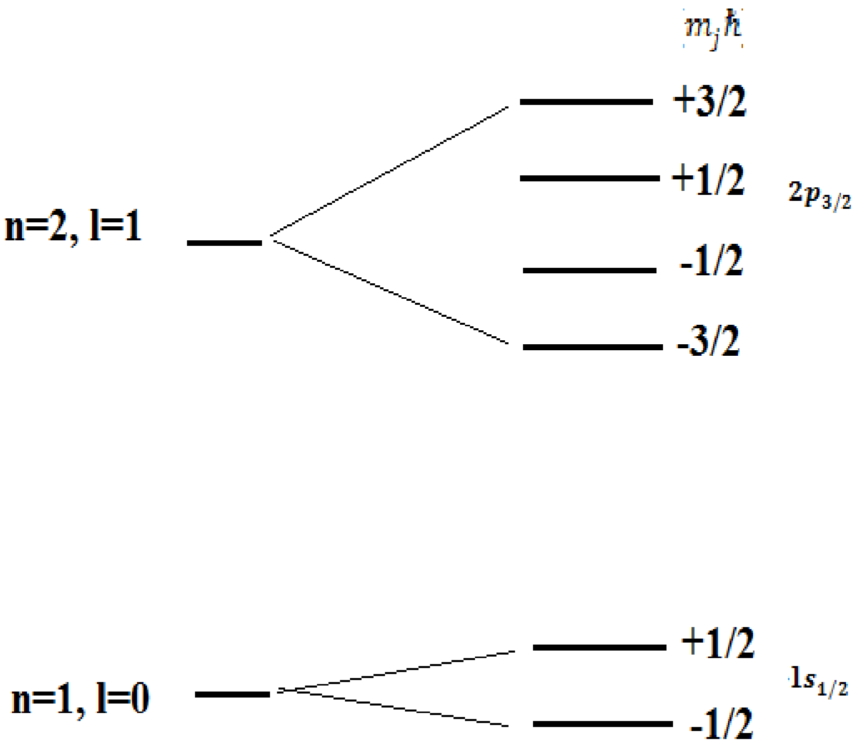
For $2p_{3/2}, l = 1, s = \frac{1}{2}, j = \frac{3}{2};\ g_{Laude} = \frac{4}{3}$

Calculate

$$\Delta E = g_{Lande}\frac{eB_z}{2m}(m_j\hbar) = \frac{2eB_z\hbar}{2m}(m_j) = \frac{2eB_z\hbar}{2m}\left(\pm\frac{1}{2}\right)$$

$$\Delta E = g_{Lande}\frac{eB_z}{2m}(m_j\hbar) = \left(\frac{4}{3}\right)\frac{eB_z\hbar}{2m}(m_j) = \frac{2eB_z\hbar}{3m}\left(\pm\frac{3}{2}, \pm\frac{1}{2}\right)$$

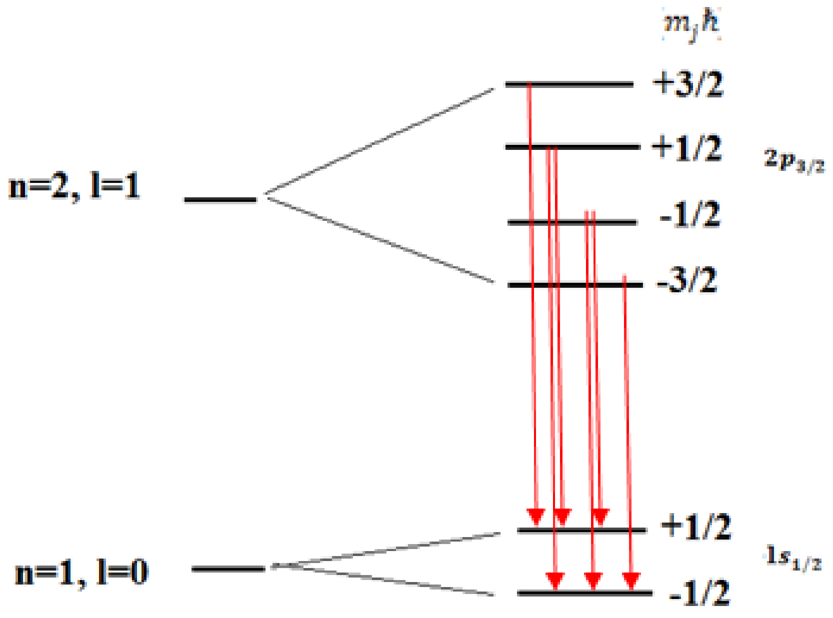
: $2p_{3/2} \rightarrow 1s_{1/2}$ in a weak external magnetic field of .05T



Selection Rule

$$\Delta m_j = 0, \pm 1$$

: $2p_{3/2} \rightarrow 1s_{1/2}$ in a weak external magnetic field of .05T



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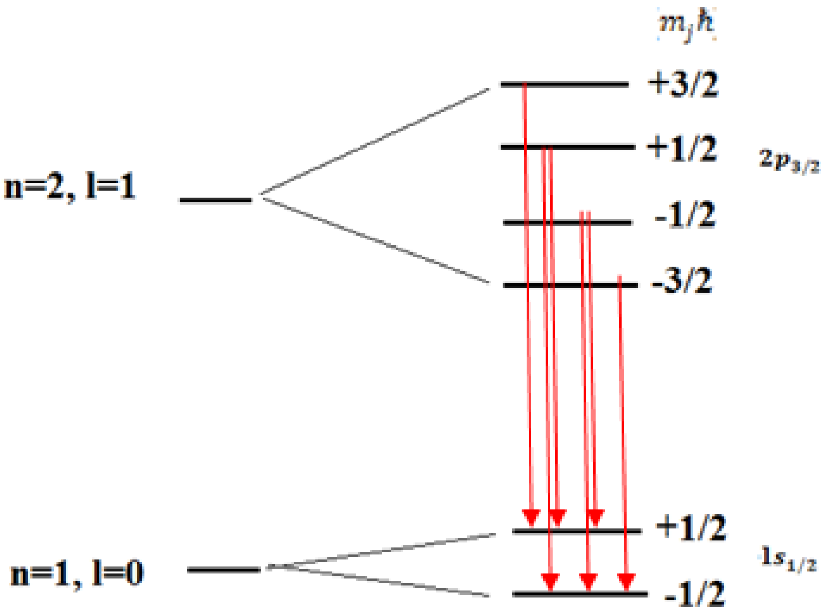
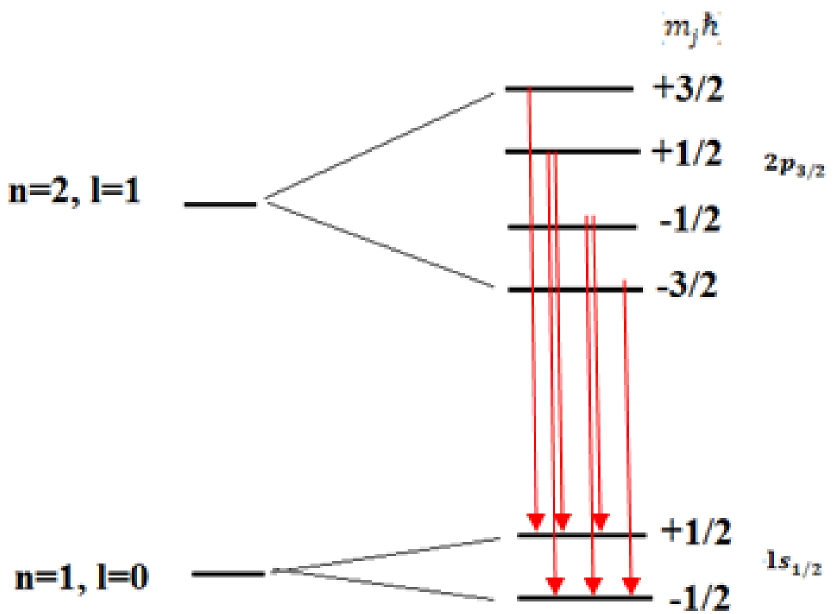


$$E_2 + \frac{2eB_z\hbar}{3m}\left(\pm\frac{3}{2}, \pm\frac{1}{2}\right)$$

$$E_1 + \frac{2eB_z\hbar}{2m}\left(\pm\frac{1}{2}\right)$$

Calculate photon energy:

: $2p_{3/2} \rightarrow 1s_{1/2}$ in a weak external magnetic field of .05T



$$E_2 + \frac{2eB_z\hbar}{3m}\left(\pm\frac{3}{2}, \pm\frac{1}{2}\right)$$

$$E_1 + \frac{2eB_z\hbar}{2m}\left(\pm\frac{1}{2}\right)$$

Calculate photon energy:

$$\Delta E = \Delta E_{without\ magnetic\ field} + \frac{2eB_z\hbar}{2m}\left(\pm\frac{5}{3}, \pm\frac{3}{3}, \pm\frac{1}{3}\right)$$