Announcement

- Homework (10%) + Class participation (10%)
- HW5 due: 03/08 and HW6 due: 03/15
- Midterm 1 (20%) + Midterm 2 (20%)
- Final (40%) on 3/22, Fri, 12-3 pm
  - Bring calculator, open-book and open-notes
  - 30% Nuclear
  - 30% Elementary
  - 40% Atomic, statistical, and solid state focusing on common underlying ideas:
    - Energy, Hamiltonian, conservation, system, etc.

- 95 <= A+
- 90 <= A < 90
- 87 <= A- < 90
- 84 <= B+ < 87
- 80 <= B < 84

Lecture 18 Topics

- Nuclear reactions
  - Fission
    - Spontaneous fission
    - Induced fission
      - Happened big time as weapon
      - Happens in nuclear power plants
  - Fusion
    - Happens in stars
    - Has yet to happen for everyday use
- Fundamental forces
  - New definition involving mediating bosons
- Antiparticles
  - Klein-Gordon equation

Nuclear Reactions

- Chemical reactions

$$2H_2 + O_2 \rightarrow 2H_2O$$

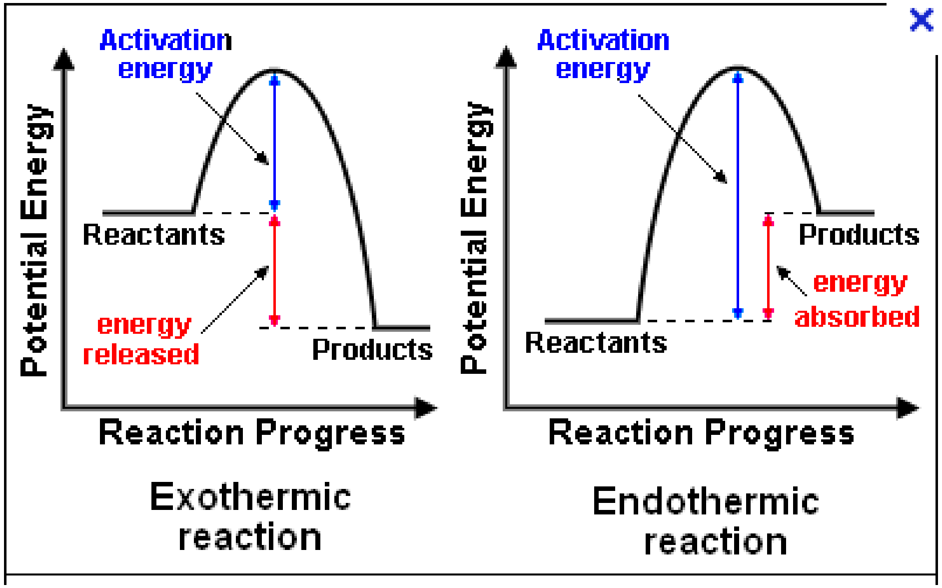
- Nuclear reactions

$$^{10}_{5}B + ^{1}_{0}n \rightarrow ^{7}_{3}Li + ^{4}_{2}He$$

Nuclear Reactions

- Chemical reactions

$$2H_2 + O_2 \rightarrow 2H_2O$$

- 

Nuclear Reactions

- Chemical reactions

$$2H_2 + O_2 \rightarrow 2H_2O$$

- Nuclear reactions

Exothermic $${}^{10}_{5}B + {}^{1}_{0}n \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He$$

Endothermic $${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow {}^{7}_{4}Be + {}^{1}_{0}n$$

Nuclear Reactions

- Chemical reactions

$$2H_2 + O_2 \rightarrow 2H_2O$$

- Nuclear reactions

Exothermic $${}^{10}_{5}B + {}^{1}_{0}n \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He$$

Endothermic $${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow {}^{7}_{4}Be + {}^{1}_{0}n$$

Released kinetic energy (Q)

Nuclear Reactions

- Chemical reactions

$$2H_2 + O_2 \rightarrow 2H_2O$$

- Nuclear reactions

Exothermic $${}^{10}_{5}B + {}^{1}_{0}n \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He$$

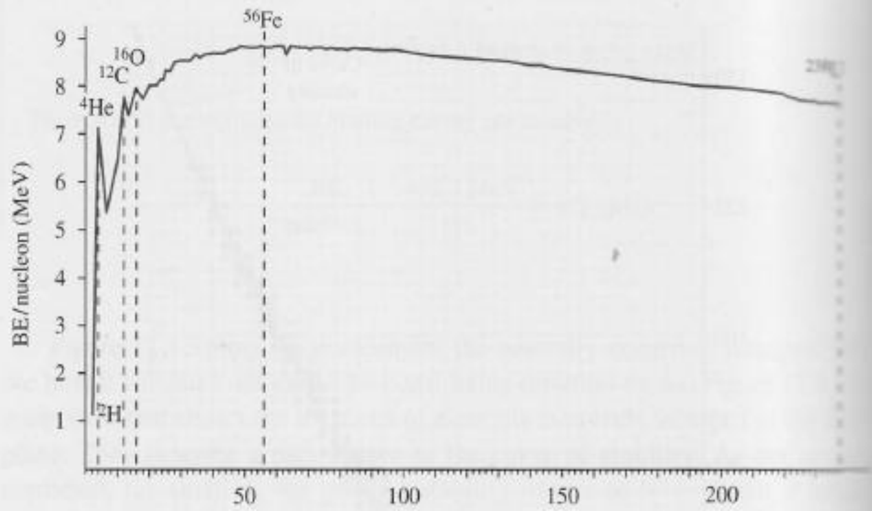
Q $$= (10.012937 + 1.008665 - 7.016003 - 4.002603)\, uc^2 = 2.79\ MeV$$

Endothermic $${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow {}^{7}_{4}Be + {}^{1}_{0}n$$

Q $$= (7.016003 + 1.007825 - 7.016928 - 1.008665)uc^2 = -1.64\ MeV$$

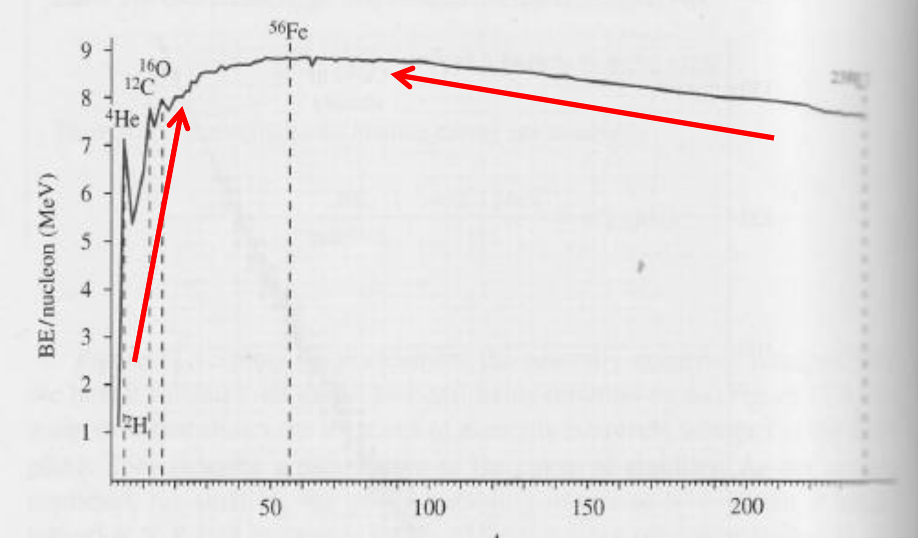
Binding energy/nucleon vs. A

Figure 11.14 Binding energy per nucleon versus *A*.



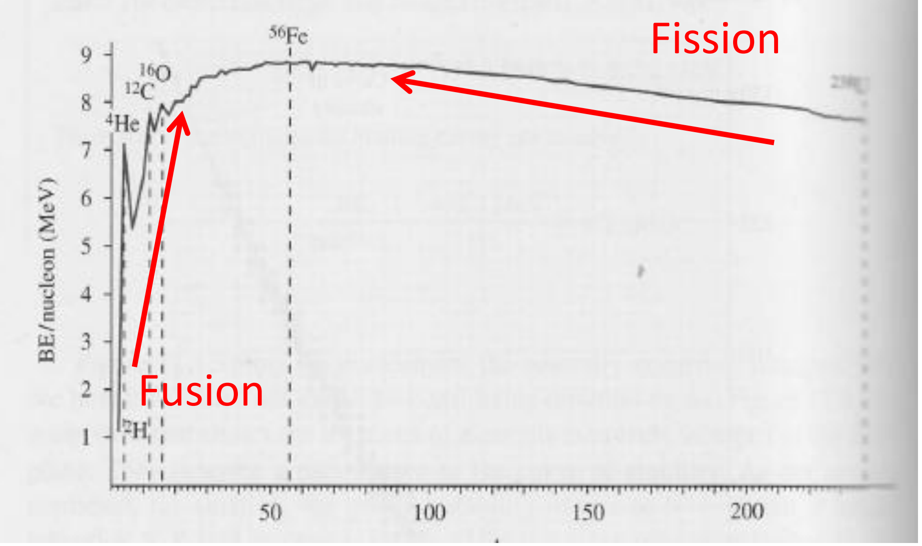
Binding energy/nucleon vs. A

Figure 11.14 Binding energy per nucleon versus *A*.



Binding energy/nucleon vs. A

Figure 11.14 Binding energy per nucleon versus *A*.



Nuclear fission

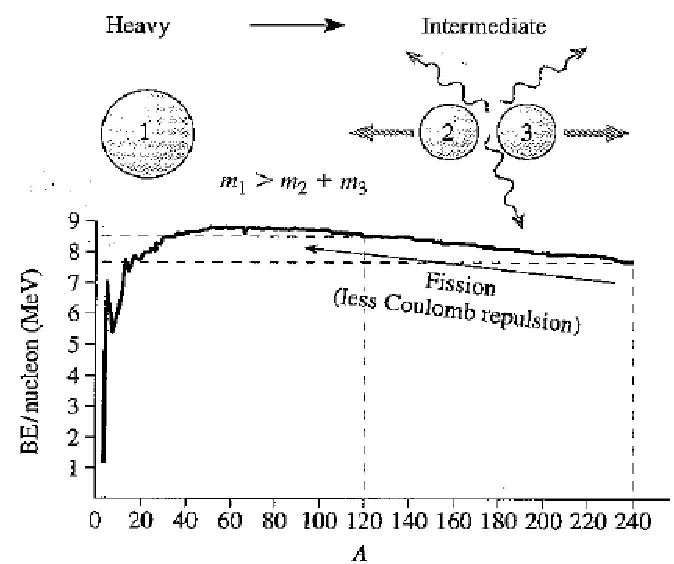
A heavy nucleus breaks into smaller nuclei

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{141}_{56}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3\,^{1}_{0}\mathrm{n}$$

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{140}_{54}\mathrm{Xe} + ^{94}_{38}\mathrm{Sr} + 2\,^{1}_{0}\mathrm{n}$$

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{132}_{50}\mathrm{Sn} + ^{101}_{42}\mathrm{Mo} + 3\,^{1}_{0}\mathrm{n}$$

Figure 11.27 Decreasing BE/nucleon via fission.



What is the driving force?

Why neutrons are released?

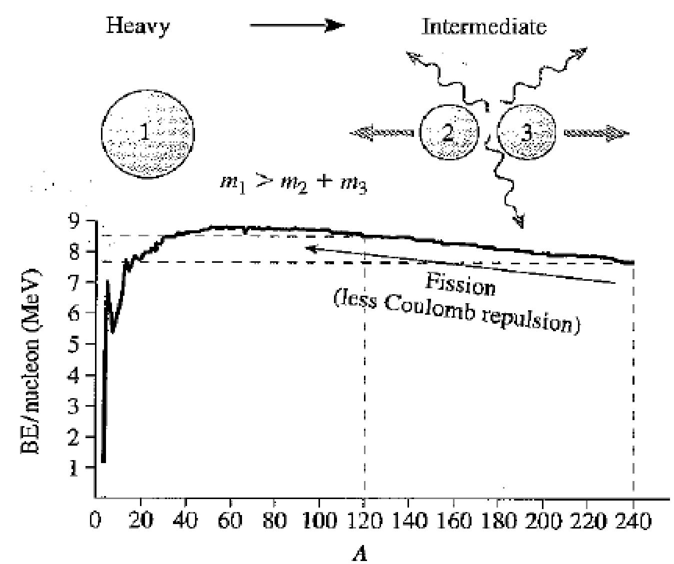
How much energy can be released if we assume the bonding energy difference between A=240 nuclei and A=120 nuclei is about 0.9 MeV?

Nuclear fission

A heavy nucleus breaks into smaller nuclei

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n$$

Figure 11.27 Decreasing BE/nucleon via fission.



What is the driving force?

Why neutrons are released?

How much energy can be released if we assume the bonding energy difference between A=240 nuclei and A=120 nuclei is about 0.9 MeV?

At the order of 200 MeV….consider that
-typical chemical reactions are at the order of 1-10 eV.
-spontaneous decays are at the order of a few MeV

Liquid drop model: Fission

When a large nucleus is excited…

FIGURE 24.11 The oscillations of a liquid drop.

Surface tension due to strong force vs. Repulsive Coulomb force

Will balance as gamma rays are emitted and
the nucleus finds a stable energy state.

Liquid drop model: Fission

FIGURE 24.11 The oscillations of a liquid drop.

When a large nucleus is TOO excited…

FIGURE 24.12 Nuclear fission according to the liquid-drop model.

Surface tension due to strong force < Repulsive Coulomb force

How to excite a large nucleus? Shoot highly energetic neutrons!!!

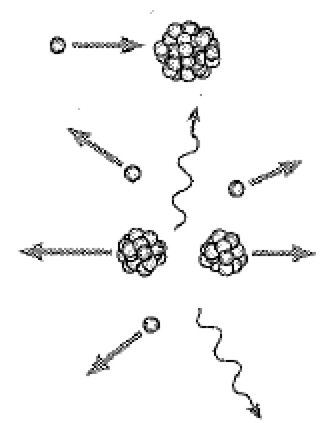
Fission: Chain reaction

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n$$

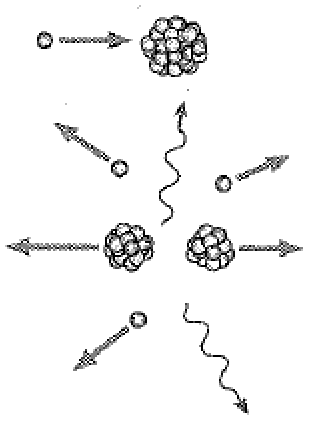
Figure 11.28 Neutron-induced fission, freeing more neutrons.



Fission: Chain reaction

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n$$

Figure 11.28 Neutron-induced fission, freeing more neutrons.



Since each nuclear fission reaction generates highly energetic neutrons, each of which also can induce another fission reaction with another large nucleus.

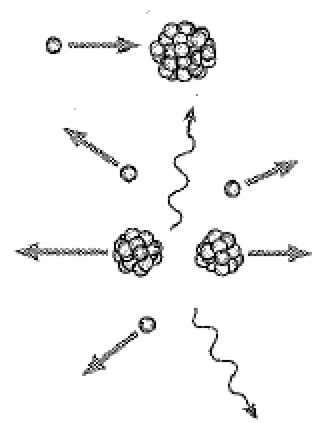
$$E_j = E_0 n^j$$

where E_0 is energy released for the fission reaction
n neutrons are generated in each fission
By the jth time, released energy can be up to E_j

Fission: Chain reaction

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n$$

Figure 11.28 Neutron-induced fission, freeing more neutrons.



Since each nuclear fission reaction generates highly energetic neutrons, each of which also can induce another fission reaction with another large nucleus.

$$E_j = E_0 n^j$$

where E_0 is energy released for the fission reaction
n neutrons are generated in each fission
By the jth time, released energy can be up to E_j

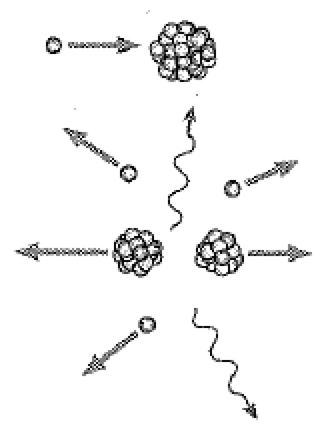
$$\boxed{E_j = E_0 k^j}$$

Since there are multiple fission pathways, consider k as a net number of neutrons generated, then

Fission: Chain reaction

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n$$
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n$$

Figure 11.28 Neutron-induced fission, freeing more neutrons.



Since each nuclear fission reaction generates highly energetic neutrons, each of which also can induce another fission reaction with another large nucleus.

$$E_j = E_0 n^j$$

where E_0 is energy released for the fission reaction
n neutrons are generated in each fission
By the jth time, released energy can be up to E_j

k=1
k<1
k>1

$$E_j = E_0 k^j$$

Since there are multiple fission pathways, consider k as a net number of neutrons generated, then

Nuclear fusion

Small nuclei fuse to become a larger nucleus

Example:

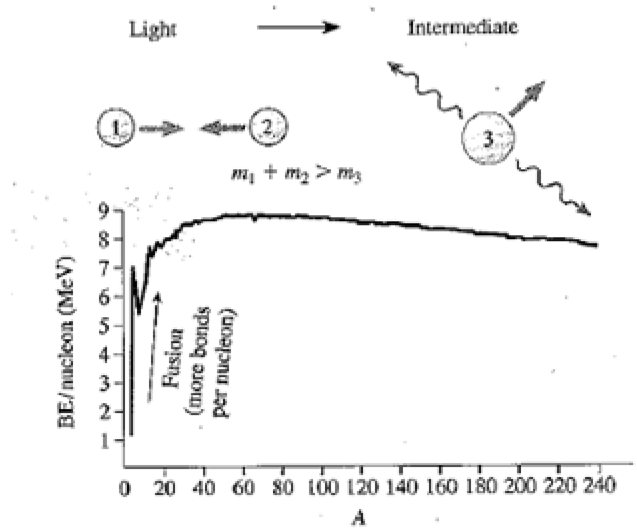
$$^2_1H + ^1_1H \rightarrow ^3_2He + \gamma$$

$Q =$

What is the driving force?

Is chain reaction possible?

Figure 11.30 Decreasing BE/nucleon via fusion.



Nuclear fusion

Small nuclei fuse to become a larger nucleus

Example:

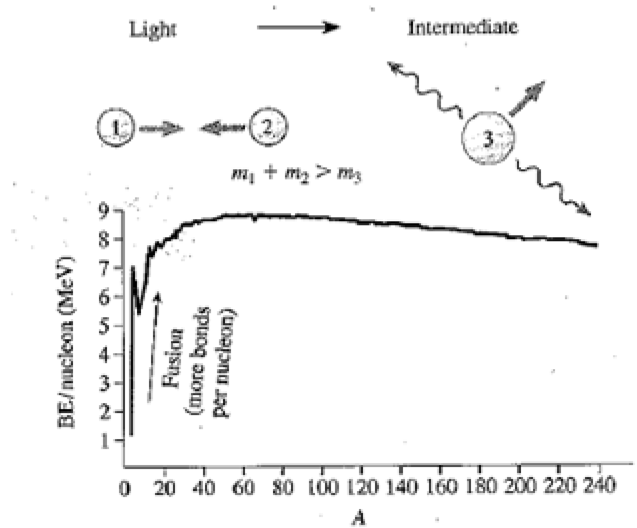
$$^2_1H + ^1_1H \rightarrow ^3_2He + \gamma$$

$$Q = (2.0141 + 1.0078 - 3.0160)uc^2$$
$$= 5.48\ MeV$$

What is the driving force?

Is chain reaction possible?

Figure 11.30 Decreasing BE/nucleon via fusion.



Proton-Proton cycle

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + _{+1}^{0}\beta + \nu$ (Q=0.42 MeV)
Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Proton-Proton cycle

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + _{+1}^{0}\beta + \nu$ (Q=0.42 MeV)
Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + _{+1}^{0}\beta + \nu$ (Q=0.42 MeV)
Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Proton-Proton cycle

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{\ 0}_{+1}\beta + \nu$ (Q=0.42 MeV)

Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Step 3: $^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H$ (Q=12.9 MeV)

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{\ 0}_{+1}\beta + \nu$ (Q=0.42 MeV)

Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Net Results:

Proton-Proton cycle

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{+1}\beta + \nu$ (Q=0.42 MeV)

Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Step 3: $^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H$ (Q=12.9 MeV)

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{+1}\beta + \nu$ (Q=0.42 MeV)

Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Net Results: Ins and Outs

Proton-Proton cycle

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{+1}\beta + \nu$ (Q=0.42 MeV)

Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Step 3: $^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H$ (Q=12.9 MeV)

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{+1}\beta + \nu$ (Q=0.42 MeV)

Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Net Results:

4 protons are fused to create one He nucleus
Two positrons and two neutrinos are generated
Total energy generated = 24.7 MeV

Proton-Proton cycle

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{+1}\beta + \nu$ (Q=0.42 MeV)

Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Step 3: $^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H$ (Q=12.9 MeV)

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{+1}\beta + \nu$ (Q=0.42 MeV)

Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

Step 1 is a relatively slow process, why?

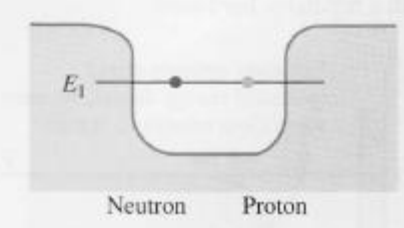


Figure 11.5 The deuteron's neutron and proton bound in a well resulting from their attractive potential energy.

Carbon cycle

He nuclei in abundance can be fused to create Carbon. Once Carbon is available,

Step 1: $^{1}_{1}H + ^{12}_{6}C \rightarrow ^{13}_{7}N$

Step 2: $^{13}_{7}N \rightarrow ^{13}_{6}C + ^{0}_{+1}\beta + \nu$

Step 3: $^{1}_{1}H + ^{13}_{6}C \rightarrow ^{14}_{7}N$

Step 4: $^{1}_{1}H + ^{14}_{7}N \rightarrow ^{15}_{8}O$

Step 5: $^{15}_{8}O \rightarrow ^{15}_{7}N + ^{0}_{+1}\beta + \nu$

Step 6: $^{1}_{1}H + ^{15}_{7}N \rightarrow ^{12}_{6}C + ^{4}_{2}He$

Net effect:

Carbon cycle

He nuclei in abundance can be fused to create Carbon. Once Carbon is available,

Step 1: $^{1}_{1}H + ^{12}_{6}C \rightarrow ^{13}_{7}N$

Step 2: $^{13}_{7}N \rightarrow ^{13}_{6}C + ^{0}_{+1}\beta + \nu$

Step 3: $^{1}_{1}H + ^{13}_{6}C \rightarrow ^{14}_{7}N$

Step 4: $^{1}_{1}H + ^{14}_{7}N \rightarrow ^{15}_{8}O$

Step 5: $^{15}_{8}O \rightarrow ^{15}_{7}N + ^{0}_{+1}\beta + \nu$

Step 6: $^{1}_{1}H + ^{15}_{7}N \rightarrow ^{12}_{6}C + ^{4}_{2}He$

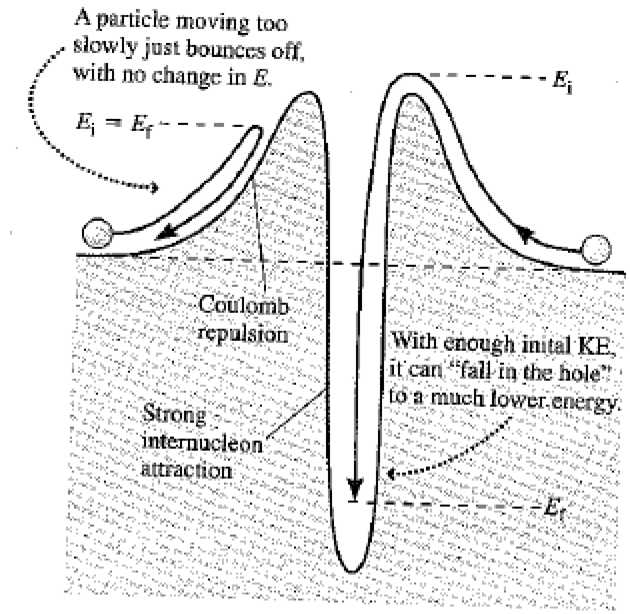
Net effect:

4 protons are fused to create one He nucleus
Two positrons and two neutrinos are generated

Spontaneous fusion not possible!

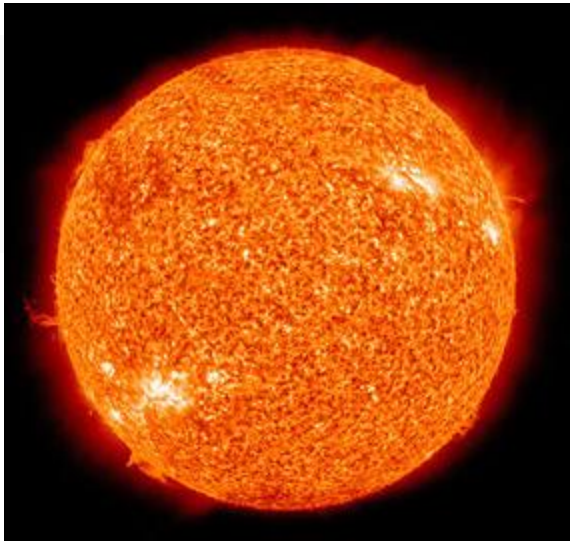
Increase in Z means that
-protons will need to be fused or added to existing nucleus.
-Coulomb repulsion should be overcome before both nuclei are bound by strong force.

Figure 11.32 Nuclear fusion: over the Coulomb hurdle, then into the strong force well.



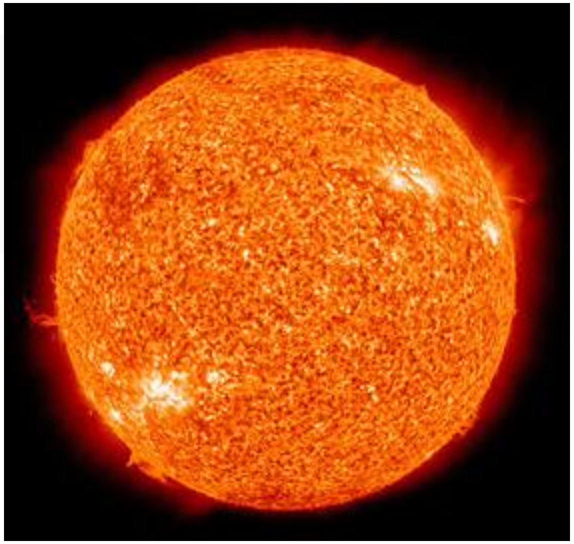
Fusion does not occur in ordinary conditions

Temperature	Center (modeled): ~1.57×10^7 K [1] Photosphere (effective): 5,778 K [1] Corona: ~5×10^6 K



Fusion does not occur in ordinary conditions

Temperature	Center (modeled): ~1.57×10^7 K [1] Photosphere (effective): 5,778 K [1] Corona: ~5×10^6 K



For nuclear fusion to occur in a Hydrogen Bomb, an atomic bomb is initially detonated.

Fusion/Fission: Clear energy solutions?

	Fission	**Fusion**
Fuel	Uranium and Thorium: Not rare and should be mined $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$ $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n$ $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n$	Deuterium: Abundant and non-toxic $^{2}_{1}D + ^{3}_{1}T \rightarrow ^{4}_{2}He + ^{1}_{0}n \quad Q=17.6\ \mathrm{MeV}$
Waste	Highly toxic Radioactive with long half-life Disposal is a problem	He isotopes (harmless) and Tritium (radioactive with short half life and not chemically hazardous)
Chain reaction	Chain reactions are possible, thus the fission process should be controlled	Chain reactions are not possible.

Elementary particles

What are the fundamental building blocks of the Universe (living and nonliving)?

Elementary particles

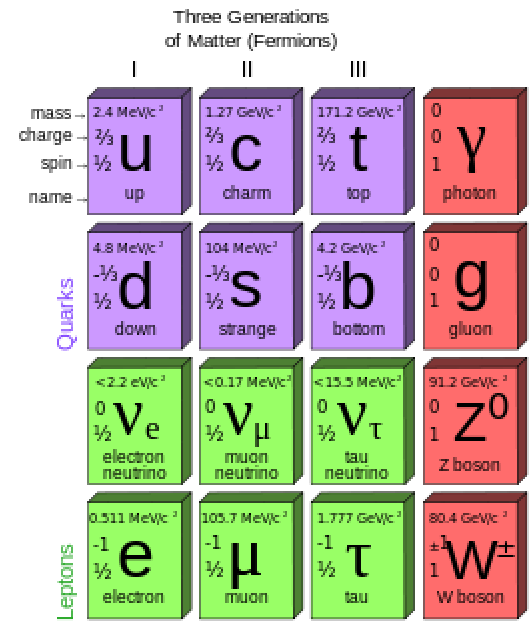
What are the fundamental building blocks of the Universe (living and nonliving)?

-Molecules
-Atoms → Elements
-Nucleus + electrons
-Nucleons (protons and neutrons) in the nucleus
-Quarks in nucleons

Elementary particles

What are the fundamental building blocks of the Universe (living and nonliving)?

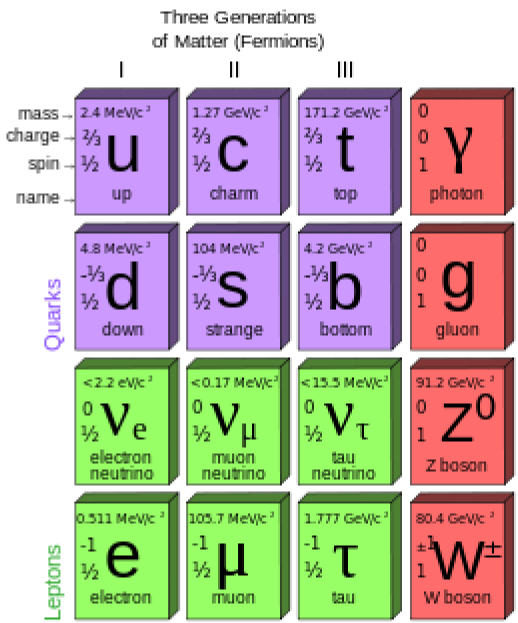
- -Molecules
- -Atoms → Elements
- -Nucleus + electrons
- -Nucleons (protons and neutrons) in the nucleus
- -Quarks in nucleons



Elementary particles

What are the fundamental building blocks of the Universe (living and nonliving)?

-Molecules
-Atoms → Elements
-Nucleus + electrons
-Nucleons (protons and neutrons) in the nucleus
-Quarks in nucleons



Three classes of particles:
- Six quarks → strong force
- Six leptons → electroweak force
- Four mediating particles called field quanta
  - → Graviton: gravitational force
  - → Photon: electromagnetic force
  - → W^+, W^-, Z^0: weak force
  - → Gluon: strong force

New definition for force

Force is exerted by exchanging a mediating particle.

Anna is throwing a snowball on a frictionless frozen pond.

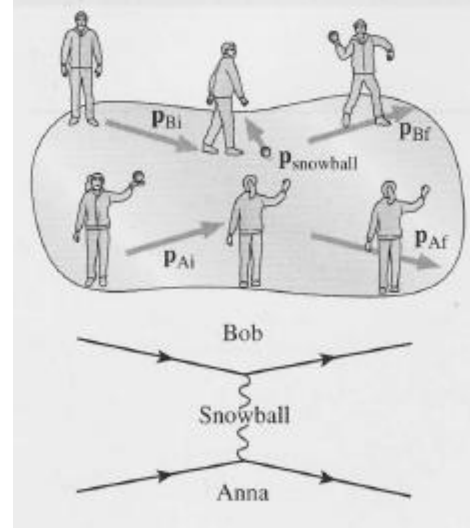


Figure 12.1 A force between students conveyed by exchange of a snowball.

New definition for force

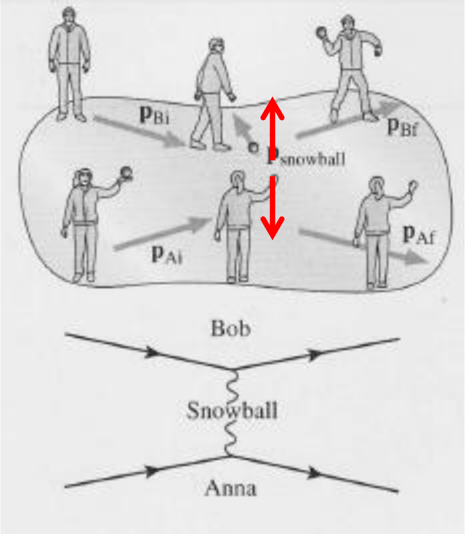
Force is exerted by exchanging a mediating particle.

Anna is throwing a snowball on a frictionless frozen pond.

$$\vec{p}_{Anna\,(after\ snowball\ throwing)} = \vec{p}_{Anna} - \vec{p}_{snowball}$$

$$\vec{p}_{Bob\,(after\ snowball\ receiving)} = \vec{p}_{Bob} + \vec{p}_{snowball}$$

Figure 12.1 A force between students conveyed by exchange of a snowball.



New definition for force

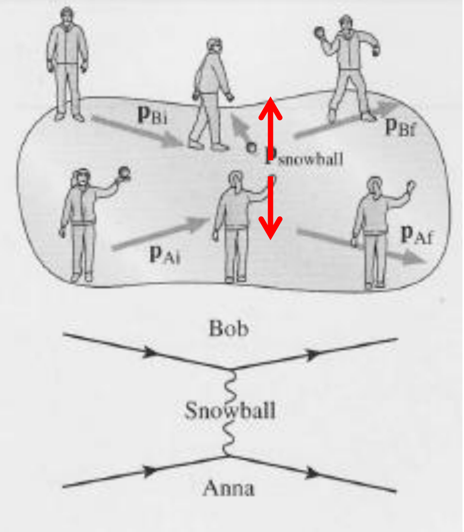
Force is exerted by exchanging a mediating particle.

Anna is throwing a snowball on a frictionless frozen pond.

$$\vec{p}_{Anna\,(after\ snowball\ throwing)} = \vec{p}_{Anna} - \vec{p}_{snowball}$$

$$\vec{p}_{Bob\,(after\ snowball\ receiving)} = \vec{p}_{Bob} + \vec{p}_{snowball}$$

Figure 12.1 A force between students conveyed by exchange of a snowball.



Limitations of snowball analogy:

-Attractive fundamental force cannot be shown

-Mediating particles exist during the exchange, not before or after

-Mediating particles do not act like snowball (classical physical entities)

Force range

From uncertainty principle

$$\Delta t \Delta E \approx \hbar \longrightarrow \Delta t \approx \frac{\hbar}{\Delta E}$$

Energy of the mediating particle

$$\Delta E = mc^2$$

$$\Delta t \approx \frac{\hbar}{\Delta E} \approx \frac{\hbar}{mc^2}$$

Force range

$$\Delta x \approx c\Delta t \approx \frac{\hbar}{c}\frac{1}{m}$$

Force range

From uncertainty principle

$$\Delta t \Delta E \approx \hbar \quad \longrightarrow \quad \Delta t \approx \frac{\hbar}{\Delta E}$$

Energy of the mediating particle

$$\Delta E = mc^2$$

$$\Delta t \approx \frac{\hbar}{\Delta E} \approx \frac{\hbar}{mc^2}$$

Force range

$$\Delta x \approx c\Delta t \approx \frac{\hbar}{c}\frac{1}{m}$$

- When the mediating particle has mass
- When it doesn't

Force range

TABLE 12.1 Fundamental forces and particles

Force	Gravitation	Electroweak			Strong	Residual
Property	Mass/energy	Charge/weak charge			Color charge	
Strength	$\sim 10^{-39}$	$\sim 10^{-2}$	$\sim 10^{-6}$		1	
Range	$1/r^2$	$1/r^2$	10^{-3} fm		short	1 fm
Mediating Bosons	Graviton?	Photon, γ	W^+, W^-	Z^0	Gluon	$\pi^\pm, \pi^0$
Spin	2?	1	1	1	1	0
Mass	0?	$< 6 \times 10^{-22}$	80.4×10^3	91.2×10^3	< 10	140, 135
Charge	—	0	+1, −1	0	0	$\pm 1, 0$
Color charge	—	—	—	—	r, g, or b + $\bar{r}$, $\bar{g}$, or $\bar{b}$	Neutral

$$\Delta x \approx c\Delta t \approx \frac{\hbar}{c}\frac{1}{m}$$

- When the mediating particle has mass
- When it doesn't

Antiparticles

- Antiparticle: particle with the same properties except charge. p and $\bar{p}$, n and $\bar{n}$
  e^- and e^+, μ^+ and μ^-.
- Experimentally detected.
- Particle-antiparticle can be produced and disappear simultaneously with the energy involved.

Schrodinger Equation

Hamiltonian operator (H)

$$H\Psi(x,t) = E\Psi(x,t)$$

Since H = Total Energy = Kinetic energy (T) + Potential energy (U)

$$\left(\frac{p^2}{2m} + U\right)\Psi(x,t) = E\Psi(x,t)$$

$$p = \frac{\hbar}{i}\frac{\partial}{\partial x} \quad \text{and} \quad \mathrm{E} = i\hbar\,\frac{\partial}{\partial t}$$

Time-Dependent Schrodinger Equation

$$\frac{-\hbar^2}{2m}\,\frac{\partial^2\Psi\,(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\,\frac{\partial\,\Psi(x,t)}{\partial t}$$

Klein-Gordon Equation

$$E = mc^2$$

Klein-Gordon Equation

$$E = mc^2$$

$$E^2 = m^2c^4 = \frac{{m_0}^2}{1 - \frac{v^2}{c^2}}c^4$$

$$= {m_0}^2c^4\left(\frac{1}{1-\frac{v^2}{c^2}}\right) = {m_0}^2c^4\left(\frac{1}{1-\frac{v^2}{c^2}} - 1 + 1\right)$$

$$= {m_0}^2c^4 + {m_0}^2c^4\left(\frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}\right) = {m_0}^2c^4 + c^2v^2\frac{{m_0}^2}{1-\frac{v^2}{c^2}}$$

$$= {m_0}^2c^4 + c^2v^2m^2 = {m_0}^2c^4 + p^2c^2$$

$$E^2 = {m_0}^2c^4 + p^2c^2$$

Klein-Gordon Equation

$$E^2 = {m_0}^2 c^4 + p^2 c^2$$

$$E^2 \psi = {m_0}^2 c^4 \psi + p^2 c^2 \psi$$

Using operator notations
$p = -i\hbar\nabla$ and $E = i\hbar \frac{\partial}{\partial t}$

$$-c^2 \hbar^2 \nabla^2 \psi + {m_0}^2 c^4 \psi = -\hbar^2 \frac{\partial^2}{\partial t^2} \psi$$

Klein-Gordon equation works for spinless particles
Use Dirac equation for particles with spin

Schrodinger vs. Klein-Gordon Eq

	Schrodinger Eq (free particle)	Klein-Gordon Eq (free particle)
Energy relation	$E = T$	$E^2 = m_0{}^2c^4 + p^2c^2$
Equation	$-\frac{\hbar^2}{2m}\nabla^2\,\psi = i\hbar\frac{\partial\psi}{\partial t}$	$-c^2\hbar^2\nabla^2\psi + m_0{}^2c^4\psi = -\hbar^2\frac{\partial^2}{\partial t^2}\psi$
Energy	$E = \frac{p^2}{2m}$, E is positive	Positive and negative
$\psi^*\psi$	Probability density Does not change with time	If formulated in such a way that it does not change with time, it represents charge density and both positive and negative values are possible: positive for particle and negative for antiparticle (homework: chapter 12 #15)

Particle and antiparticle

The antiparticle state is similar to a hole in a sea of allowed but filled negative energy states of the particle.

-If filled, the antiparticle state cannot be observed.

-When a hole is available, a particle can be annihilated, emitting photon energy twice of the particle's energy.

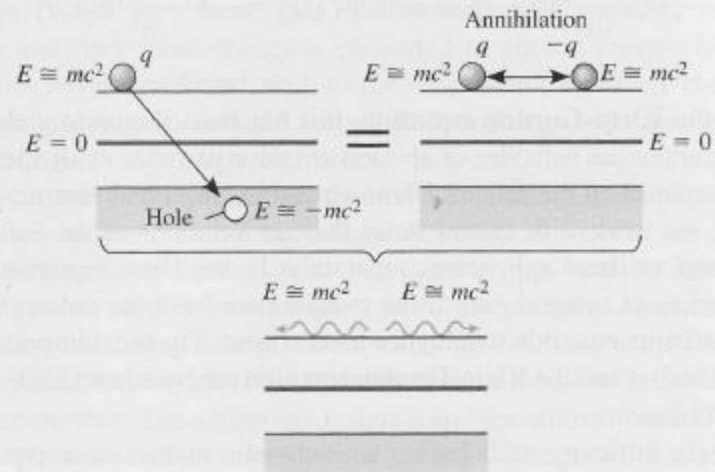


Figure 12.2 Whether a positive-energy particle drops into an empty negative-energy state, or two positive-energy particles annihilate, the result is the same.