Lecture 17 Topics

- Nuclear models
  - Liquid Drop model
  - Shell model
- Radioactive decay
  - Alpha decay (α = He nucleus)
  - Beta decay: β^+ decay, β^- decay, electron capture
  - Gamma decay (γ): photon

Binding energy

$$\text{Binding energy} = (\text{mass of individual nucleons} - \text{mass of nucleus})c^2$$

$$\text{Deuteron} = 1 \text{ proton} + 1 \text{ neutron}$$

$$\text{Binding Energy} = (\text{proton mass} + \text{neutron mass} - \text{Deuteron mass})c^2$$

m_P =1.007276 u
m_N=1.008665 u
m_D=2.013553 u
uc^2=931.5 MeV

Binding energy

$$Binding\ Energy = \left(Zm_H + Nm_n - M_{^A_ZX}\right)c^2$$

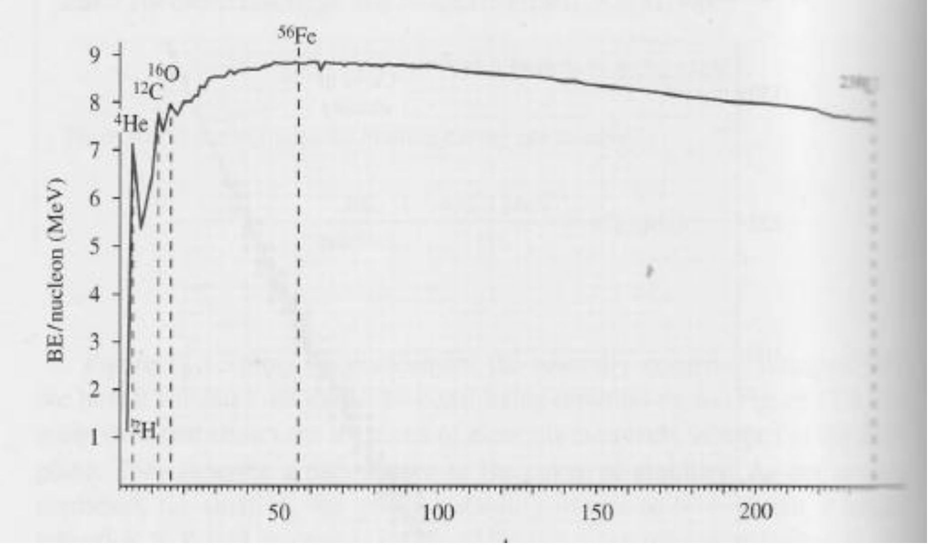
Where m_H= atomic mass of hydrogen

m_n= neutron mass

$M_{^A_ZX}$ = atomic mass of the nucleus

Binding energy/nucleon vs. A

Figure 11.14 Binding energy per nucleon versus A.



Magic numbers= 2, 8, 20, 28, 50, 82, 126

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

$$C_1 = 15.8$$
$$C_2 = 17.8$$
$$C_3 = 0.71$$
$$C_4 = 23.7$$

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

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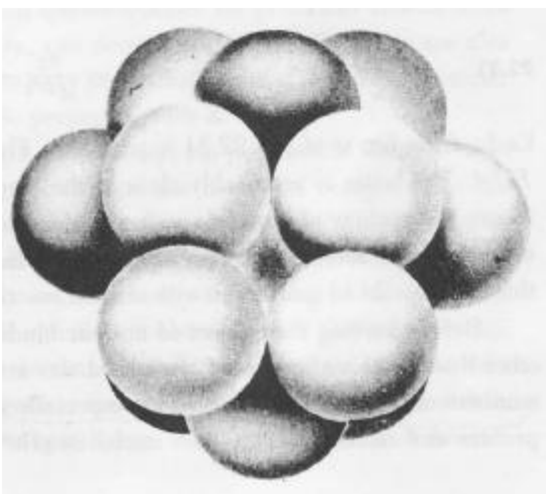

$C_1 = 15.8$
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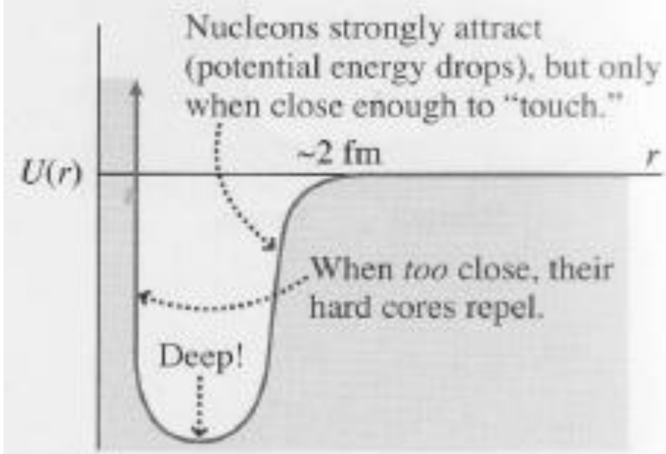
$C_4 = 23.7$

Figure 11.4 The basic elements of the internucleon (strong force) potential energy—a strong, short-range attraction with a repulsive hard core.



Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

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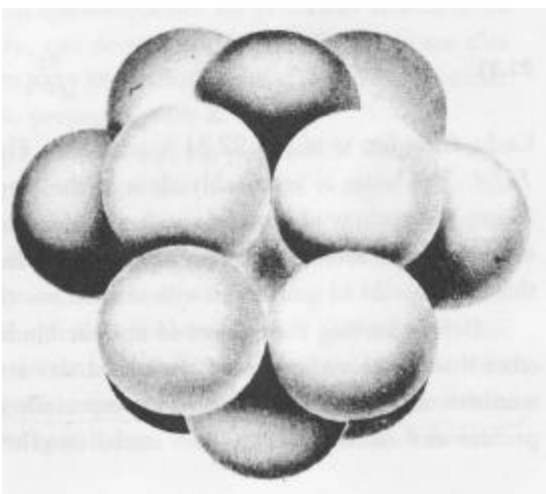
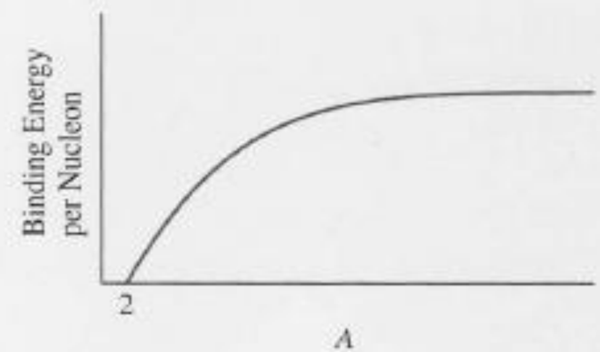


$C_1 = 15.8$
$C_2 = 17.8$
$C_3 = 0.71$
$C_4 = 23.7$

Figure 11.6 Binding energy per nucleon due to the strong internucleon attraction only. The smallest nuclei have few bonds per nucleon. In large nuclei, many nucleons are surrounded.



Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

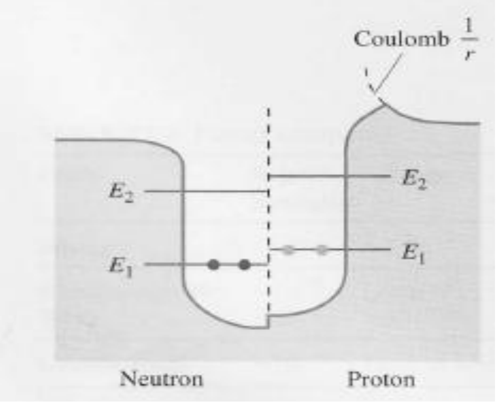


Figure 11.7 Coulomb repulsion raises proton energies.

$$C_1 = 15.8$$
$$C_2 = 17.8$$
$$C_3 = 0.71$$
$$C_4 = 23.7$$

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

$C_1 = 15.8$
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Figure 11.7 Coulomb repulsion raises proton energies.

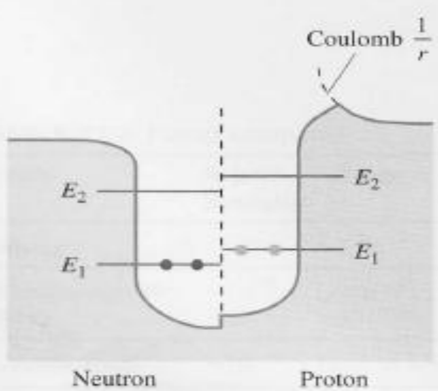
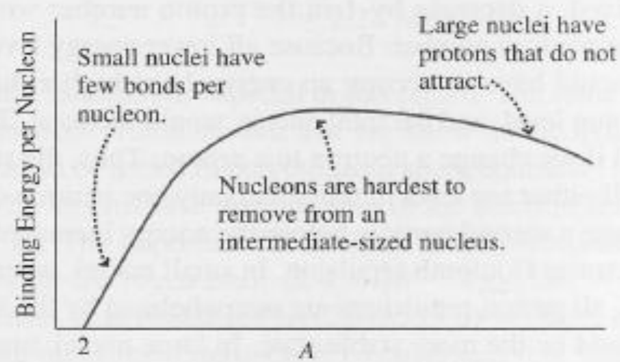


Figure 11.8 Binding energy per nucleon due to both the strong internucleon attraction and Coulomb repulsion.

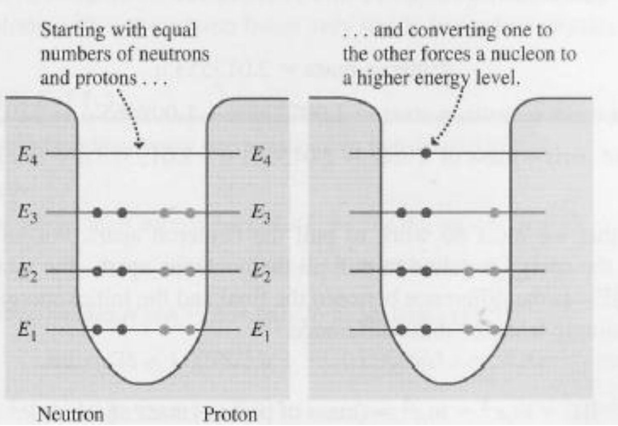


Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

Figure 11.9 Ignoring Coulomb repulsion, the exclusion principle argues that for a given number of nucleons, the lowest energy should have $N = Z$.



$$C_1 = 15.8$$
$$C_2 = 17.8$$
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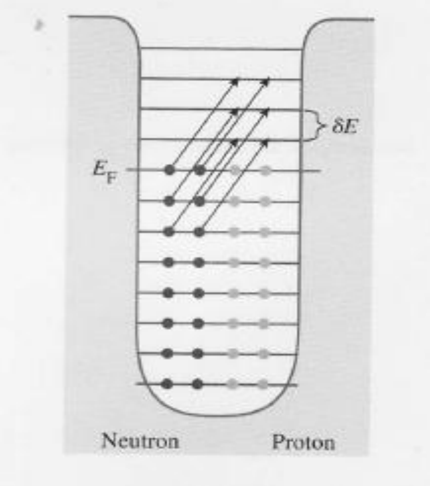
$$C_4 = 23.7$$

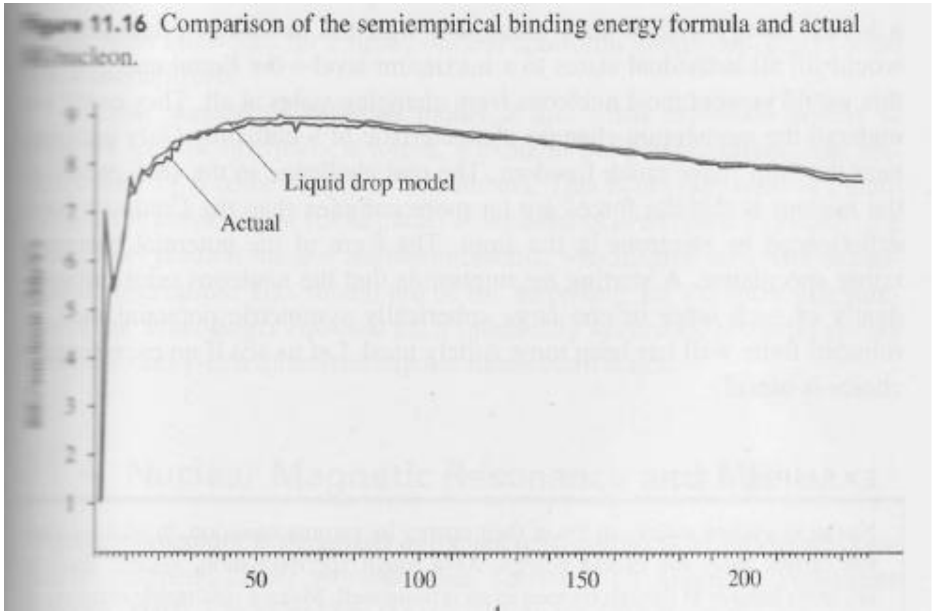
Figure 11.15 If j neutrons become protons, the energy increases by $\frac{1}{2} j^2 \delta E$.



Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

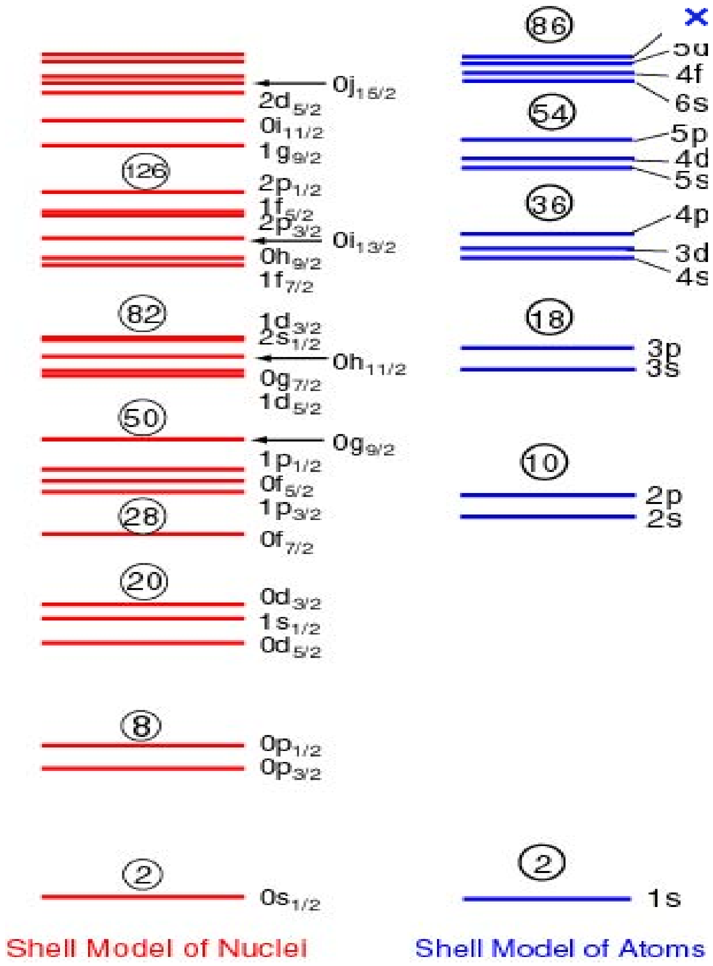


Magic numbers= 2, 8, 20, 28, 50, 82, 126

Shell Model

	Atomic Shell Model	**Nuclear Shell Model**
Potential	Electrostatic between nucleus and electrons	Net effect of all the forces nucleons experience in a nucleus
Magic numbers	Atoms with closed electronic shells are stable such as He (2 electrons), Ne (10), Ar (18), Kr (36), Xe (54), Rn (86).	Nuclei are particularly stable when the number of nucleons is 2, 8, 20, 28, 50, 82, and 126.
Exclusion principle	Electrons follow the Exclusion principle	Protons and neutrons separately follow the Exclusion Principle
Movement	Electrons move in orbitals	Nucleons do not move like electrons because most nucleons fill states to a maximum level, preventing them from changing momentum.

Electrons: Atomic Orbital
Nucleons: Nuclei Orbital



3 dimensional harmonic oscillator

$$\frac{-\hbar^2}{2m}\nabla^2\psi(x) + \frac{1}{2}\mu\omega^2 x^2\,\psi(x) = \mathrm{E}\,\psi(x)$$

3 dimensional harmonic oscillator

$$\frac{-\hbar^2}{2m}\nabla^2\psi(x) + \frac{1}{2}\mu\omega^2 x^2\,\psi(x) = \mathrm{E}\,\psi(x)$$

$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

$n \equiv 2k + l$

- For every even $n, l = 0, 2, \ldots n-2, n$
- For every odd $n, l = 1, 3, \ldots n-2, n$
- $-l \leq m \leq l$
- Every n and l, there are $2l + 1$ energy degeneracies, which can accommodate $2(2l + 1)$ nucleons

3 dimensional harmonic oscillator

$$n \equiv 2k + l$$

even $n, l \; = \; 0, 2, \ldots n-2, n$

odd $n, l = \; 1, 3, \ldots n-2, n$

$$E = \hbar\omega \left(2k + l + \frac{3}{2}\right)$$

n	k	l	Energy
0	0	0	$\frac{3}{2}\hbar\omega$
1	0	1	$\frac{5}{2}\hbar\omega$
2	0	2	$\frac{7}{2}\hbar\omega$
	1	0	
3	0	3	$\frac{9}{2}\hbar\omega$
	1	1	
4	0	4	$\frac{11}{2}\hbar\omega$
	1	2	
	2	0	
5	0	5	$\frac{13}{2}\hbar\omega$
	1	3	
	2	1	

Not Magic Numbers!!!

3 dimensional harmonic oscillator

$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

$$n \equiv 2k + l$$

2(2l+1)

n	k	l	No. of nucleons in (n, k, l)	Energy
0	0	0	2	$\frac{3}{2}\hbar\omega$
1	0	1	6	$\frac{5}{2}\hbar\omega$
2	0	2	10	$\frac{7}{2}\hbar\omega$
	1	0	2	
3	0	3	14	$\frac{9}{2}\hbar\omega$
	1	1	6	
4	0	4	18	$\frac{11}{2}\hbar\omega$
	1	2	10	
	2	0	2	
5	0	5	22	$\frac{13}{2}\hbar\omega$
	1	3	14	
	2	1	6	

Not Magic Numbers!!!

3 dimensional harmonic oscillator

$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

2(2l+1)

$$n \equiv 2k + l$$

n	k	l	No. of nucleons in (n, k, l)	No. of nucleons in n	Total nucleons	Energy
0	0	0	2	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	6	6	8	$\frac{5}{2}\hbar\omega$
2	0	2	10	12	20	$\frac{7}{2}\hbar\omega$
	1	0	2			
3	0	3	14	20	40	$\frac{9}{2}\hbar\omega$
	1	1	6			
4	0	4	18	30	70	$\frac{11}{2}\hbar\omega$
	1	2	10			
	2	0	2			
5	0	5	22	42	112	$\frac{13}{2}\hbar\omega$
	1	3	14			
	2	1	6			

Not Magic Numbers!!!

LS coupling

n	k	l	j	Energy
0	0	0	½	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	$\frac{5}{2}\hbar\omega$
2	0	2	5/2 3/2	$\frac{7}{2}\hbar\omega$
	1	0	½	
3	0	3	7/2 5/2	$\frac{9}{2}\hbar\omega$
	1	1	3/2 1/2	
4	0	4	9/2 7/2	$\frac{11}{2}\hbar\omega$
	1	2	5/2 3/2	
	2	0	½	
5	0	5	11/2 9/2	$\frac{13}{2}\hbar\omega$
	1	3	7/2 5/2	
	2	1	3/2 1/2	

LS coupling

2j+1

n	k	l	j	No. of nucleons in (n, j)		Energy
0	0	0	½	2		$\frac{3}{2}\hbar\omega$
1	0	1	3/2	4		$\frac{5}{2}\hbar\omega$
			1/2	2		
2	0	2	5/2	6		$\frac{7}{2}\hbar\omega$
			3/2	4		
	1	0	½	2		
3	0	3	7/2	8		$\frac{9}{2}\hbar\omega$
			5/2	6		
	1	1	3/2	4		
			1/2	2		
4	0	4	9/2	10		$\frac{11}{2}\hbar\omega$
			7/2	8		
	1	2	5/2	6		
			3/2	4		
	2	0	½	2		
5	0	5	11/2	12		$\frac{13}{2}\hbar\omega$
			9/2	10		
	1	3	7/2	8		
			5/2	6		
	2	1	3/2	4		
			1/2	2		

LS coupling

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in (n, $\|\vec{j}\|$)	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2	6	$\frac{5}{2}\hbar\omega$
2	0	2	5/2 3/2	6 4	10	$\frac{7}{2}\hbar\omega$
	1	0	½	2	2	
3	0	3	7/2 5/2	8 6	14	$\frac{9}{2}\hbar\omega$
	1	1	3/2 1/2	4 2	6	
4	0	4	9/2 7/2	10 8	18	$\frac{11}{2}\hbar\omega$
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5	0	5	11/2 9/2	12 10	22	$\frac{13}{2}\hbar\omega$
	1	3	7/2 5/2	8 6	14	
	2	1	3/2 1/2	4 2	6	

LS coupling

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in $(n, \lvert\vec{J}\rvert)$	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2	6	$\frac{5}{2}\hbar\omega$
2	0	2	5/2 3/2	6 4	10	$\frac{7}{2}\hbar\omega$
	1	0	½	2	2	
3	0	3	7/2 5/2	8 6	14	$\frac{9}{2}\hbar\omega$
	1	1	3/2 1/2	4 2	6	
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	1	2	5/2 3/2	6 4	10	
	2	0	½	2	2	
5	0	5	11/2 9/2	12 10	22	$\frac{13}{2}\hbar\omega$
	1	3	7/2 5/2	8 6	14	
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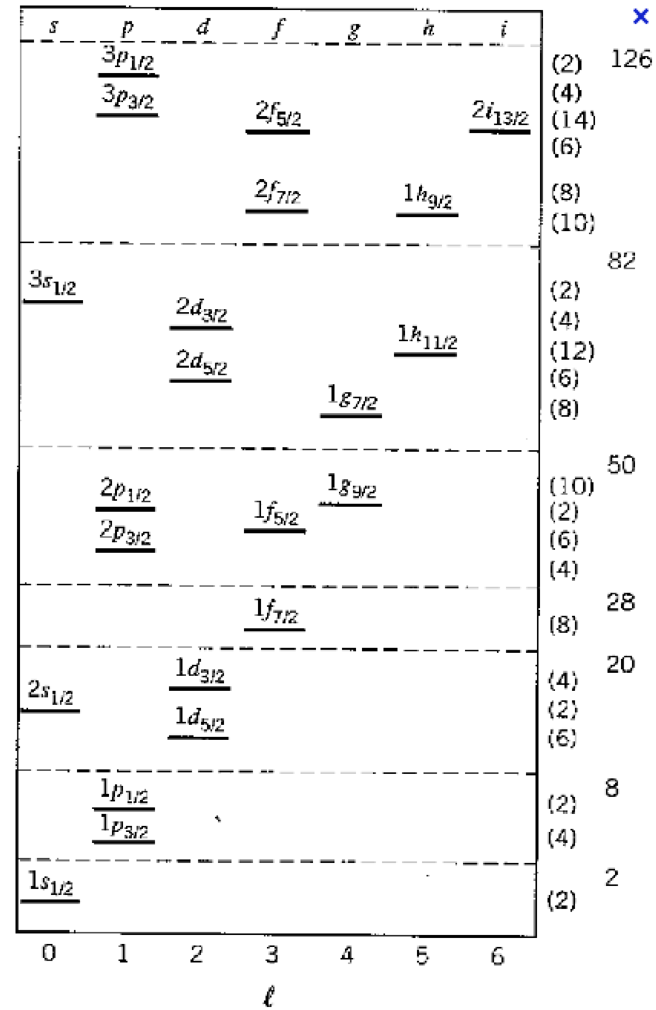

LS coupling

K+1 is the name of energy level

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in (n,$\|\vec{J}\|$)	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2	6	$\frac{5}{2}\hbar\omega$
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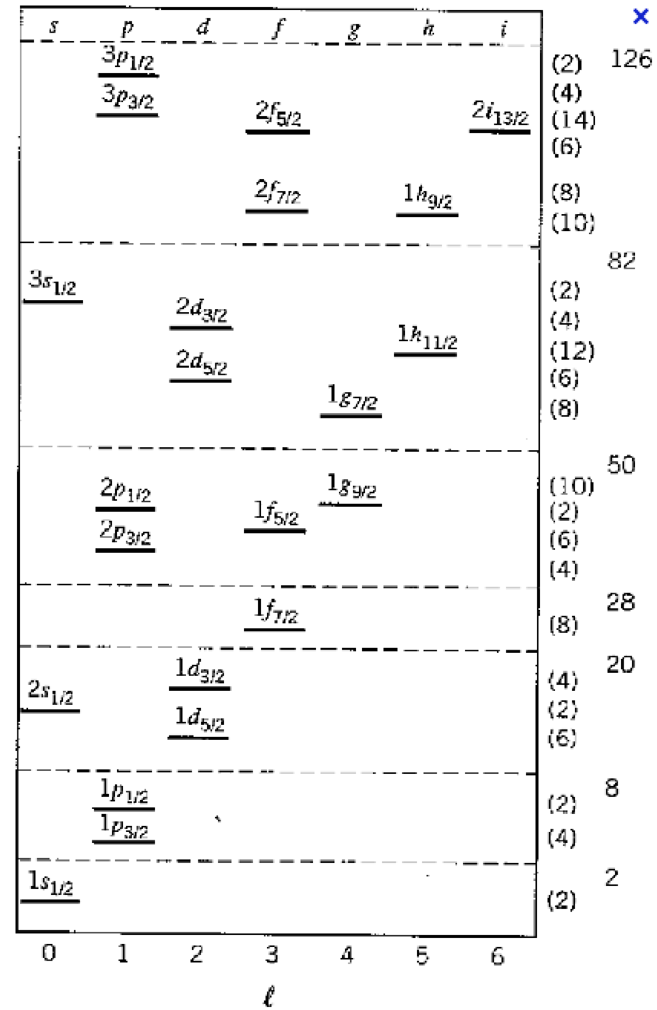

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0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
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	1	2	5/2 3/2	6 4	10	
	2	0	½	2	2	
5	0	5	11/2 9/2	12 10	22	$\frac{13}{2}\hbar\omega$
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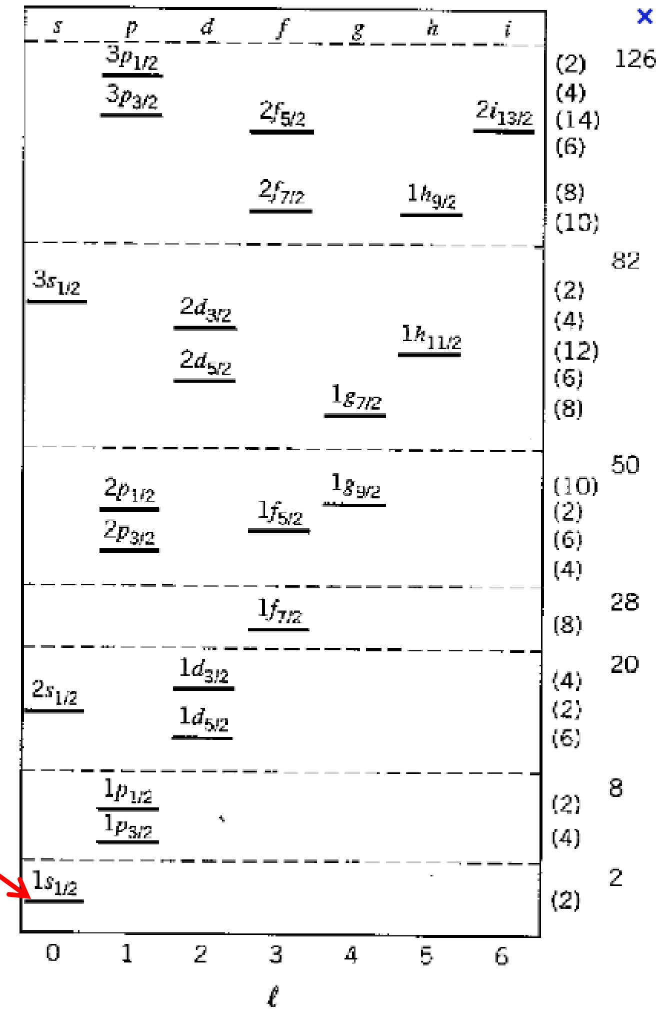

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2	0	2	5/2 3/2	6 4	10	$\frac{7}{2}\hbar\omega$
	1	0	½	2	2	
3	0	3	7/2 5/2	8 6	14	$\frac{9}{2}\hbar\omega$
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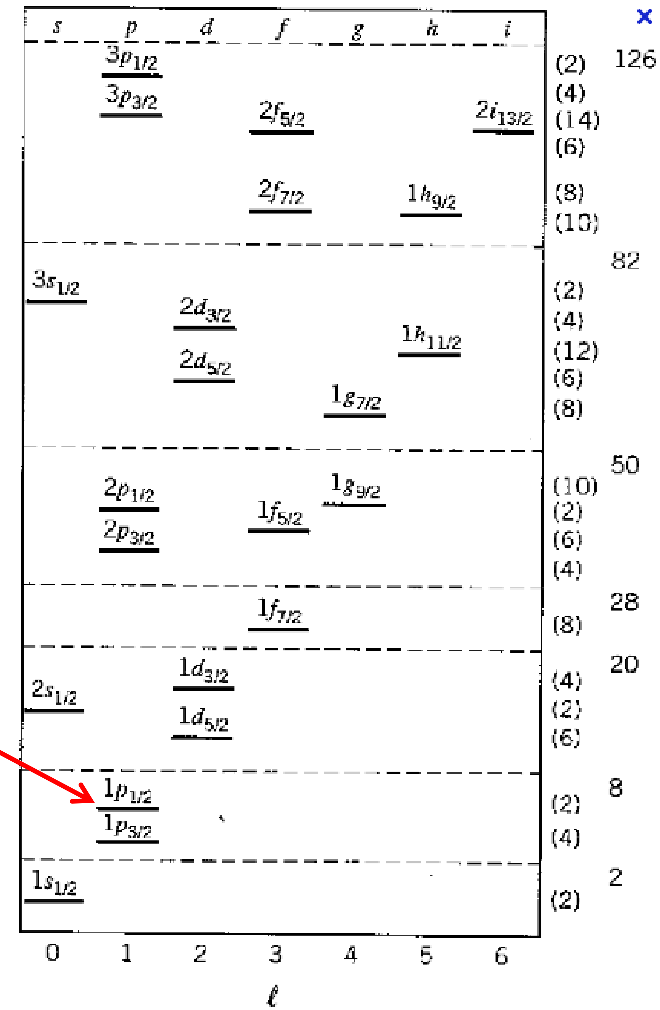

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n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in (n,$\lvert\vec{J}\rvert$)	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
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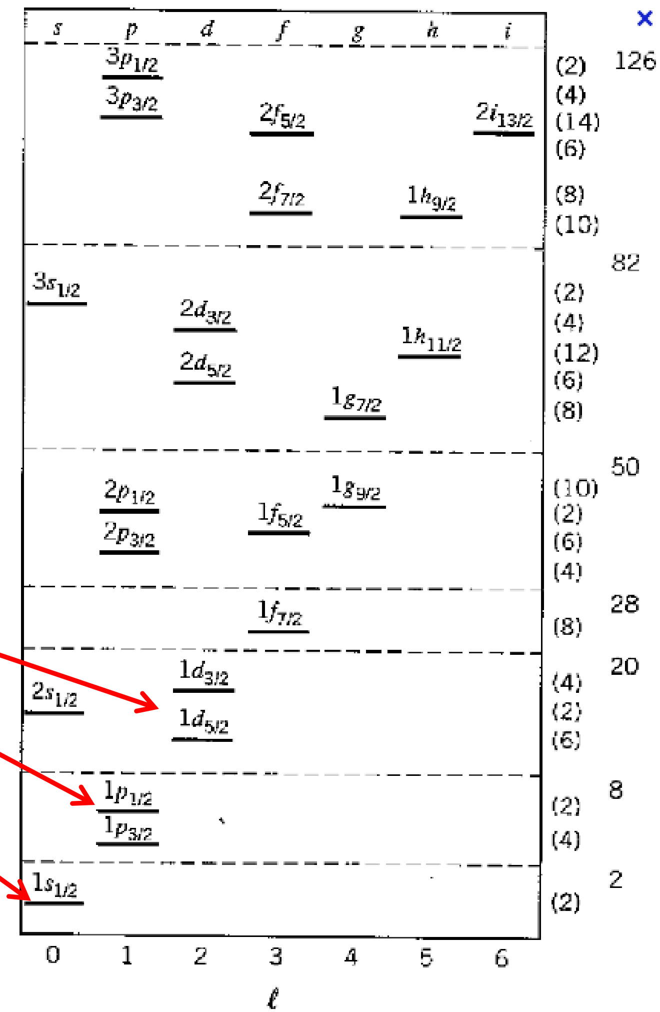
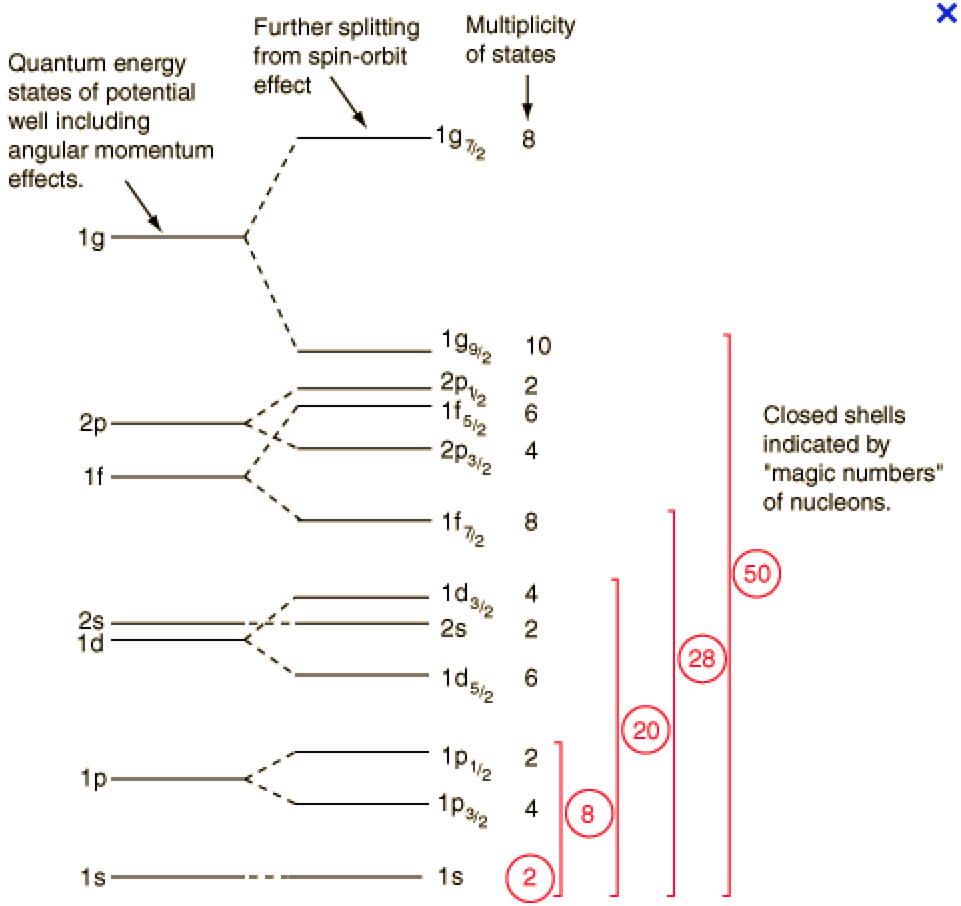
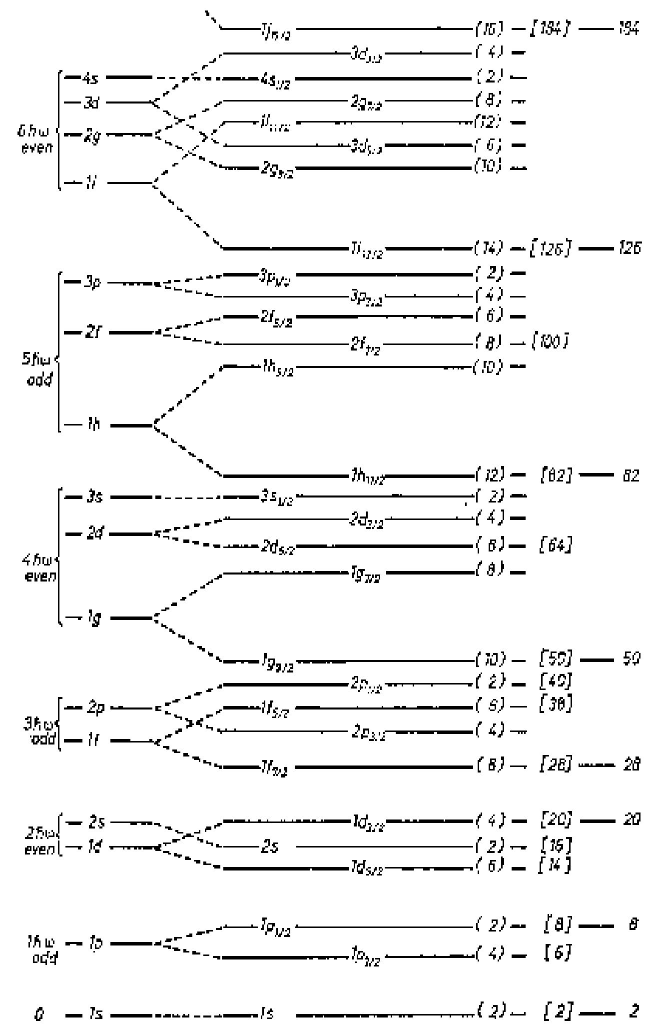
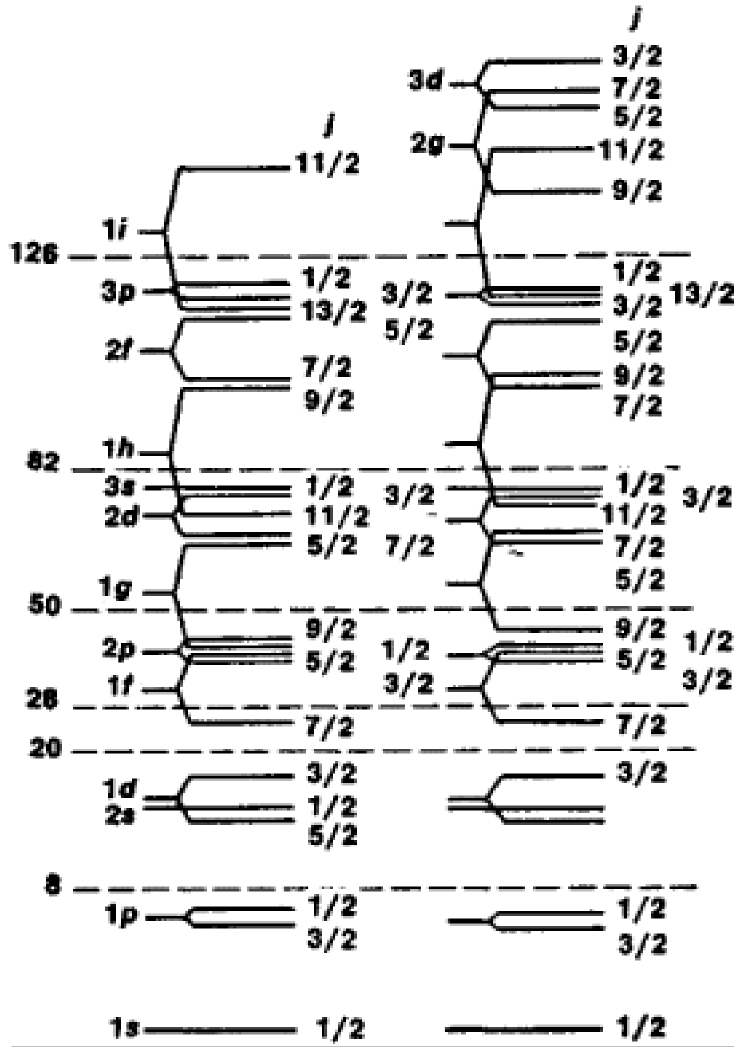



Quantum energy states of potential well including angular momentum effects.
Further splitting from spin-orbit effect
Multiplicity of states
1g
2p
1f
2s
1d
1p
1s
$1g_{7/2}$ 8
$1g_{9/2}$ 10
$2p_{1/2}$ 2
$1f_{5/2}$ 6
$2p_{3/2}$ 4
$1f_{7/2}$ 8
$1d_{3/2}$ 4
2s 2
$1d_{5/2}$ 6
$1p_{1/2}$ 2
$1p_{3/2}$ 4
1s
2
8
20
28
50
Closed shells indicated by "magic numbers" of nucleons.


6ħω even
5ħω odd
4ħω even
3ħω odd
2ħω even
1ħω odd
0
[184] 184
[126] 126
[82] 82
[50] 50
[28] 28
[20] 20
[8] 8
[2] 2



Proton Neutron

Radioactive decay

- An unstable nucleus (parent nucleus) can spontaneously emit small particles or energies to become a nucleus (daughter nucleus) in a more stable state.
  - Alpha decay (α = He nucleus)
  - Beta decay: β^+ decay, β^- decay, electron capture
  - Gamma decay (γ = photons)

Alpha decay

- Emits an alpha particle
- Alpha particle is He nucleus=2 protons +2 neutrons

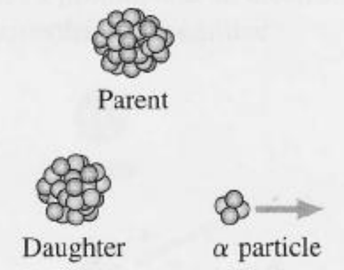


Figure 11.18 Alpha decay.

$$^{A}_{Z}X(parent) \rightarrow {}^{A-4}_{Z-2}Y(daughter) + {}^{4}_{2}He$$

- $Z_{daughter} = Z_{parent} - 2$
- $N_{daughter} = N_{parent} - 2$
- $A_{daughter} = A_{parent} - 4$

Released kinetic energy (Q)

$$Q = (m_{parent} - m_{daughter} - m_{^{4}_{2}He})\, c^2$$

m: atomic mass

$^{238}_{92}U \rightarrow$

Alpha decay

- Emits an alpha particle
- Alpha particle is He nucleus=2 protons +2 neutrons

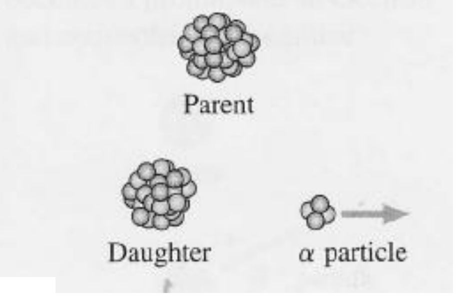


Figure 11.18 Alpha decay.

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Released kinetic energy (Q)

$$Q = (m_{parent} - m_{daughter} - m_{^{4}_{2}He})\,c^2$$

m: atomic mass

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$$

$^{238}_{92}U$ 234.050784 u

$^{234}_{90}Th$ 234.043593 u

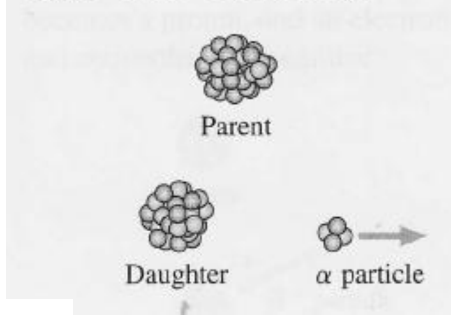
$^{4}_{2}He$ 4.002603 u

Uc^2 =931.5 MeV

Alpha decay

- Emits an alpha particle
- Alpha particle is He nucleus=2 protons +2 neutrons

Figure 11.18 Alpha decay.



$$^{A}_{Z}X(parent) \rightarrow {}^{A-4}_{Z-2}Y(daughter) + {}^{4}_{2}He$$

- $Z_{daughter} = Z_{parent} - 2$
- $N_{daughter} = N_{parent} - 2$
- $A_{daughter} = A_{parent} - 4$

Released kinetic energy (Q)

$$Q = (m_{parent} - m_{daughter} - m_{^{4}_{2}He})\, c^2$$

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

$$Q = (m_{parent} - m_{daughter} - m_{^{4}_{2}He})\, c^2$$
$$= (238.050784 - 234.043593 - 4.002603)uc^2$$
$$= 0.004588 \text{ x } 931.5\, MeV = 4.27\, MeV$$

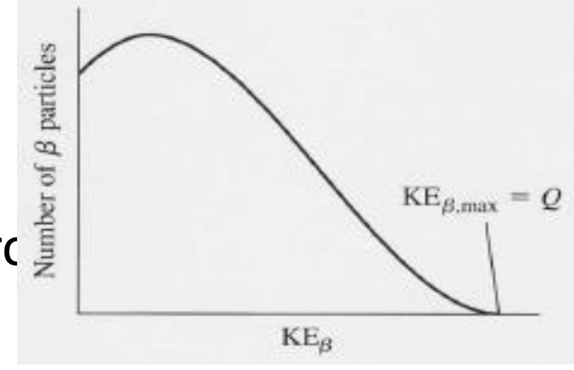
$^{238}_{92}U$	234.050784 u
$^{234}_{90}Th$	234.043593 u
$^{4}_{2}He$	4.002603 u

Uc^2 =931.5 MeV

Beta decay

- Emits β^+, β^- particles (= positron, electron)
- Another particle is involved!
- To satisfy:
  - Energy conservation
    - Beta particles carry kinetic energies fro
      allowed
  - Charge conservation
    - β^+, β^- carries +1 and -1
  - Angular momentum conservation
    - β^+, β^- : spin ½

Figure 11.19 The mysterious variation in β particle energies.



Beta decay

- Emits β^+, β^- particles (= positron, electron)
- Neutrino (ν) is involved!
- To satisfy:
  - Energy conservation
    - Beta particles carry kinetic energies from C
      allowed Neutrino: mass negligible
  - Charge conservation
    - β^+, β^- carries +1 and -1 Neutrino: neutral
  - Angular momentum conservation
    - β^+, β^- : spin ½ Neutrino spin=1/2

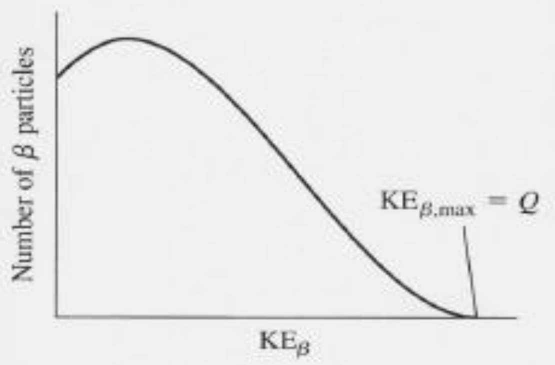
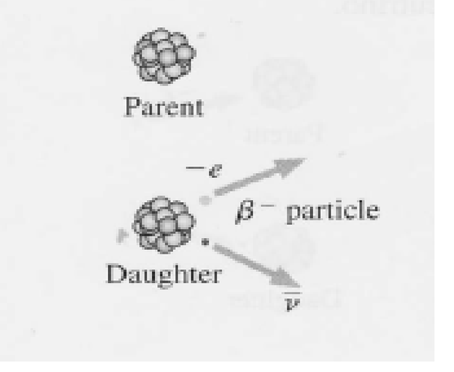


Figure 11.19 The mysterious variation in β particle energies.

β^- decay

- Emits an electron and an anti-neutrino.
- Changes a neutron inside the nucleus into a proton.

$${}^{A}_{Z}X(parent) \rightarrow {}_{Z+1}^{\;\;\;A}Y(daughter) + {}_{-1}^{\;\;0}\beta^- + \bar{\nu}$$



- $Z_{daughter} = Z_{parent} + 1$
- $N_{daughter} = N_{parent} - 1$
- $A_{daughter} = A_{parent}$

Released kinetic energy

$$Q = (m_{parent} - m_{daughter})\, c^2$$

Example:

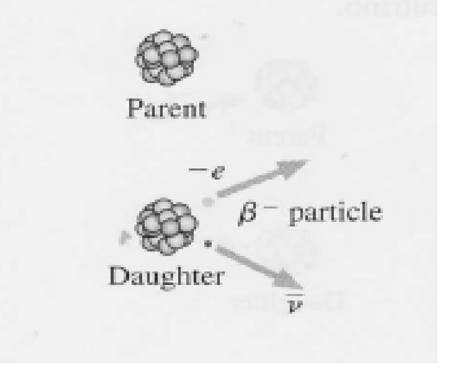
$${}^{12}_{5}B \rightarrow$$

$$Q =$$

β^- decay

- Emits an electron and an anti-neutrino.
- Changes a neutron inside the nucleus into a proton.

$${}^{A}_{Z}X(parent) \rightarrow {}_{Z+1}^{A}Y(daughter) + {}_{-1}^{0}\beta^{-} + \bar{\nu}$$



- $Z_{daughter} = Z_{parent} + 1$
- $N_{daughter} = N_{parent} - 1$
- $A_{daughter} = A_{parent}$

Released kinetic energy

$$Q = (m_{parent} - m_{daughter})\, c^2$$

Example:

$${}^{12}_{5}B \rightarrow {}^{12}_{6}C + {}_{-1}^{0}\beta^{-} + \bar{\nu}$$

$Q =$

${}^{12}_{5}B$ =12.014532 u

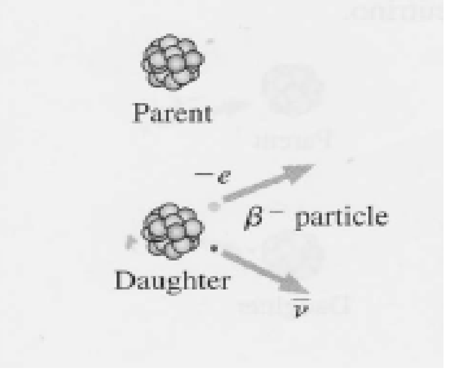
${}^{12}_{6}C$ =12 u

uc^2= 931.5 MeV

β^- decay

- Emits an electron and an anti-neutrino.
- Changes a neutron inside the nucleus into a proton.

$$^{A}_{Z}X(parent) \rightarrow {}^{A}_{Z+1}Y(daughter) + {}^{0}_{-1}\beta^- + \bar{\nu}$$



- $Z_{daughter} = Z_{parent} + 1$
- $N_{daughter} = N_{parent} - 1$
- $A_{daughter} = A_{parent}$

Released kinetic energy

$$Q = (m_{parent} - m_{daughter})\, c^2$$

Example:

$$^{12}_{5}B \rightarrow {}^{12}_{6}C + {}^{0}_{-1}\beta^- + \bar{\nu}$$

$$Q = (12.014352 - 12)uc^2 = 13.4\ MeV$$

$^{12}_{5}B$ =12.014532 u

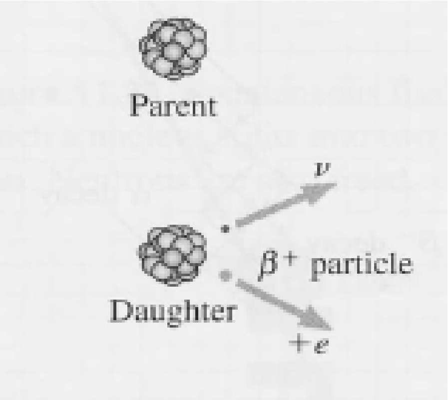
$^{12}_{6}C$ =12 u

uc^2= 931.5 MeV

β^+ decay

- Emits a positron and a neutrino
- Changes a proton inside the nucleus into a neutron.

$$ {}_{Z}^{A}X(parent) \rightarrow {}_{Z-1}^{A}Y(daughter) + {}_{+1}^{0}\beta^+ + \nu $$



- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$
- $A_{daughter} = A_{parent}$

Released kinetic energy

$$ Q = (m_{parent} - m_{daughter} - 2m_{electron})\, c^2 $$

Example $\quad {}_{7}^{12}N \rightarrow$

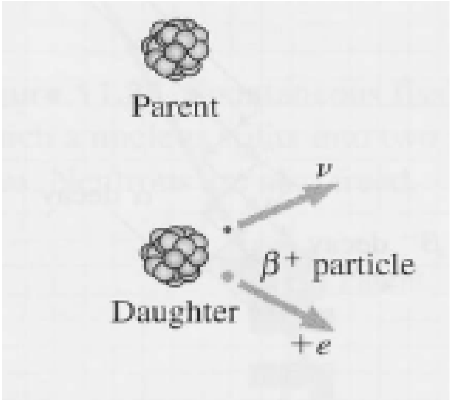
$Q =$

β^+ decay

- Emits a positron and a neutrino
- Changes a proton inside the nucleus into a neutron.

$$^{A}_{Z}X(parent) \rightarrow {}_{Z-1}^{A}Y(daughter) + {}_{+1}^{0}\beta^{+} + \nu$$

- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$
- $A_{daughter} = A_{parent}$



Released kinetic energy

$$Q = (m_{parent} - m_{daughter} - 2m_{electron})\,c^2$$

Example $\quad {}^{12}_{7}N \rightarrow {}^{12}_{6}C + {}_{+1}^{0}\beta^{+} + \nu$

${}^{12}_{7}N$ = 12.018613

${}^{12}_{6}C$ =12

uc^2=931.5 MeV

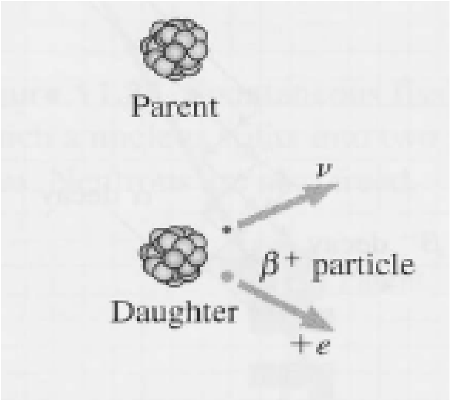
$Q =$

β^+ decay

- Emits a positron and a neutrino
- Changes a proton inside the nucleus into a neutron.

$$^{A}_{Z}X(parent) \rightarrow {}_{Z-1}^{A}Y(daughter) + {}_{+1}^{0}\beta^+ + \nu$$

- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$
- $A_{daughter} = A_{parent}$



Released kinetic energy

$$Q = (m_{parent} - m_{daughter} - 2m_{electron})\, c^2$$

Example

$$^{12}_{7}N \rightarrow {}^{12}_{6}C + {}_{+1}^{0}\beta^+ + \nu$$

$^{12}_{7}N$ = 12.018613

$^{12}_{6}C$ =12

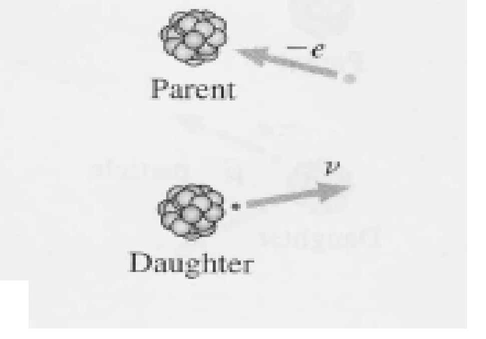
uc^2=931.5 MeV

$$Q = (12.018613 - 12 - 2 \text{ x } 0.0005486)uc^2$$
$$= 16.3\ MeV$$

Electron capture

- A nucleus with too many protons can change a proton into a neutron by capturing an electron.
- Electron capture is easier than decay since an electron is already exists for a nucleus to capture.

$${}^{A}_{Z}X(parent) + {}^{0}_{-1}\beta^{-} \rightarrow {}_{Z-1}{}^{A}Y(daughter) + \nu$$



- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$
- $A_{daughter} = A_{parent}$

Released kinetic energy

$$Q = (m_{parent} - m_{daughter})\, c^2$$

$${}^{11}_{6}C + {}^{0}_{-1}\beta^{-} \rightarrow$$

$$Q =$$

Electron capture

- A nucleus with too many protons can change a proton into a neutron by capturing an electron.
- Electron capture is easier than decay since an electron is already exists for a nucleus to capture.

$$^{A}_{Z}X(parent) + {}^{0}_{-1}\beta^{-} \rightarrow {}_{Z-1}^{A}Y(daughter) + \nu$$

- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$
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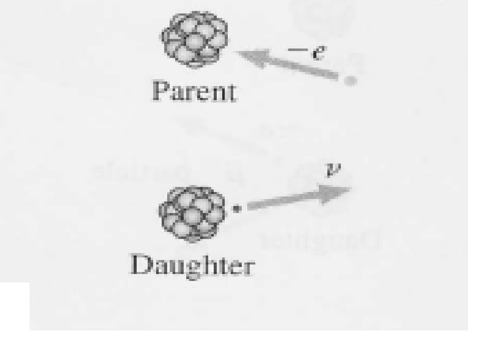

Released kinetic energy

$$Q = (m_{parent} - m_{daughter})\, c^2$$

$^{11}_{6}C$ =11.01143 u

$^{11}_{5}B$ =11.009305 u

$$^{11}_{6}C + {}^{0}_{-1}\beta^{-} \rightarrow {}^{11}_{5}B + \nu$$

$Q =$

Electron capture

- A nucleus with too many protons can change a proton into a neutron by capturing an electron.
- Electron capture is easier than decay since an electron is already exists for a nucleus to capture.

$$^{A}_{Z}X(parent) + {}^{0}_{-1}\beta^{-} \rightarrow {}_{Z-1}^{A}Y(daughter) + \nu$$

- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$
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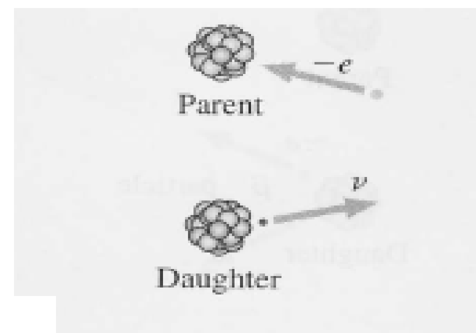

Released kinetic energy

$$Q = (m_{parent} - m_{daughter})\, c^2$$

$^{11}_{6}C$ =11.01143 u

$^{11}_{5}B$ =11.009305 u

$$^{11}_{6}C + {}^{0}_{-1}\beta^{-} \rightarrow {}^{11}_{5}B + \nu$$

$$Q = (11.01143 - 11.009305)uc^2 = 1.97\ MeV$$

Gamma decay

- A nucleus in an excited state emits photons (gamma particles, to go into a lower energy state.
- Gamma decay does not alter N or Z.
- Gamma energies are characteristic of a given isotope, and are thus used to identify the isotope.

Figure 11.24 Gamma decay.

Radioactive decay series

- An unstable nucleus can be involved in a series of decays until it finds a stable state.
- We can plot this process on a graph that represents N and Z numbers of each nucleus in the series.

Alpha decay is shown by an arrow

- $Z_{daughter} = Z_{parent} - 2$
- $N_{daughter} = N_{parent} - 2$

β^- decay

- $Z_{daughter} = Z_{parent} + 1$
- $N_{daughter} = N_{parent} - 1$

β^+ decay and electron capture

- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$

Radioactive decay series

- An unstable nucleus can | decays until it finds a stak
- We can plot this process N and Z numbers of each

Alpha decay is shown by an arrow
  - $Z_{daughter} = Z_{parent} - 2$
  - $N_{daughter} = N_{parent} - 2$

β^- **decay**
  - $Z_{daughter} = Z_{parent} + 1$
  - $N_{daughter} = N_{parent} - 1$

β^+ **decay and electron capture**
  - $Z_{daughter} = Z_{parent} - 1$
  - $N_{daughter} = N_{parent} + 1$

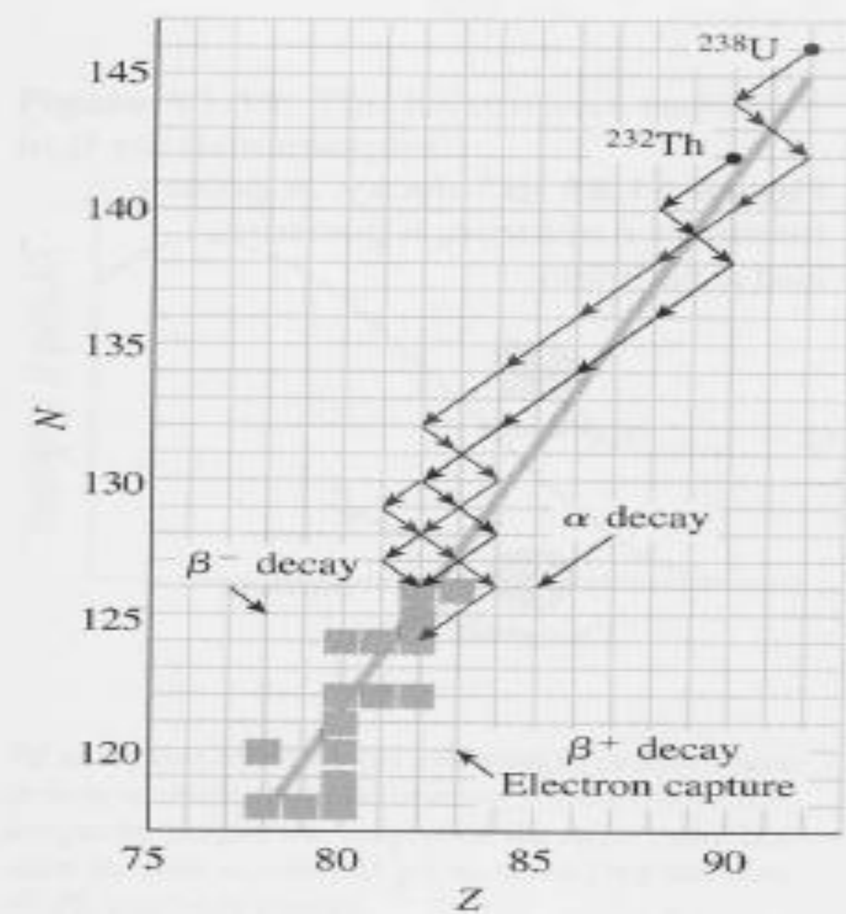

Figure 11.23 The "directions" of α and β decays, and the decay series of uranium-238 and thorium-232.

Radioactive decay series

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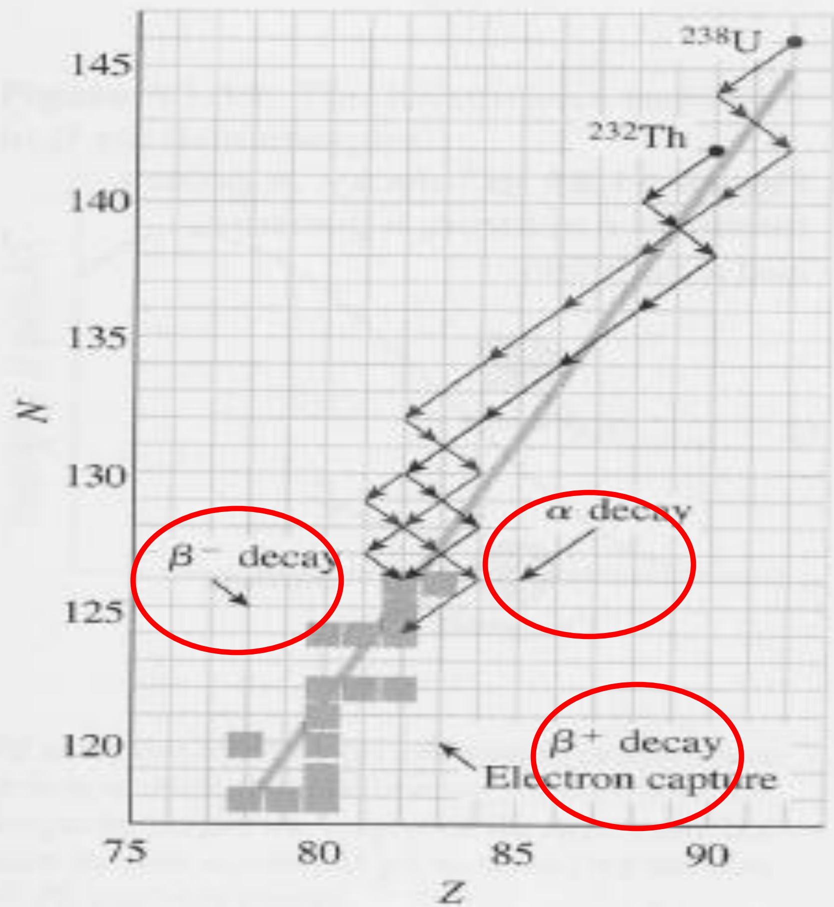
- We can plot this process c N and Z numbers of each

Alpha decay is shown by an arrow
- $Z_{daughter} = Z_{parent} - 2$
- $N_{daughter} = N_{parent} - 2$

β^- decay
- $Z_{daughter} = Z_{parent} + 1$
- $N_{daughter} = N_{parent} - 1$

β^+ decay and electron capture
- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$

Figure 11.23 The "directions" of α and β decays, and the decay series of uranium-238 and thorium-232.



Radioactive decay series

- An unstable nucleus can l… decays until it finds a stal…
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Alpha decay is shown by an arrow
- $Z_{daughter} = Z_{parent} - 2$
- $N_{daughter} = N_{parent} - 2$

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- $Z_{daughter} = Z_{parent} + 1$
- $N_{daughter} = N_{parent} - 1$

β^+ decay and electron capture
- $Z_{daughter} = Z_{parent} - 1$
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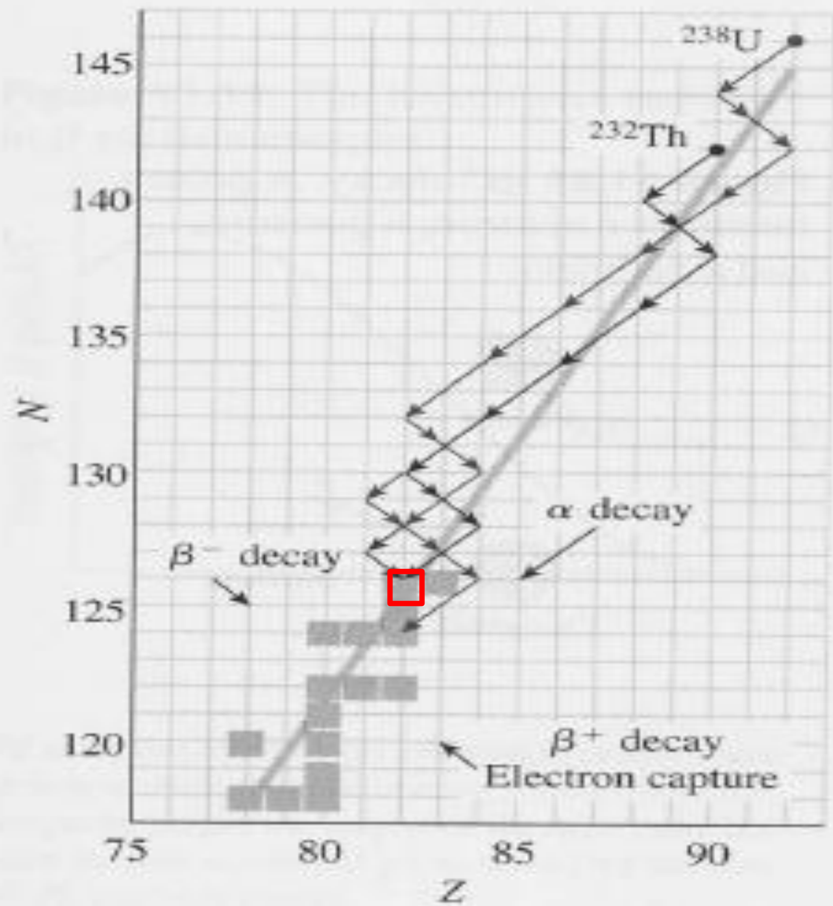
Figure 11.23 The "directions" of α and β decays, and the decay series of uranium-238 and thorium-232.



Pb=82 protons +126 neutrons

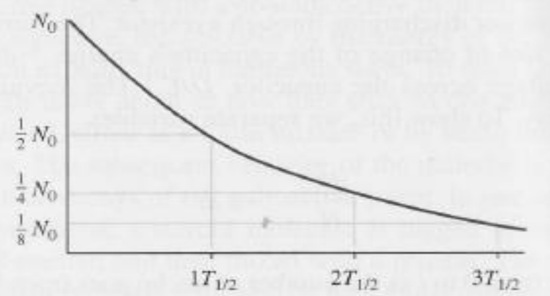
Radioactive decay law

- For all decays, the rate of decay over time will be proportional to the sample size:

$$\frac{dN}{dt} \propto N \qquad \frac{dN}{dt} = -\lambda N$$

where N=Number of nuclei; λ=decay constant

Figure 11.26 Exponential decay.



Radioactive decay law

- For all decays, the rate of decay over time will be proportional to the sample size:

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$ where N=Number of nuclei; λ=decay constant

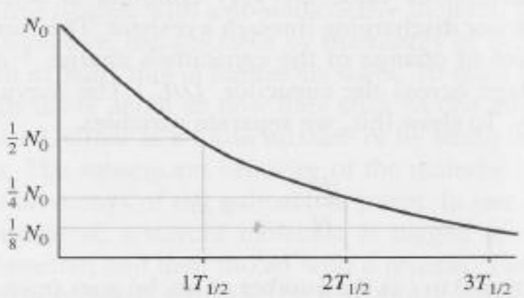
$$\frac{dN}{N} = -\lambda\, dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$ln\frac{N}{N_0} = -\lambda t$$

$$N = N_0 e^{-\lambda t}$$

Figure 11.26 Exponential decay.



Decay rate R = λN (decays per second)

Radioactive decay law

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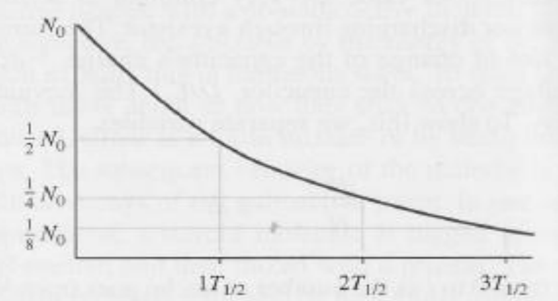

Figure 11.26 Exponential decay.

$$ln\frac{N}{N_0} = -\lambda t$$

$$N = N_0 e^{-\lambda t}$$

Decay rate R = λN (decays per second)

$$\frac{1}{2}N_0 = N_0 e^{-\lambda T_{1/2}}$$

Half-life ($T_{1/2}$)

$$\lambda = \frac{ln2}{T_{1/2}}$$

Half-life

$$\lambda = \frac{ln2}{T_{1/2}} \qquad N = N_0 e^{-\lambda t}$$

Decay rate $R = \lambda N$ (decays per second)

TABLE 11.3 Selected decays

Isotope	Decay Mode	Half-Life
$^{35}_{20}Ca$	β^+	50 ms
$^{3}_{1}H$	β^-	12.3 yr
$^{238}_{92}U$	α	4.5×10^9 yr

A vessel holds 2 μg of tritium.

(a) Initial decay rate?

Tritium's atomic mass = 3.02 u and 1 u=1.66 x $10^{-27}\,kg$

$$N = \frac{sample\ mass}{atomic\ mass\ of\ Tritium} \qquad \lambda = \frac{ln2}{T_{1/2}}$$

$$T_{1/2} = 12.3\ yr = 12.3 \times 3.16 \times 10^7\ sec$$

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} \cdot \frac{sample\ mass}{atomic\ mass\ of\ Tritium}$$

(a) Time elapse before the decay rate falls to 1% of its initial value?

$$t = \frac{-\ln\left(\frac{1}{100}\right)}{\lambda} = 2.6 \times 10^9\ \text{sec} = 81.7\ years$$

Half-life

$$\lambda = \frac{ln2}{T_{1/2}} \qquad N = N_0 e^{-\lambda t}$$

Decay rate R = λN (decays per second)

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Tritium's atomic mass = 3.02 u and 1 u=1.66 x $10^{-27}\,kg$

N= $\frac{sample\,mass}{atomic\,mass\,of\,Tritium}$ $\quad \lambda = \frac{ln2}{T_{1/2}}$

$T_{1/2} = 12.3\,yr = 12.3 \times 3.16 \times 10^7\,sec$

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} \cdot \frac{sample\,mass}{atomic\,mass\,of\,Tritium}$$

$$R = 7.1 \times 10^8\ decays/sec$$

(a) Time elapse before the decay rate falls to 1% of its initial value?

$$N = N_0 e^{-\lambda t}$$

$$\frac{1}{100} N_0 = N_0 e^{-\lambda t}$$

Half-life

$$\lambda = \frac{\ln 2}{T_{1/2}} \qquad N = N_0 e^{-\lambda t}$$

Decay rate $R = \lambda N$ (decays per second)

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$$t = \frac{-\ln\left(\frac{1}{100}\right)}{\lambda} = 2.6 \times 10^9\ \text{sec} = 81.7\ years$$

Carbon 14 Dating

- Carbon-14's decay has a half-life of 5730 years.
- Carbon-14 dating only works for formerly living organisms.
- Carbon-14's amount is constantly maintained for living organisms since living organisms exchange Carbon with the environment. Ratio of naturally produced C14/C12=1.3 x 10^{-12}
- When, a living organism dies, it stops the exchange process, thus C-14 in the dead organism decays exponentially.

Fossil age

Sample: 6 g of carbon and a decay rate (R) of 30 /min

- Since C14/C12 = 1.3×10^{-12} when the sample was alive

- Decay constant (λ)

- Current decay rate (R)=30 /min=30/60 seconds= .5 /sec

- Elapsed time $N = N_0 e^{-\lambda t}$

1 year = 3.16×10^{7} seconds

Fossil age

Sample: 6 g of carbon and a decay rate (R) of 30 /min

- Since C14/C12 = 1.3 x 10^{-12} when the sample was alive

$$N_0(C_{14}) = (1.3x\ 10^{-12}) \cdot \left(\frac{6\ g}{12\ g}\right) \cdot (6.02 \text{ x } 10^{23}) = 3.9 \text{ x } 10^{11}$$

- Decay constant (λ)

- Current decay rate (R)=30 /min=30/60 seconds= .5 /sec

- Elapsed time $N = N_0 e^{-\lambda t}$

1 year = 3.16 x 10^7 seconds

Fossil age

Sample: 6 g of carbon and a decay rate (R) of 30 /min

- Since C14/C12 = 1.3 x 10^{-12} when the sample was alive

$$N_0(C_{14}) = (1.3x\ 10^{-12}) \cdot \left(\frac{6\ g}{12\ g}\right) \cdot (6.02 \text{ x } 10^{23}) = 3.9 \text{ x } 10^{11}$$

- Decay constant (λ)

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5730\ years\ \cdot 3.16\ x 10^7 sec/year} = 3.83 \text{ x } 10^{-12}\ \ /sec$$

- Current decay rate (R)=30 /min=30/60 seconds= .5 /sec

$$R = N \cdot\ \lambda = N \cdot 3.83 \text{ x } 10^{-12}\ \ /sec = 0.5\ /sec$$

$$N = 1.31 \text{ x } 10^{11}$$

- Elapsed time $N = N_0 e^{-\lambda t}$

1 year = 3.16 x 10^7 seconds

Fossil age

Sample: 6 g of carbon and a decay rate (R) of 30 /min

- Since C14/C12 = 1.3 x 10^{-12} when the sample was alive

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- Decay constant (λ)

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5730\ years\ \cdot 3.16\, x 10^7 sec/year} = 3.83 \text{ x } 10^{-12}\ \ /sec$$

- Current decay rate (R)=30 /min=30/60 seconds= .5 /sec

$$R = N \cdot\ \lambda = N \cdot 3.83 \text{ x } 10^{-12}\ \ /sec = 0.5\, /sec$$

$$N = 1.31 \text{ x } 10^{11}$$

- Elapsed time $N = N_0 e^{-\lambda t}$

1 year = 3.16 x 10^7 seconds

$$t = -\frac{1}{\lambda} \ln \frac{N}{N_0} = -\frac{1}{3.83 \text{ x } 10^{-12}\ \ /sec} \ln \frac{1.31 \text{ x } 10^{11}}{3.9 \text{ x } 10^{11}} = 2.86 \text{ x } 10^{11}\ sec \sim 9000\ years$$