**PH102, 2013W, Lecture Notes: February 28, Thurs, Class 16**
**Nuclear Physics: Nuclear Structure, Properties, and Models**
**……………………….**

**Structure of Nuclei**

Nuclei consist of nucleons, i.e. protons and neutrons.

- Z= number of protons
- N= number of neutrons
- A=mass number=total number of nucleons = Z + N

| | charge | Mass in kg | Mass in u |
|---|---|---|---|
| Proton | +e | $1.6726217 \times 10^{-27}$ | 1.007276 |
| Neutron | 0 | $1.6749273 \times 10^{-27}$ | 1.008665 |
| Electron | -e | $9.109 \times 10^{-31}$ | 0.0005486 |

Where 1u (atomic mass) = 1/12th of the mass of C-12

- Isotopes= Nuclei with the same number of protons but different numbers of neutrons. Isotopes that are not present in nature are unstable. Some isotopes that are abundant in nature are unstable.
- Hydrogen isotopes
  - Hydrogen's nucleus has one proton and is stable
  - Deuterium's nucleus has one proton and one neutron and is stable, 0.015% of the hydrogen atoms found in nature are Deuterium.
  - Tritium's nucleus has one proton and two neutrons and is unstable.

Figure 11.2 Isotopes of hydrogen.

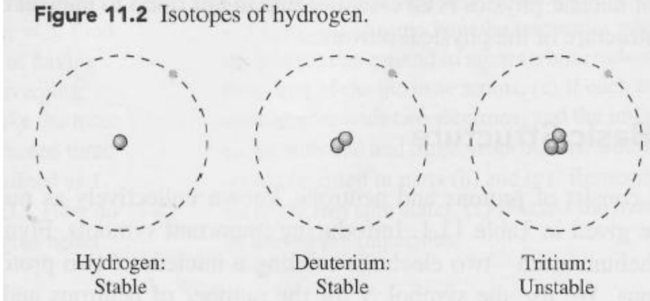
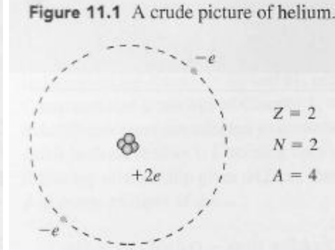



Figure 11.1 A crude picture of helium.

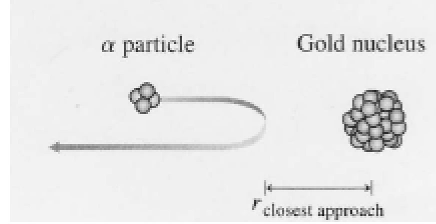



Z=1, N=0, A=1; Z=1, N=1, A=2; Z=1, N=2, A=3; Z=2, N=2, A=4

**Size of Nucleus**

- Rutherford's experiment with $\alpha$ particles (=He's nuclei after electrons are removed) fired toward the thin Gold film.
  - The nuclei are roughly spherical.
  - Most of alpha particles do not scatter
  - The head-on approach would enable alpha particles to approach the gold nuclei closest. The experiment showed that $r = A^{1/3}R_0$ where $R_0 = 1.2\ x\ 10^{-15}m$.

Figure 11.3 Probing for the radius of the nucleus.



- The nuclear dimension is about $10^{-15}$meter (femto-meter)

- Nuclear volume

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(A^{1/3}R_0)^3 = \frac{4}{3}\pi A R_o^3 \sim A\ (proportional\ to\ mass\ number)$$

- Nuclear density is about the same for all nuclei since volume is proportional to mass (number).

$$density = \frac{mass}{volume} = \frac{A \text{ x } mass\ of\ nucleon}{A \text{ x } \frac{4}{3}\pi R_o^3} \cong 10^{17} kg/m^3$$

- Nucleons in a nucleus is closely packed and each nucleon's volume in a nucleus is

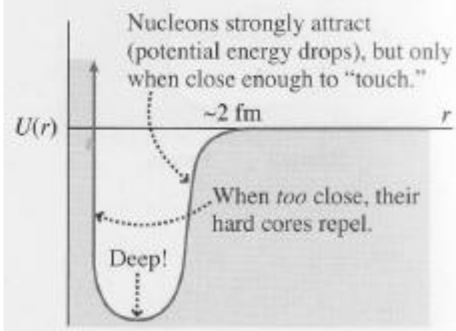
$$\frac{V}{A} = \frac{4}{3}\pi R_o^3$$

Thus, the effective radius of a nucleon is $R_0$.

**Strong Force:**

- Nucleons inside a nucleus are held together by strong force
- Strong force is attractive
- Strong force is short-ranged (about 2 fm) and strong
- Strong force is nearly identical between n-n, n-p, and p-p.
- The potential energy for strong force is shown in Figure 11.4.

**Figure 11.4** The basic elements of the internucleon (strong force) potential energy—a strong, short-range attraction with a repulsive hard core.




Comparison of forces

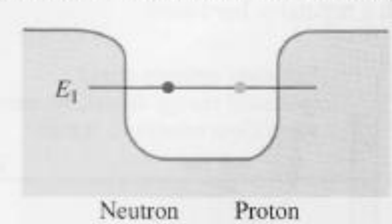
| Force | Applying to | Relative strength | Range |
|---|---|---|---|
| Strong | Between nucleons | 1 | $\sim 1\ fm$ |
| Electromagnetic | Between charges | $\sim 10^{-2}$ | $\propto 1/r^2$ (long range) |
| Weak | Related to radioactive decay | $\sim 10^{-6}$ | $\sim 10^{-3}\ fm$ |
| Gravitational | Between masses | $\sim 10^{-39}$ | $\propto 1/r^2$ (long range) |

**Nuclear binding:** not all nucleon arrangements are stable.

Two-nucleon nuclei:

- Protons and neutrons are fermions and are subject to the Exclusion Principle.
- Parallel spin arrangements are more stable.
- Proton-neutron can have a parallel spin arrangement in the ground spatial state while proton-proton and neutron-neutron cannot do this.
- Proton-Neutron thus is stable and has one bound state. And Deuterium's nucleus has a total spin of 1.

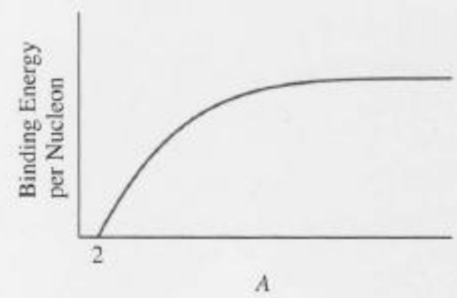
**Figure 11.5** The deuteron's neutron and proton bound in a well resulting from their attractive potential energy.



Arbitrary number of nucleon nuclei:

- Strong force
  - As more and more nucleons are present in nuclei, the number of bonds among nucleons increases, e.g. (2 nucleons→ 1 bond, 3 nucleons → 3 bonds, 4 nucleons → 6 bonds, etc.). This indicates that the binding energy of a nucleus will increase as the number of nucleons increases.
  - However, the strong force is short ranged and attracting only the nearby nucleons, making each surrounding nucleon has the same number of bonds. Nucleons at the surface are not completely surrounded. The ratio of surface nucleons over total number of nucleons diminishes as A grows $\propto 1/r$. Figure 11.6 shows this trend.

**Figure 11.6** Binding energy per nucleon due to the strong internucleon attraction only. The smallest nuclei have few bonds per nucleon. In large nuclei, many nucleons are surrounded.




- Coulomb repulsion
  - Proton-proton will repel if they are not in contact with one another. See Figure 11.7 where proton's energy levels are slightly elevated due to repulsive force.
  - Coulomb force becomes more and more act as a destabilizing force as nuclei get larger.

**Figure 11.7** Coulomb repulsion raises proton energies.

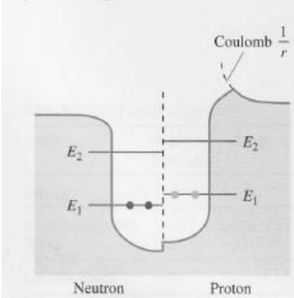
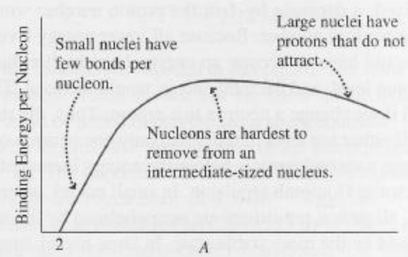



**Figure 11.8** Binding energy per nucleon due to both the strong internucleon attraction and Coulomb repulsion.




- The Exclusion Principle
  - Protons and neutrons obey the exclusion principle independently.
  - For smaller nuclei, the lowest energy state can be obtained with equal numbers of protons and neutrons. See Figure 11.9
  - For larger nuclei, the lower energy state can be obtained when N > Z due to the increase in repulsive force between protons. See Figure 11-10.

**Figure 11.9** Ignoring Coulomb repulsion, the exclusion principle argues that for a given number of nucleons, the lowest energy should have $N = Z$.

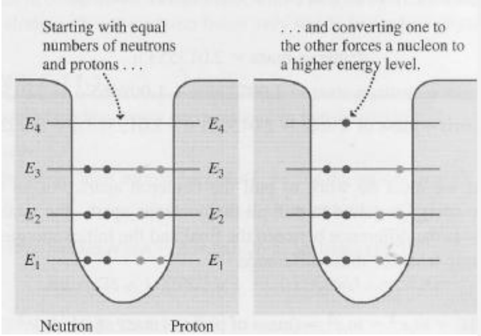
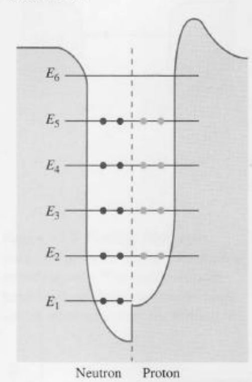



**Figure 11.10** In large nuclei, when Coulomb repulsion becomes significant, the lowest energy should have $N > Z$.



**Liquid Drop Model for Nucleus**:

Binding energy= volume term + surface term + Coulomb term + Asymmetry term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

where

$$C_1 = 15.8$$
$$C_2 = 17.8$$
$$C_3 = 0.71$$
$$C_4 = 23.7$$

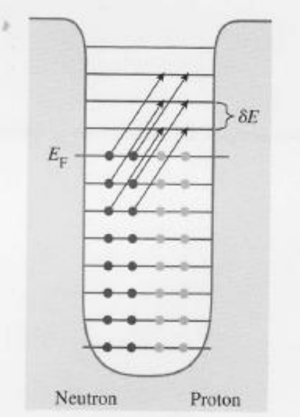


**Figure 11.15** If $j$ neutrons become protons, the energy increases by $\frac{1}{2} j^2 \delta E$.

The binding energy formula above is called the semiempirical binding energy formula because the formula is based on theoretical argument with empirical evidence.

Example: Fe-56

Binding energy = 15.8 x 56 – 17.8 x $56^{2/3}$ -0.71 $\frac{26\ x\ 25}{56^{1/3}} - 23.7 \frac{(30-26)^2}{56}$ = 496.9 MeV

Binding energy/nucleon = 496.9 MeV/56 = 8.87 MeV

Figure 11.16 shows the actual data with the semiempirocal binding energy formula plot. Fit is good except very light nuclei such as He's nucleus.

**Binding Energy**

Binding energy = (mass of individual nucleons – mass of nucleus)$c^2$

Example: Deuteron's binding energy

Deuteron = 1 proton + 1 neutron

Binding Energy = (proton mass + neutron mass – Deuteron mass)$c^2$

$$= (1.007276\ u + 1.008665\ u - 2.013553\ u)c^2 = 0.002388\ uc^2$$
$$= 0.002388\ x\ 1.661\ x\ 10^{-27} kg\ x\ (3x10^8 m/sec)^2 = 2.22\ MeV$$

Compare the binding energy of electron in the hydrogen atom (=13.6 eV) and the binding energy of nucleons in the deuterium $\sim 10^{-5}$

$$Binding\ Energy = \left( Zm_H + Nm_n - M_{{}^A_Z X} \right) c^2$$

Where $m_H$= atomic mass of hydrogen

$m_n$= neutron mass

$M_{{}^A_Z X}$ = atomic mass of the nucleus

For example: $^{56}_{26}Fe = 55.934939\ u$

Binding energy = (26 x 1.007825 u +30 x 1.008665 u – 55.934939 u) $c^2$=0.528461 u$c^2$ =492.3 MeV   (consider $c^2 = 931.5\ MeV/u$).

The curve of stability: Binding energy/nucleon for naturally occurring isotopes
Figure 11.12 and Figure 11.13

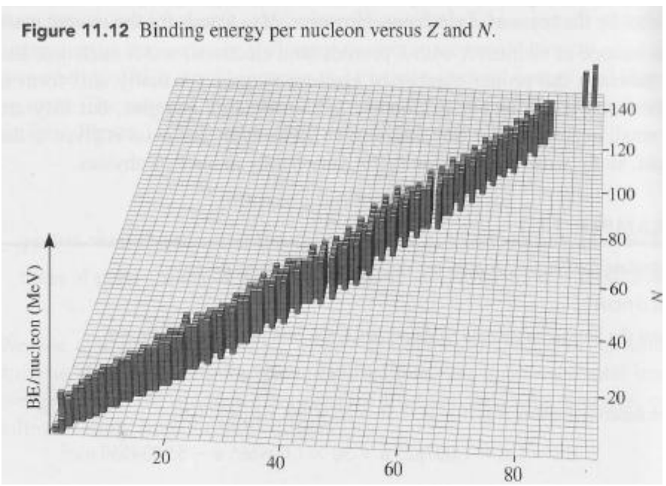



**Figure 11.12** Binding energy per nucleon versus *Z* and *N*.

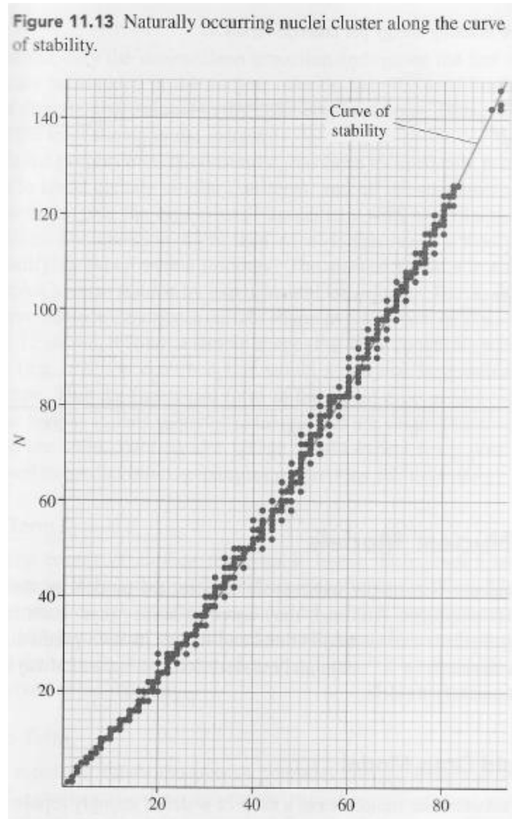



**Figure 11.13** Naturally occurring nuclei cluster along the curve of stability.

Binding energy per nucleon over A: Figure 11.14

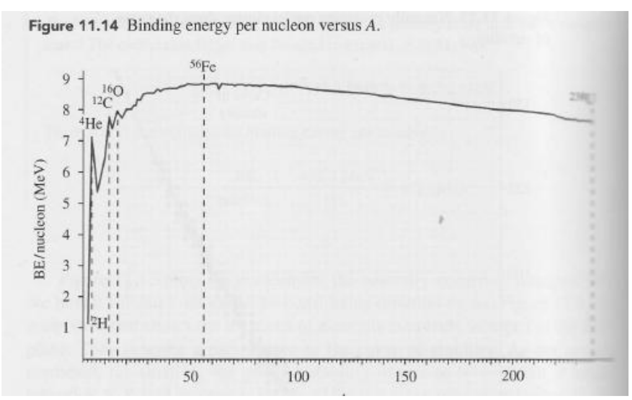
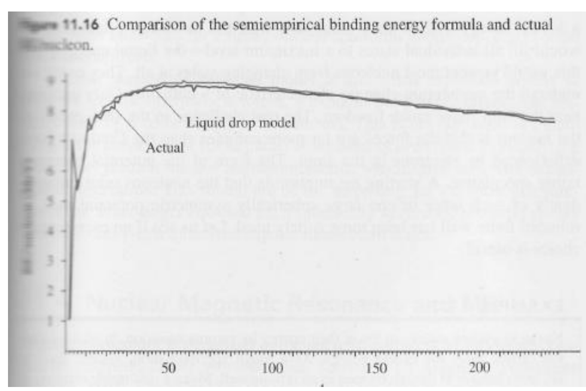



**Figure 11.14** Binding energy per nucleon versus *A*.




11.16 Comparison of the semiempirical binding energy formula and actual ... nucleon.

**Nuclear Physics: Nuclear Shell Model**

The Liquid Drop model can predict the nuclear binding energy reasonably well. The Liquid Drop model is based on the Strong Force between two adjacent nucleons in a nucleus. However, the Liquid Drop model cannot explain magic numbers: 2, 8, 20, 28, 50, 82, 126 where nuclei with magic numbers of nucleons are particularly stable. The Shell Model explains the magic numbers, the tendency for Z and N to be simply even numbers, and the model can explain angular momenta of nuclei.

| | Atomic Shell Model | Nuclear Shell Model |
|---|---|---|
| Potential | Electrostatic between nucleus and electrons | Net effect of all the forces nucleons experience in a nucleus |
| Magic numbers | Atoms with closed electronic shells are stable such as He (2 electrons), Ne (10), Ar (18), Kr (36), Xe (54), Rn (86). | Nuclei are particularly stable when the number of nucleons is 2, 8, 20, 28, 50, 82, and 126. |
| Exclusion principle | Electrons follow the Exclusion principle | Protons and neutrons separately follow the Exclusion Principle |
| Movement | Electrons move in orbitals | Nucleons do not move like electrons because most nucleons fill states to a maximum level, preventing them from changing momentum. |

Atomic Energy States:

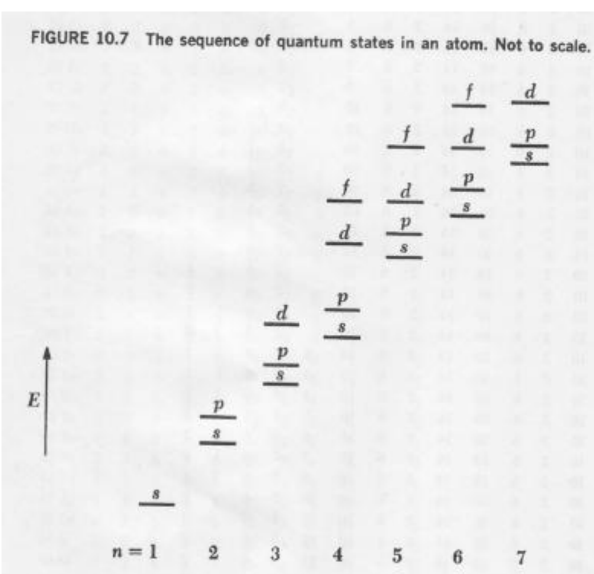
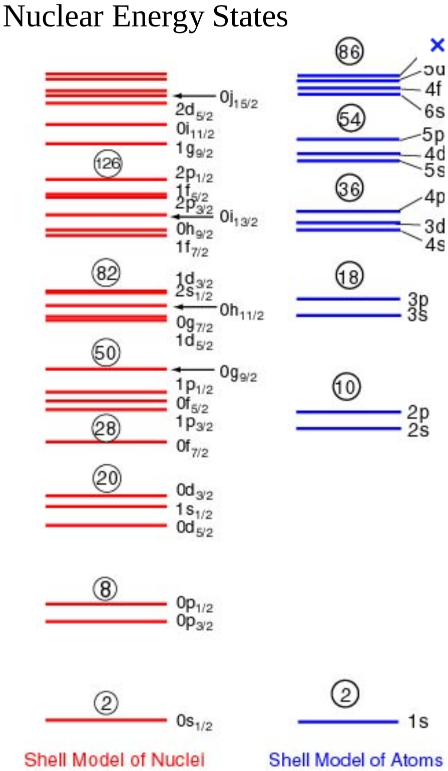


FIGURE 10.7 The sequence of quantum states in an atom. Not to scale.

Nuclear Energy States




The nuclear shell model is based on three dimensional harmonic oscillator solutions.

$$\frac{-\hbar^2}{2m}\nabla^2\psi(x) + \frac{1}{2}\mu\omega^2 x^2\ \psi(x) = \mathrm{E}\ \psi(x)$$

$$\psi_{klm}(r,\theta,\phi) = N_{kl} r^l e^{-\nu r^2} L_k^{(l+\frac{1}{2})}(2\nu r^2) Y_{lm}(\theta,\phi)$$

where

$$N_{kl} = \sqrt{\sqrt{\frac{2\nu^3}{\pi}} \frac{2^{k+2l+3}\ k!\ \nu^l}{(2k+2l+1)!!}}$$ is a normalization constant.

$$\nu \equiv \frac{\mu\omega}{2\hbar}$$

$L_k^{(l+\frac{1}{2})}(2\nu r^2)$ are generalized Laguerre polynomials. The order $k$ of the polynomial is a non-negative integer.

$Y_{lm}(\theta,\phi)$ is a spherical harmonic function.

The energy eigenvalue is

$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

The energy is usually described by the single quantum number

$$n \equiv 2k + l$$

- For every even $n, l = 0, 2, \ldots n-2, n$
- For every odd $n, l = 1, 3, \ldots n-2, n$
- $-l \leq m \leq l$
- Every $n$ and $l$, there are $2l+1$ energy degeneracies, which can accommodate $2(2l+1)$ nucleons

Using these rules, we can obtain the following table:

| n | k | l | No. of nucleons in (n, k, l) | No. of nucleons in n | Total nucleons | Energy |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 2 | 2 | 2 | $\frac{3}{2}\hbar\omega$ |
| 1 | 0 | 1 | 6 | 6 | 8 | $\frac{5}{2}\hbar\omega$ |
| 2 | 0 | 2 | 10 | 12 | 20 | $\frac{7}{2}\hbar\omega$ |
| | 1 | 0 | 2 | | | |
| 3 | 0 | 3 | 14 | 20 | 40 | $\frac{9}{2}\hbar\omega$ |
| | 1 | 1 | 6 | | | |
| 4 | 0 | 4 | 18 | 30 | 70 | $\frac{11}{2}\hbar\omega$ |
| | 1 | 2 | 10 | | | |
| | 2 | 0 | 2 | | | |
| 5 | 0 | 5 | 22 | 42 | 112 | $\frac{13}{2}\hbar\omega$ |
| | 1 | 3 | 14 | | | |
| | 2 | 1 | 6 | | | |

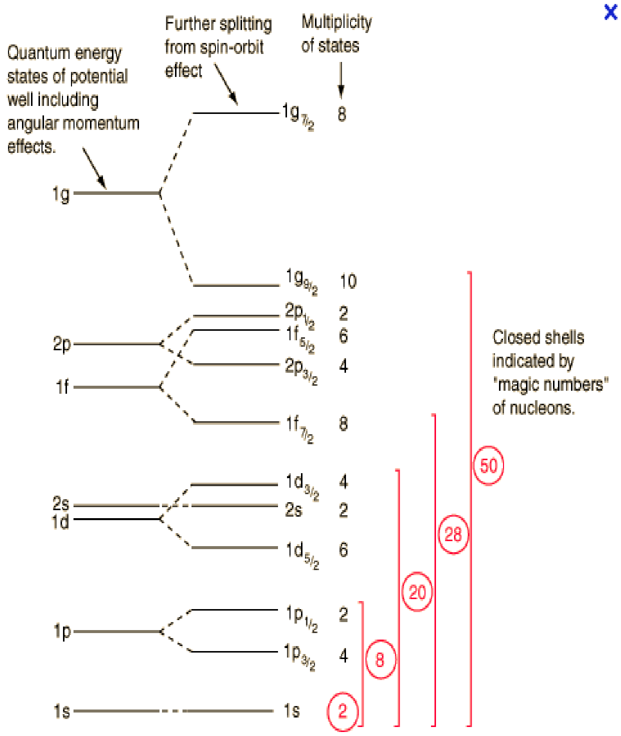
The above table predicts the magic numbers of 2, 8, and 20, but not higher than these numbers. To account for this, we need to consider the L-S coupling, which is similar to the L-S coupling of electrons.

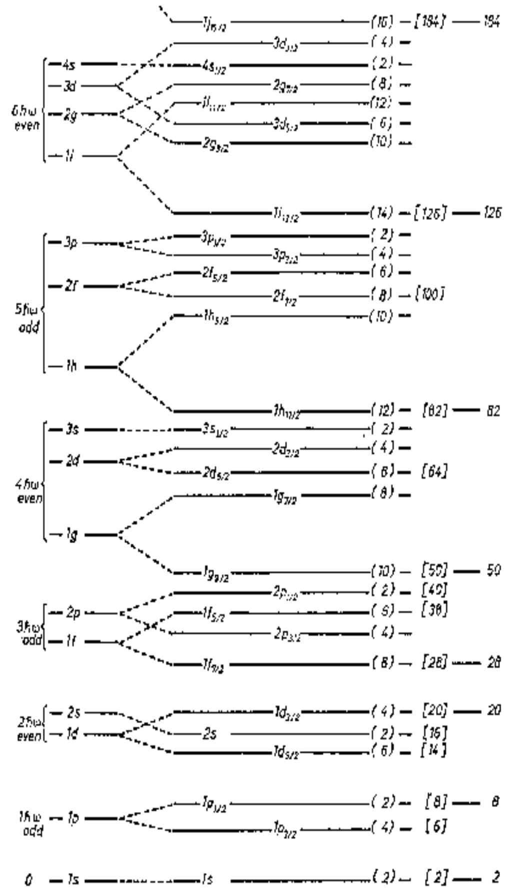
$$\vec{J} = \vec{L} + \vec{S}$$

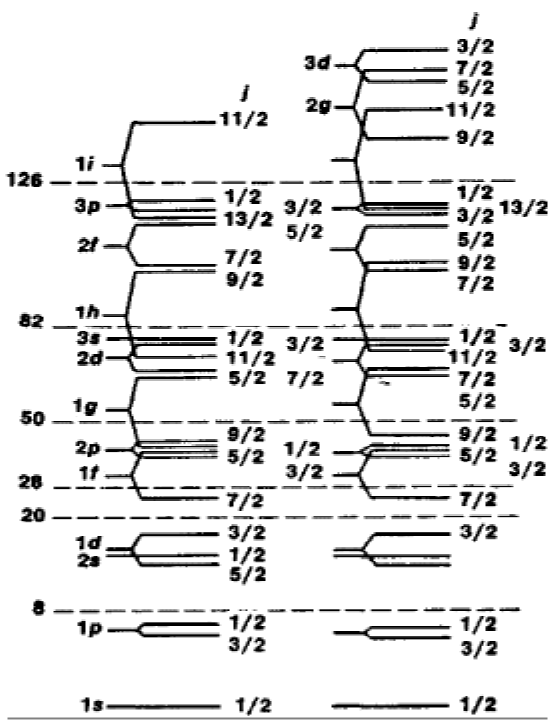
This means that, instead of $l$ and $m_l$ and spin (as two times of each state), we need to consider $j$ and $m_j$. This will result in each (n,k,l) energy state to split into two more states.

| n | k | l | j | No. of nucleons in (n, j) | No. of nucleons in (n,$\vert\vec{J}\vert$) | Energy |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | ½ | 2 | 2 | $\frac{3}{2}\hbar\omega$ |
| 1 | 0 | 1 | 3/2<br>1/2 | 4<br>2 | 6 | $\frac{5}{2}\hbar\omega$ |
| 2 | 0 | 2 | 5/2<br>3/2 | 6<br>4 | 10 | $\frac{7}{2}\hbar\omega$ |
| | 1 | 0 | ½ | 2 | 2 | |
| 3 | 0 | 3 | 7/2<br>5/2 | 8<br>6 | 14 | $\frac{9}{2}\hbar\omega$ |
| | 1 | 1 | 3/2<br>1/2 | 4<br>2 | 6 | |
| 4 | 0 | 4 | 9/2<br>7/2 | 10<br>8 | 18 | $\frac{11}{2}\hbar\omega$ |
| | 1 | 2 | 5/2<br>3/2 | 6<br>4 | 10 | |
| | 2 | 0 | ½ | 2 | 2 | |
| 5 | 0 | 5 | 11/2<br>9/2 | 12<br>10 | 22 | $\frac{13}{2}\hbar\omega$ |
| | 1 | 3 | 7/2<br>5/2 | 8<br>6 | 14 | |
| | 2 | 1 | 3/2<br>1/2 | 4<br>2 | 6 | |

Then, each $(n, k, l)$ state splits into two j states, except $j = 0$. When it does, the split becomes greater as $l$ gets greater for the same $k$. In each split, the higher $j$ state is lower than the lower $j$ state.




Empirical evidence

Proton energy levels Neutron energy levels