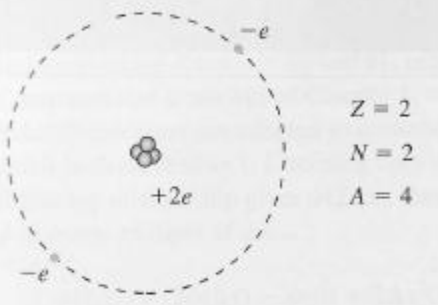
Lecture 16 Topics

- Structure of nucleus
- Size of nucleus
- Strong force and potential
- Nuclear binding energy (theoretical/empirical)
  - Strong force
  - Electrostatic force-repulsive force
  - Exclusion principle
- Nuclei models
  - Liquid Drop model
  - Shell model

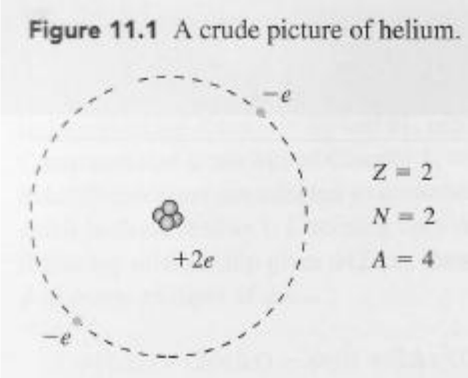
Nucleus

Figure 11.1 A crude picture of helium.



	charge	Mass in kg	Mass in u
Proton	+e	$1.6726217 \times 10^{-27}$	1.007276
Neutron	0	$1.6749273 \times 10^{-27}$	1.008665
Electron	-e	9.109×10^{-31}	0.0005486

Nucleus



Z= number of protons
N=number of neutrons
A=mass number=number of nucleons=Z+N

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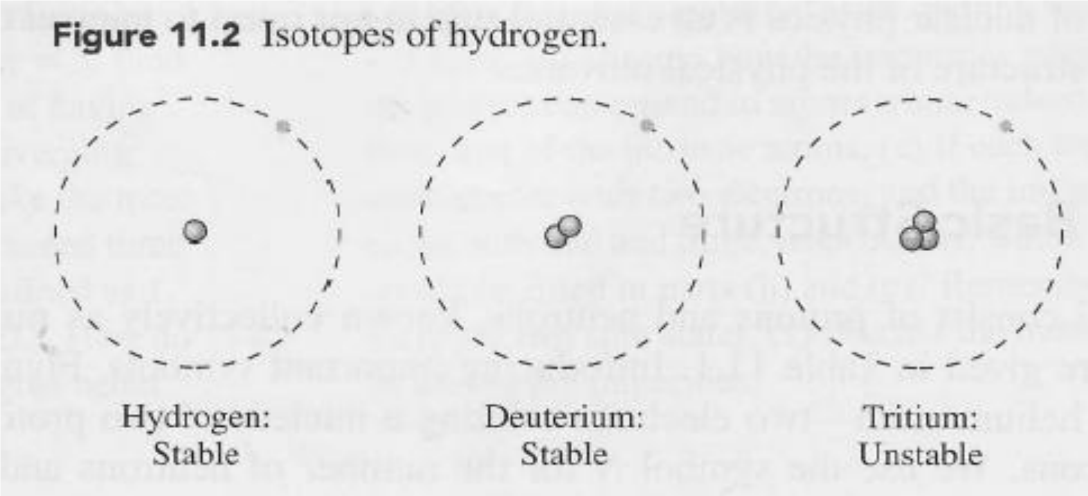
Isotopes



Nuclei	Hydrogen	Deuterium	Tritium
Z			
N			
A			
Electron number			

Isotopes

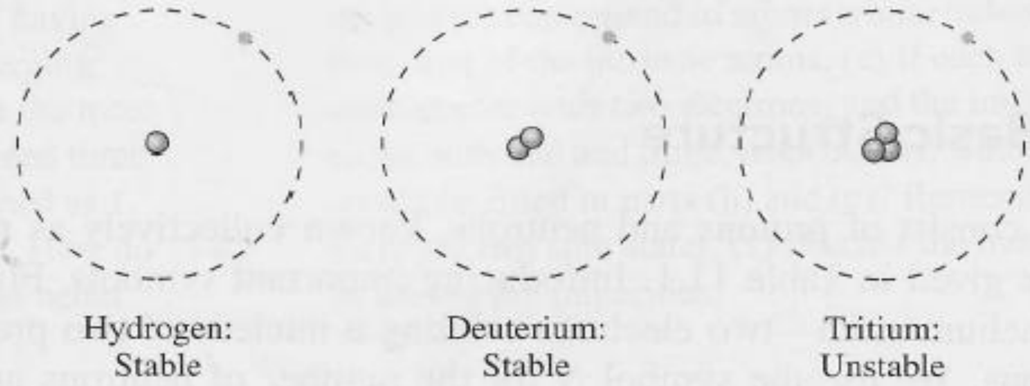


Figure 11.2 Isotopes of hydrogen.

Nuclei	Hydrogen	Deuterium	Tritium
Z	1	1	1
N	0	1	2
A	1	2	3
Electron number	1	1	1

Stable

Nucleus size

Rutherford's alpha particle experiment on Gold foil

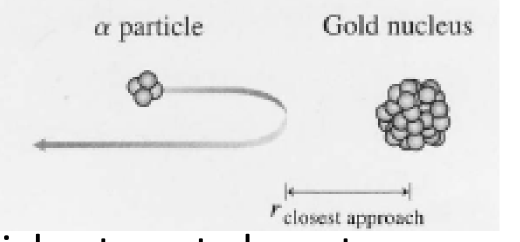


Figure 11.3 Probing for the radius of the nucleus.

→The nucleus is roughly spherical
→The nucleus is centered at the middle of the atom
→The nucleus occupies a relatively small space
→The head-on approach would enable the alpha particles to get closest

$$r = A^{1/3} R_0 \qquad R_0 = 1.2\ x\ 10^{-15} m.$$

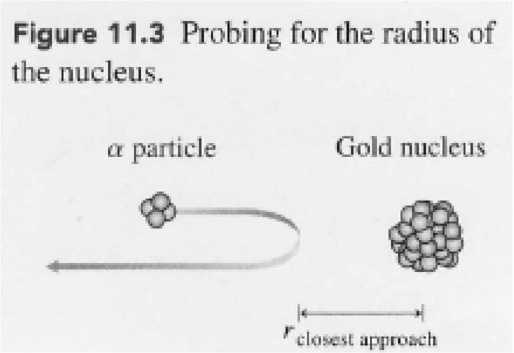
Nucleus Volume =

Nuclear Density=

Nucleon volume=

Nucleus size

Rutherford's alpha particle experiment on Gold foil



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Nuclear Volume = $V = \frac{4}{3}\pi r^3 =$

Nuclear Density= $density = \frac{mass}{volume} =$

Nucleon volume= $\frac{V}{A} =$

Nucleus size

Rutherford's alpha particle experiment on Gold foil

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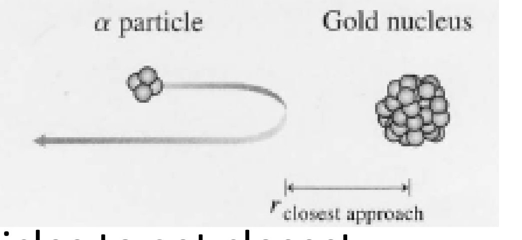

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$$r = A^{1/3} R_0 \qquad R_0 = 1.2 \ x\ 10^{-15} m.$$

Nuclear Volume = $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (A^{1/3} R_0)^3 = \frac{4}{3}\pi A R_o^3 \sim A$

Nuclear Density= $density = \frac{mass}{volume} = \frac{A \text{ x } mass\ of\ nucleon}{A \text{ x } \frac{4}{3}\pi R_o^3} \cong 10^{17} kg/m^3$

Nucleon volume= $\frac{V}{A} = \frac{4}{3}\pi R_o^3$

Strong force

→ Strong

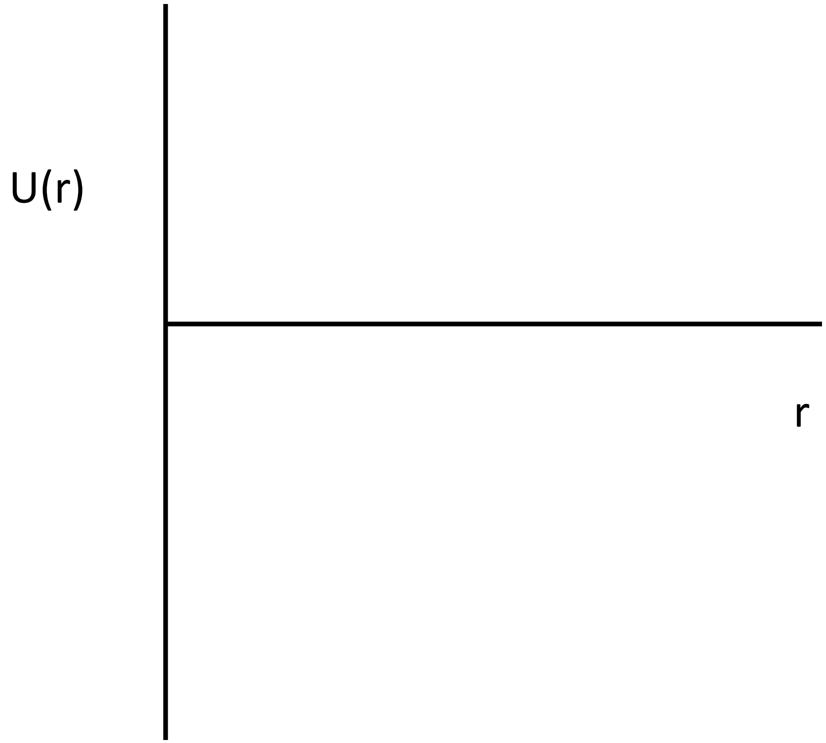
Strong force

→ Strong

Force	Applying to	Relative strength	Range
Strong	Between nucleons	1	$\sim 1\, fm$
Electromagnetic	Between charges	$\sim 10^{-2}$	$\propto 1/r^2$ (long range)
Weak	Related to radioactive decay	$\sim 10^{-6}$	$\sim 10^{-3}\, fm$
Gravitational	Between masses	$\sim 10^{-39}$	$\propto 1/r^2$ (long range)

Strong force

- →Strong
- → attractive
- →Short-ranged (about 2 fm)



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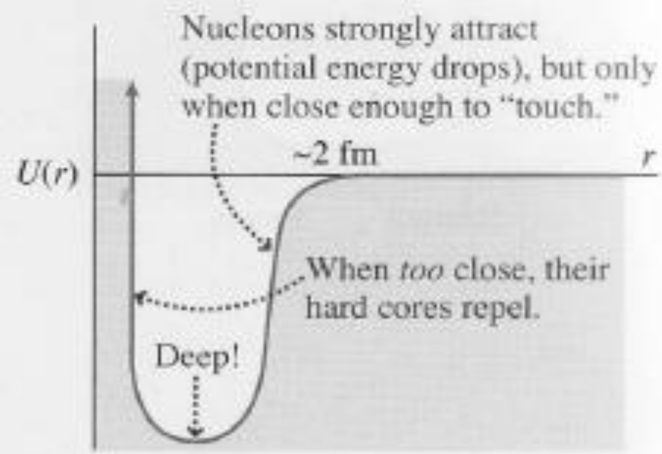
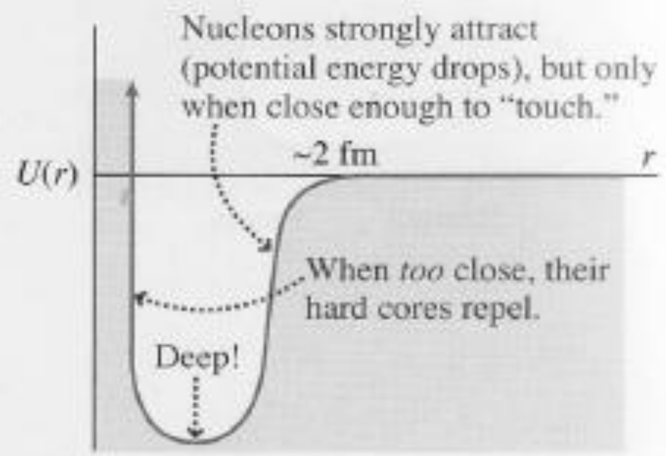


Figure 11.4 The basic elements of the internucleon (strong force) potential energy—a strong, short-range attraction with a repulsive hard core.

Strong force

→Strong
→ attractive
→Short-ranged (about 2 fm)
→Nearly identical between
  Proton-proton
  Neutron-neutron
  Proton-neutron

Figure 11.4 The basic elements of the internucleon (strong force) potential energy—a strong, short-range attraction with a repulsive hard core.



Nuclear binding: Two nucleons

- Protons and neutrons are subject to Exclusion Principle, independently
- Parallel spin arrangements in internucleon attraction are more stable

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Example:

Use two nucleon arrangements and figure which arrangement is most stable:

1. Proton-proton
2. Neutron-neutron
3. Proton-neutron

Nuclear binding: Two nucleons

- Protons and neutrons are subject to Exclusion Principle, independently
- Parallel spin arrangements are more stable

Example:

Proton-neutron pair can be most stably arranged

Experimental evidence:

- Deuterium's nucleus' (deuteron) total spin is 1
- Deuteron has one single bound state.

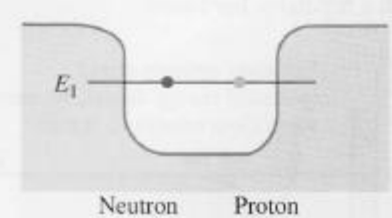
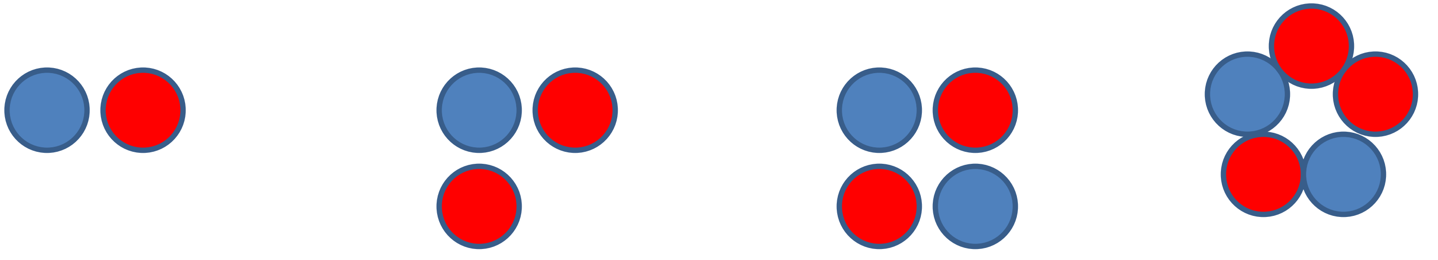


Figure 11.5 The deuteron's neutron and proton bound in a well resulting from their attractive potential energy.

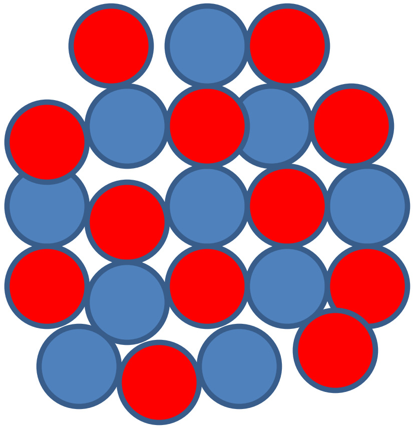
Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:



Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:
    - The number of bonds each nucleon can have increases, thus binding energy per nucleon increases
  - When the total number of nucleons is large



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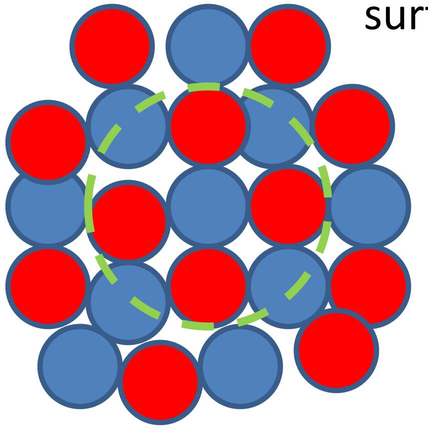


surface nucleons

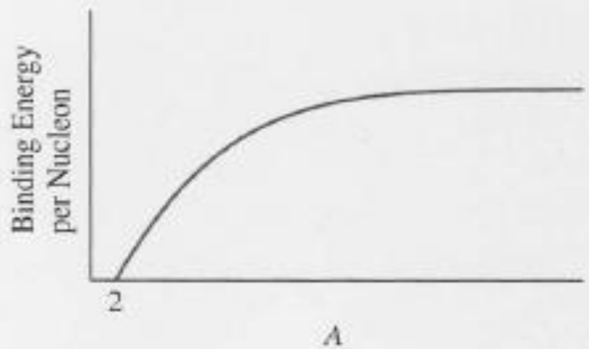
Completely-Surrounded nucleons

Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:
    - The number of bonds each nucleon can have increases, thus binding energy per nucleon increases
  - When the total number of nucleons is large
    - Since strong force is short ranged, making each nucleon has the same number of surrounding nucleons.
    - The nucleons at the surface are not completely surrounded. The proportion of surface nucleons diminish by 1/r

Only strong force is considered

Figure 11.6 Binding energy per nucleon due to the strong internucleon attraction only. The smallest nuclei have few bonds per nucleon. In large nuclei, many nucleons are surrounded.



Coulomb repulsion

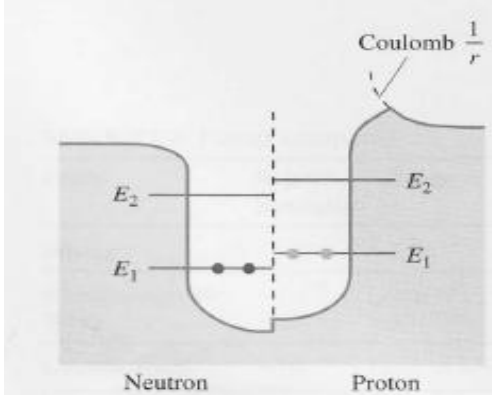
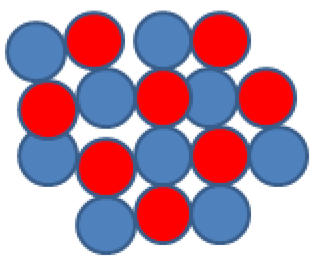


Figure 11.7 Coulomb repulsion raises proton energies.



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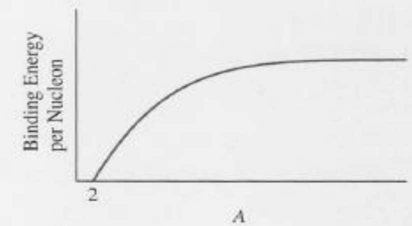

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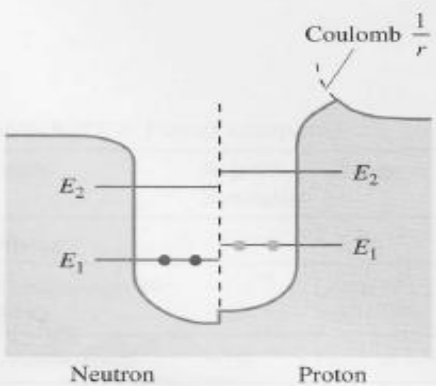
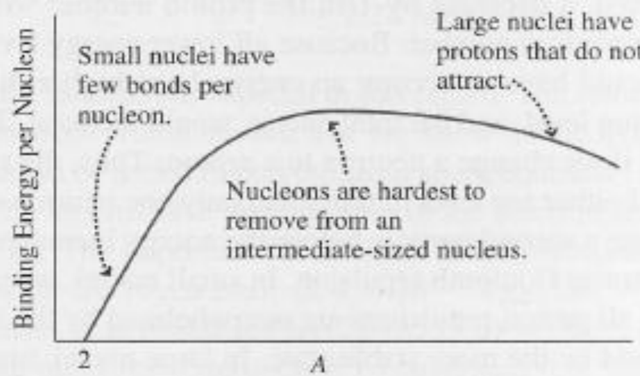
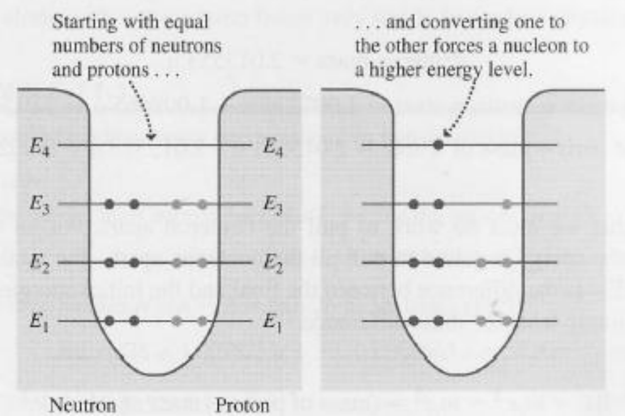


Figure 11.8 Binding energy per nucleon due to both the strong internucleon attraction and Coulomb repulsion.



Exclusion principle

Figure 11.9 Ignoring Coulomb repulsion, the exclusion principle argues that for a given number of nucleons, the lowest energy should have $N = Z$.



For small nuclei, repulsive coulomb interactions are less influential

When N=Z,

When N≠Z

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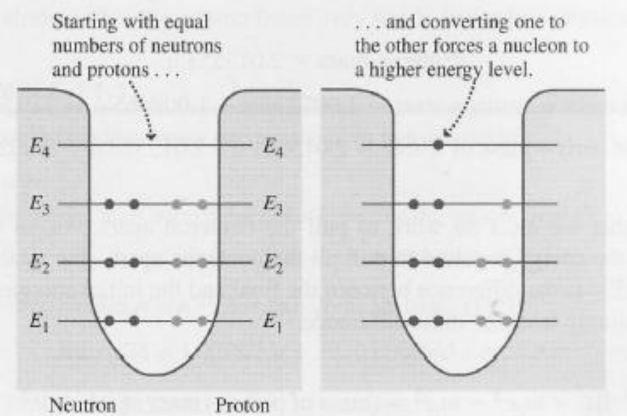

For small nuclei, repulsive coulomb interactions are less influential

When N=Z, more stable

When N≠Z, energy raised

Exclusion principle

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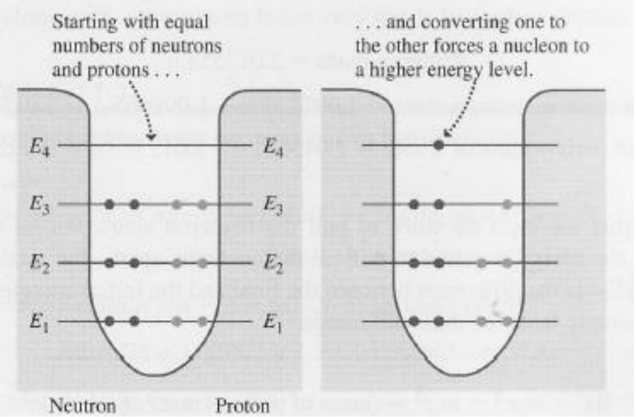
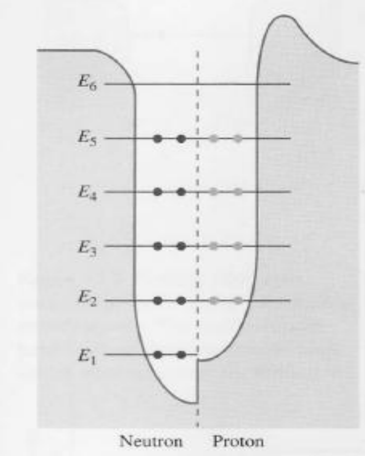


Figure 11.10 In large nuclei, when Coulomb repulsion becomes significant, the lowest energy should have $N > Z$.

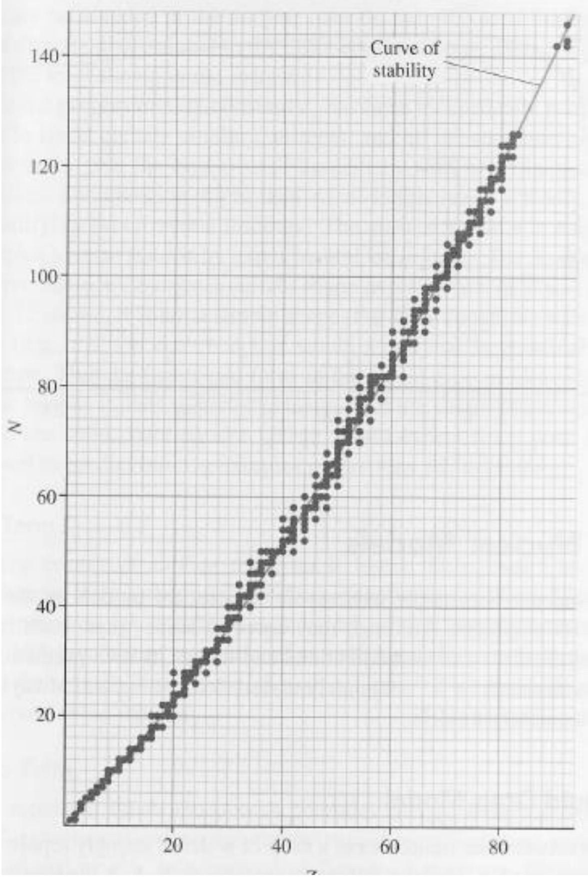


For large nuclei, repulsive force changes

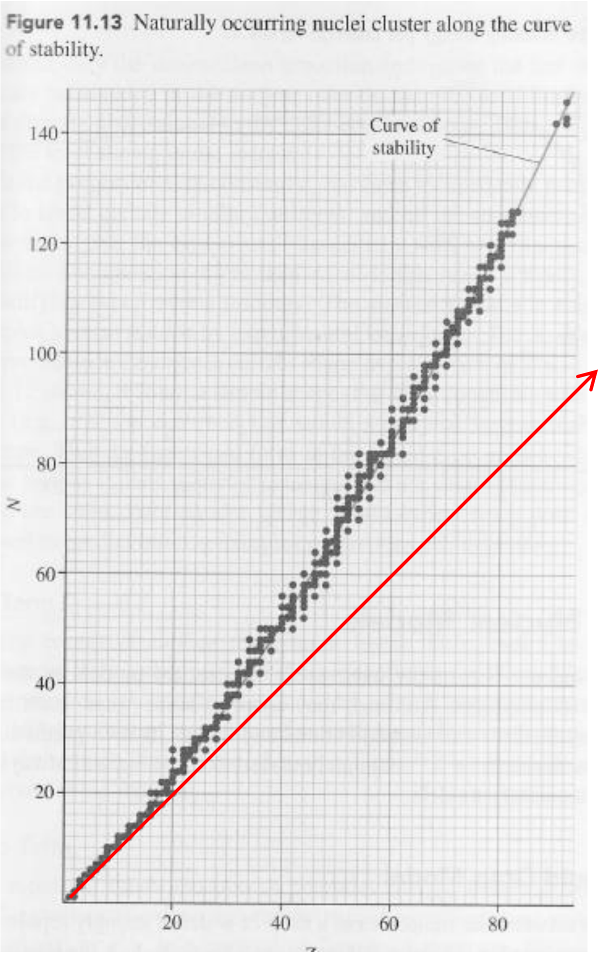
N>Z is more stable

Binding energy/nucleon over Z

Figure 11.13 Naturally occurring nuclei cluster along the curve of stability.



Curve of stability



For smaller nuclei,
N=Z to be stable

As nuclei get larger,
N > Z to be stable

Binding energy

$$\text{Binding energy} = (\text{mass of individual nucleons} - \text{mass of nucleus})c^2$$

$$\text{Deuteron} = 1\ \text{proton} + 1\ \text{neutron}$$

$$\text{Binding Energy} = (\text{proton mass} + \text{neutron mass} - \text{Deuteron mass})c^2$$

m_P =1.007276 u
m_N=1.008665 u
m_D=2.013553 u
uc^2=931.5 MeV

Binding energy

$$\text{Binding energy} = (\text{mass of individual nucleons} - \text{mass of nucleus})c^2$$

Deuteron = 1 proton + 1 neutron

$$\text{Binding Energy} = (\text{proton mass} + \text{neutron mass} - \text{Deuteron mass})c^2$$

$$= (1.007276\, u + 1.008665\, u - 2.013553\, u)c^2 = 0.002388\, uc^2$$
$$= 0.002388\, x\, 1.661\, x\, 10^{-27} kg\ x\ (3x10^8 m/sec)^2 = 2.22\ MeV$$

Binding energy

$$Binding\ Energy = \left(Zm_H + Nm_n - M_{^A_ZX}\right)c^2$$

Where m_H= atomic mass of hydrogen

m_n= neutron mass

$M_{^A_ZX}$ = atomic mass of the nucleus

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Example: $^{56}_{26}Fe = 53.934939\,u$

m_H =1.007825 u

m_N=1.008665 u

uc^2=931.5 MeV

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Example: $^{56}_{26}Fe = 53.934939\,u$

Binding energy = (26 x 1.007825 u +30 x 1.008665 u – 55.934939 u) c^2=0.528461 uc^2 =492.3 MeV (consider $c^2 = 931.5\ MeV/u$).

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

$$C_1 = 15.8$$
$$C_2 = 17.8$$
$$C_3 = 0.71$$
$$C_4 = 23.7$$

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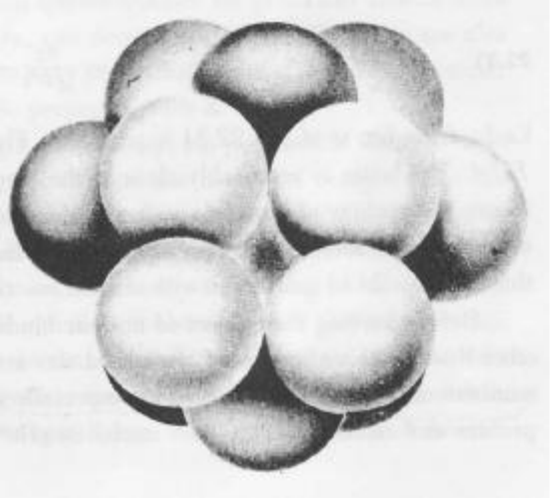

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Strong force (C1 and C2)

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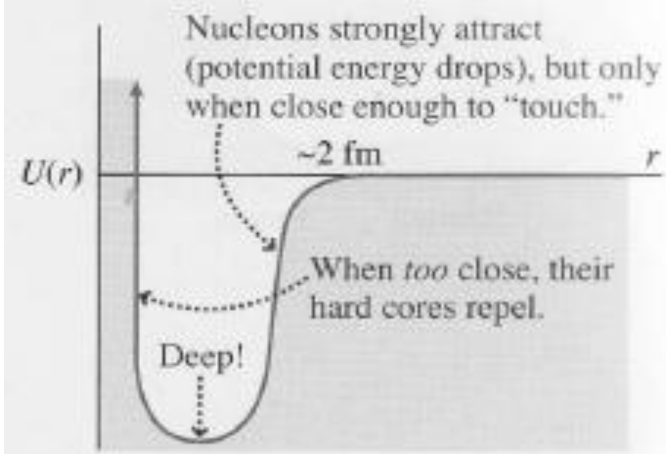

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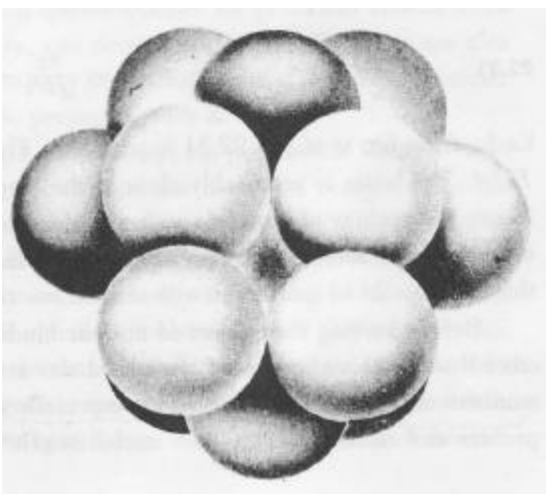

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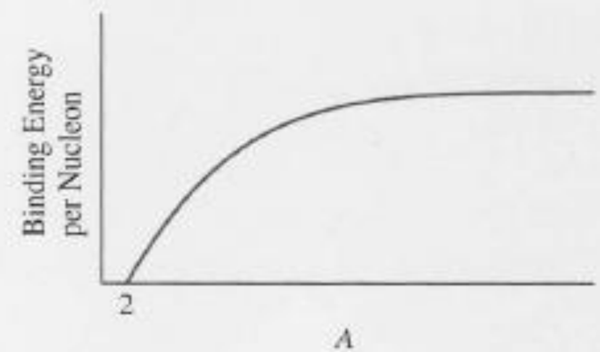

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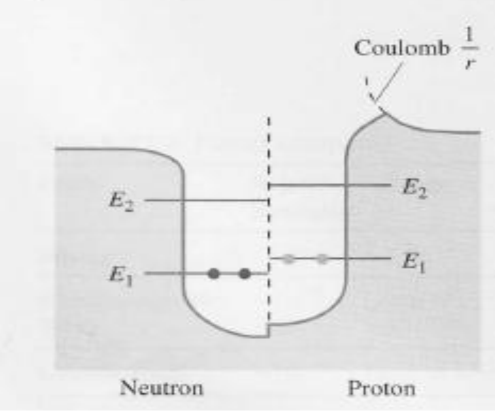
Figure 11.7 Coulomb repulsion raises proton energies.



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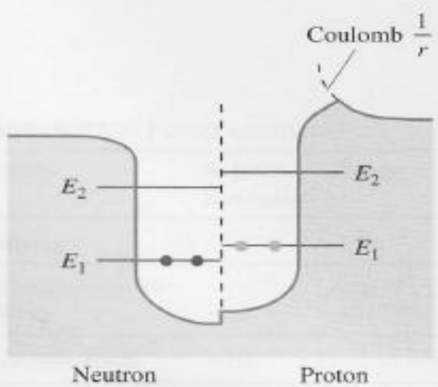

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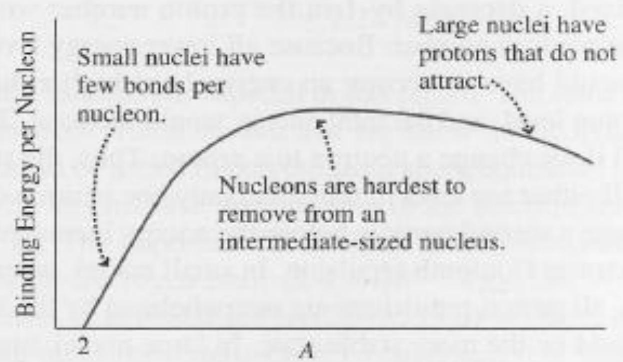

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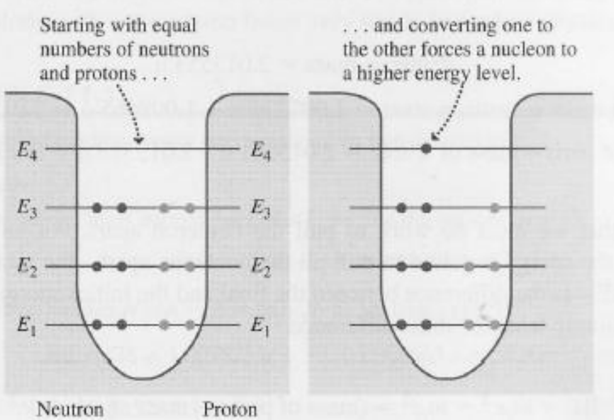
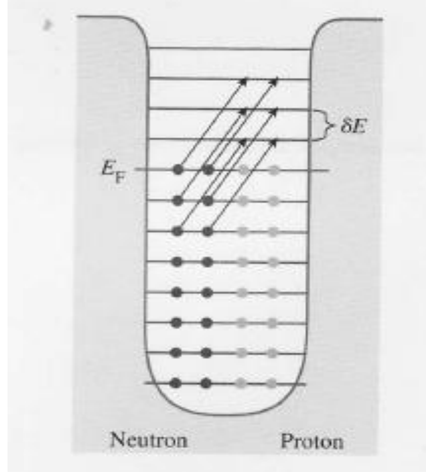


Figure 11.15 If j neutrons become protons, the energy increases by $\frac{1}{2} j^2 \delta E$.



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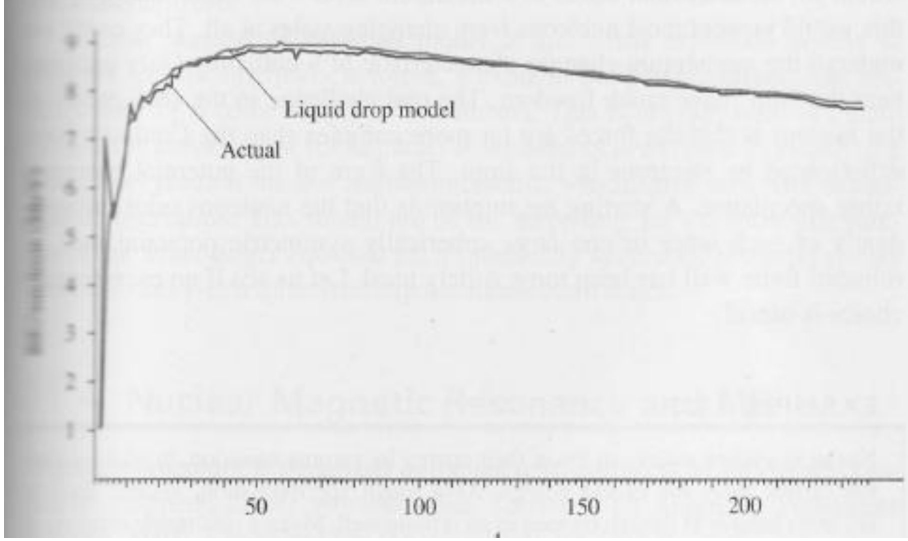
$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

11.16 Comparison of the semiempirical binding energy formula and actual
nucleon.



Magic numbers= 2, 8, 20, 28, 50, 82, 126

Shell Model

	Atomic Shell Model	Nuclear Shell Model
Potential	Electrostatic between nucleus and electrons	Net effect of all the forces nucleons experience in a nucleus
Magic numbers	Atoms with closed electronic shells are stable such as He (2 electrons), Ne (10), Ar (18), Kr (36), Xe (54), Rn (86).	Nuclei are particularly stable when the number of nucleons is 2, 8, 20, 28, 50, 82, and 126.
Exclusion principle	Electrons follow the Exclusion principle	Protons and neutrons separately follow the Exclusion Principle
Movement	Electrons move in orbitals	Nucleons do not move like electrons because most nucleons fill states to a maximum level, preventing them from changing momentum.

Electrons: Atomic Orbital

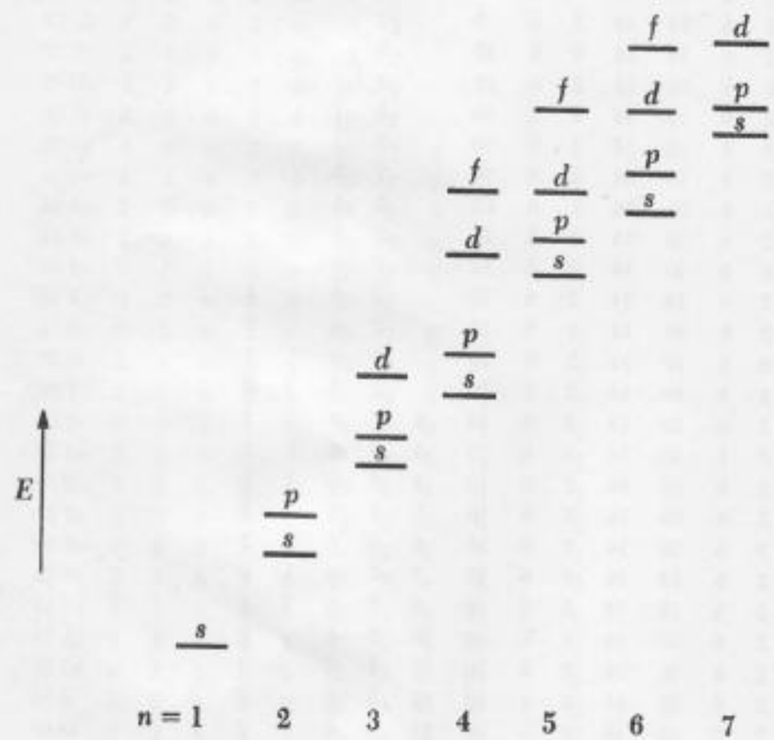


FIGURE 10.7 The sequence of quantum states in an atom. Not to scale.

Electrons: Atomic Orbital

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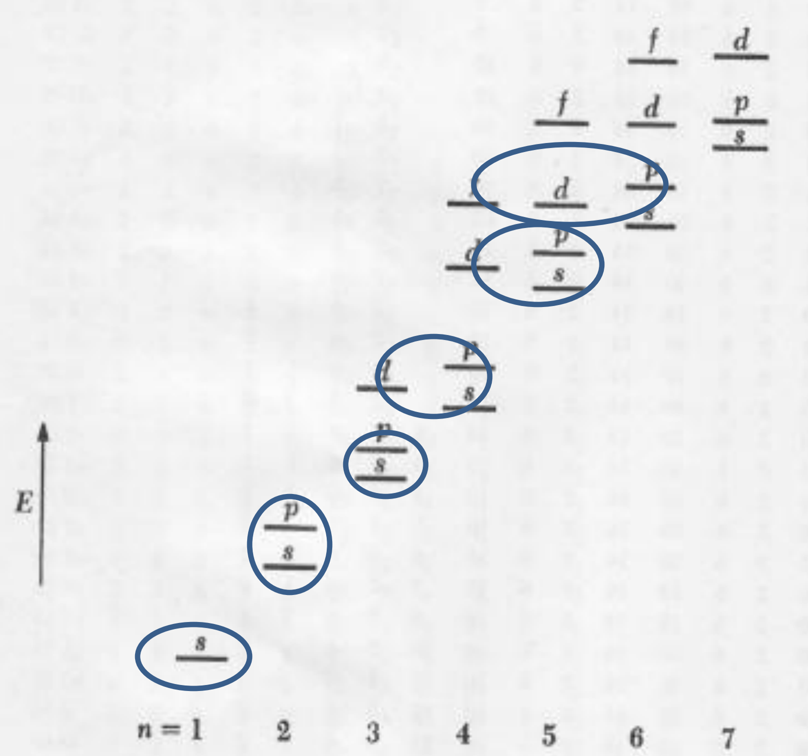


86: Rn

54: Xe

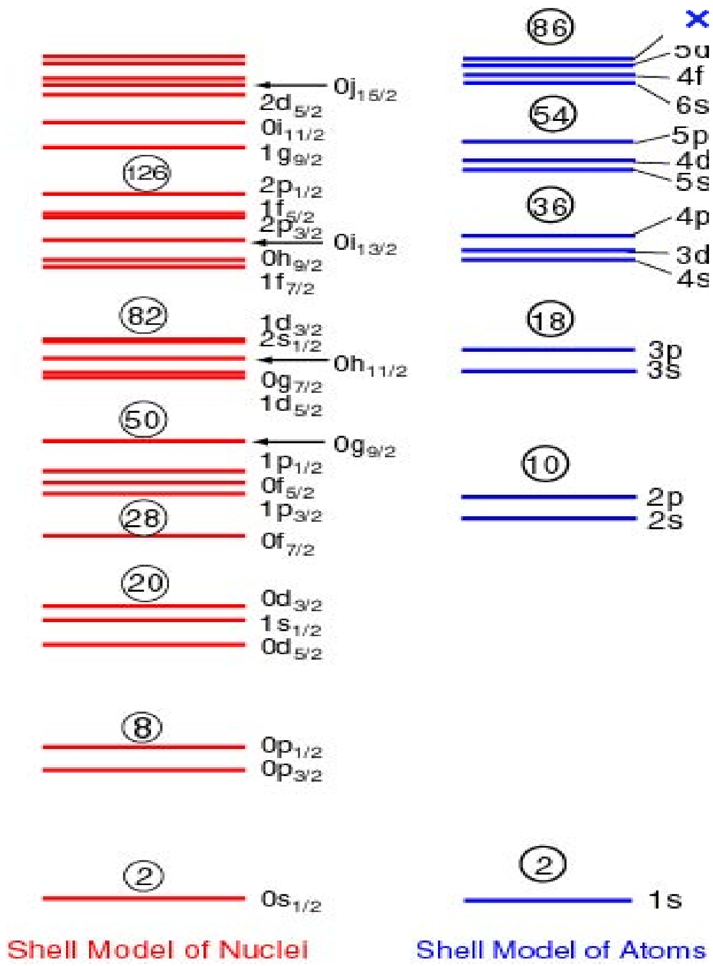
36: Kr

18: Ar

10: Ne

2: He

Nuclear Shell Model



3 dimensional harmonic oscillator

$$\frac{-\hbar^2}{2m}\nabla^2\psi(x) + \frac{1}{2}\mu\omega^2x^2\,\psi(x) = \mathrm{E}\,\psi(x)$$

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$$\psi_{klm}(r,\theta,\phi) = N_{kl} r^l e^{-\nu r^2} L_k^{(l+\frac{1}{2})}(2\nu r^2) Y_{lm}(\theta,\phi)$$

where

$$N_{kl} = \sqrt{\sqrt{\frac{2\nu^3}{\pi}} \frac{2^{k+2l+3}\,k!\,\nu^l}{(2k+2l+1)!!}}$$ is a normalization constant.

$$\nu \equiv \frac{\mu\omega}{2\hbar}$$

$L_k^{(l+\frac{1}{2})}(2\nu r^2)$ are generalized Laguerre polynomials. The order *k* of the polynomial is a non-negative integer.

$Y_{lm}(\theta,\phi)$ is a spherical harmonic function.

3 dimensional harmonic oscillator

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$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

$n \equiv 2k + l$

- For every even $n, l = 0, 2, \ldots n-2, n$
- For every odd $n, l = 1, 3, \ldots n-2, n$
- $-l \leq m \leq l$
- Every n and l, there are $2l+1$ energy degeneracies, which can accommodate $2(2l+1)$ nucleons

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	1	1	6			
4	0	4	18	30	70	$\frac{11}{2}\hbar\omega$
	1	2	10			
	2	0	2			

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	1	2	10			
	2	0	2			
5	0	5	22	42	112	$\frac{13}{2}\hbar\omega$
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Magic numbers= 2, 8, 20, 28, 50, 82, 126

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Not Magic Numbers!!!

LS coupling

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in (n, $|\vec{j}|$)	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2	6	$\frac{5}{2}\hbar\omega$

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LS coupling

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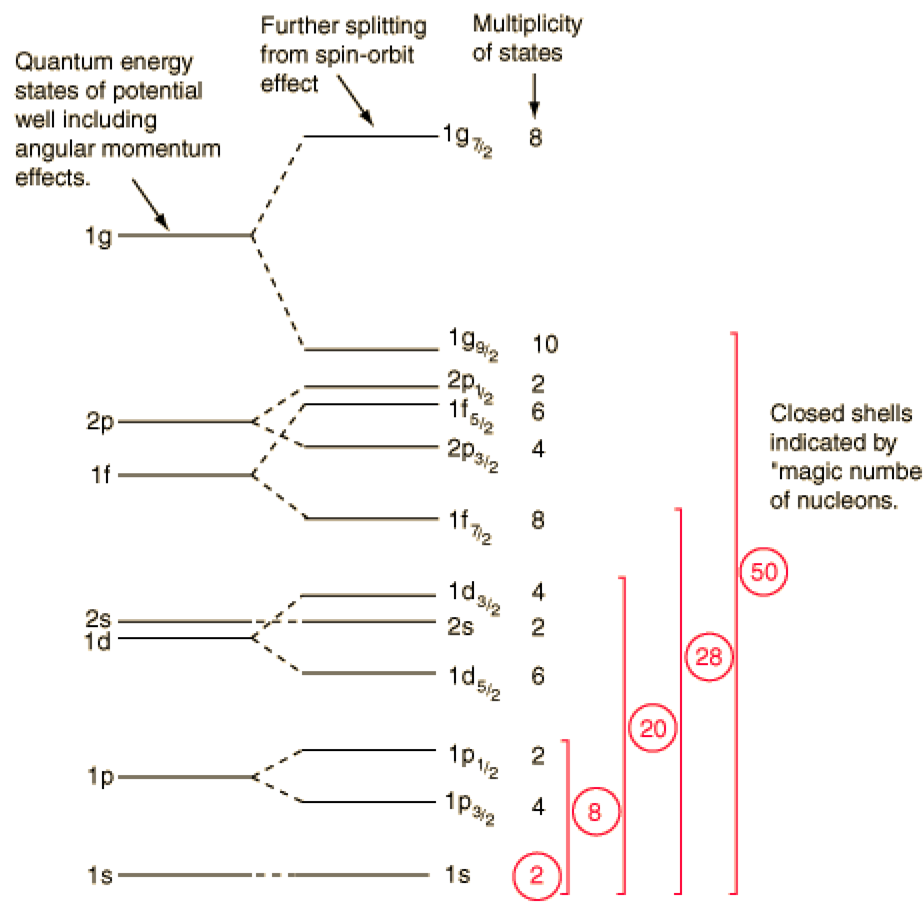
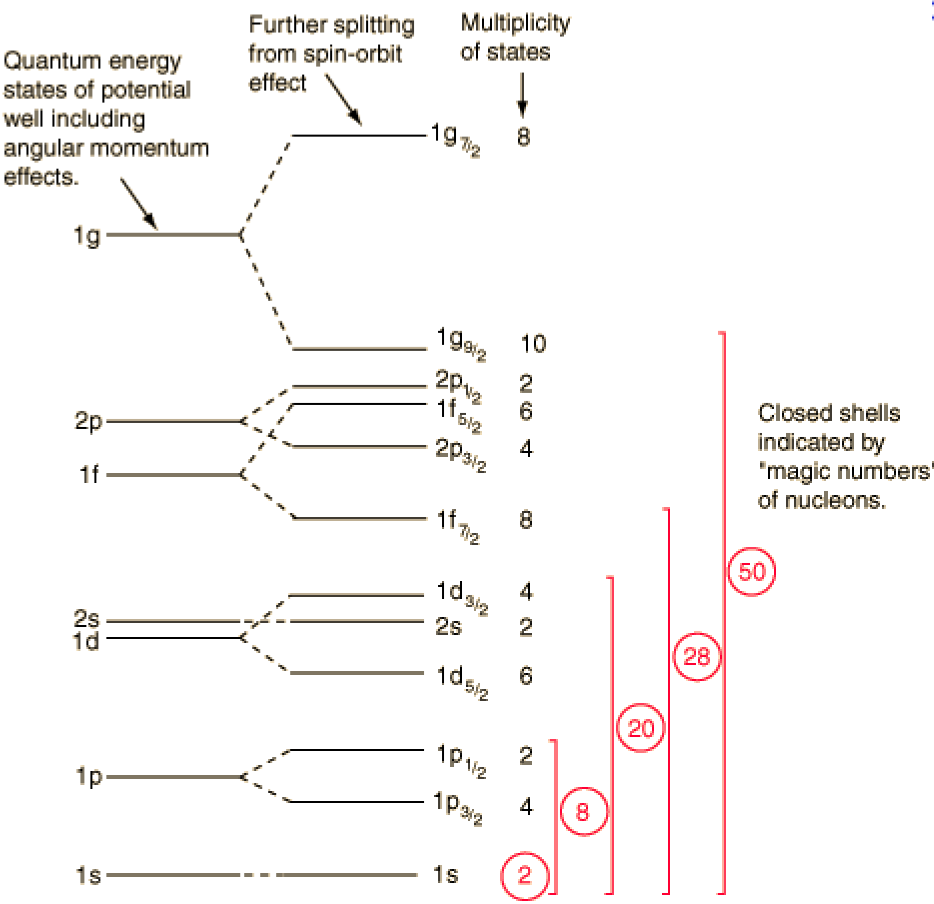
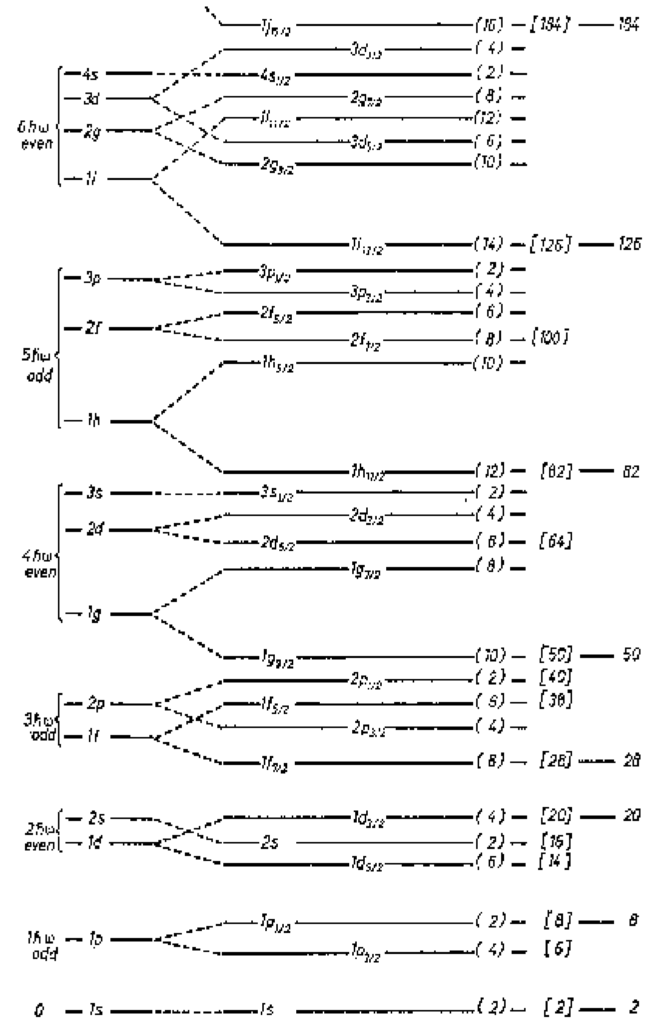
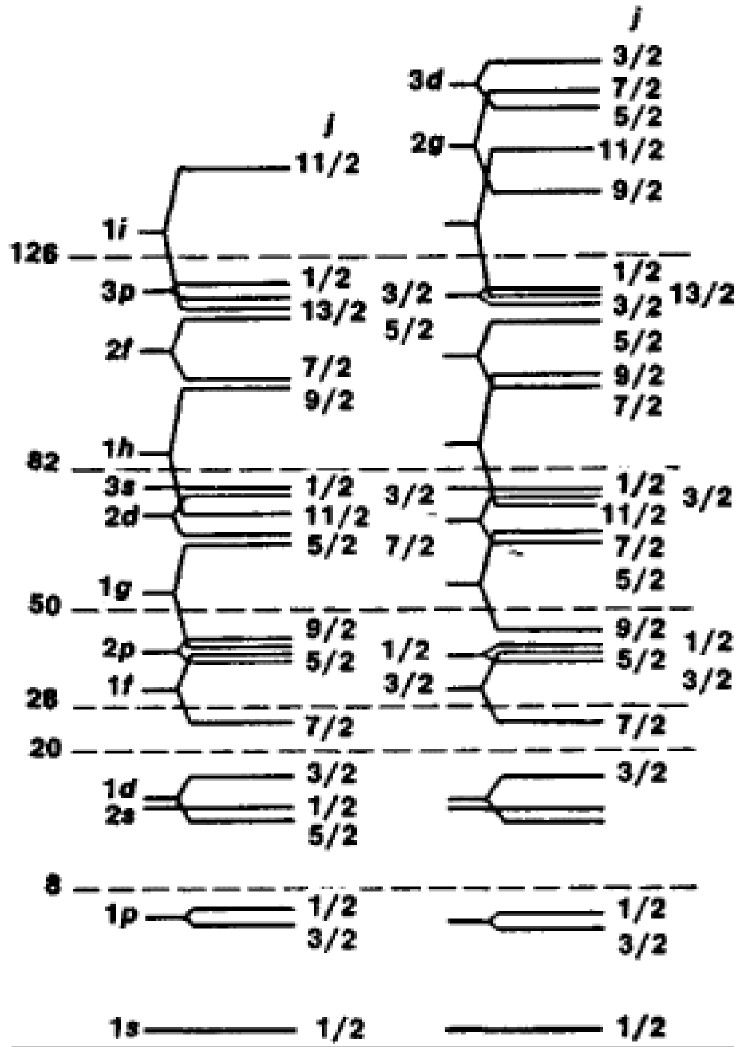




Quantum energy states of potential well including angular momentum effects.
Further splitting from spin-orbit effect
Multiplicity of states
1g
2p
1f
2s
1d
1p
1s
$1g_{7/2}$ 8
$1g_{9/2}$ 10
$2p_{1/2}$ 2
$1f_{5/2}$ 6
$2p_{3/2}$ 4
$1f_{7/2}$ 8
$1d_{3/2}$ 4
2s 2
$1d_{5/2}$ 6
$1p_{1/2}$ 2
$1p_{3/2}$ 4
1s
2
8
20
28
50
Closed shells indicated by "magic numbers" of nucleons.



184
126
82
50
28
20
8
2



Proton Neutron