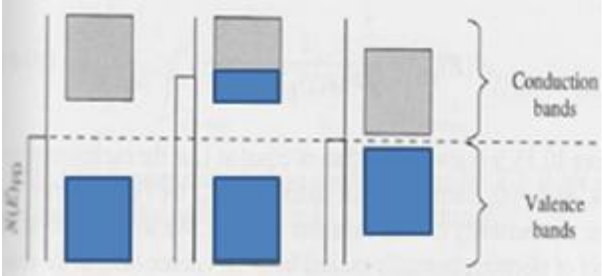
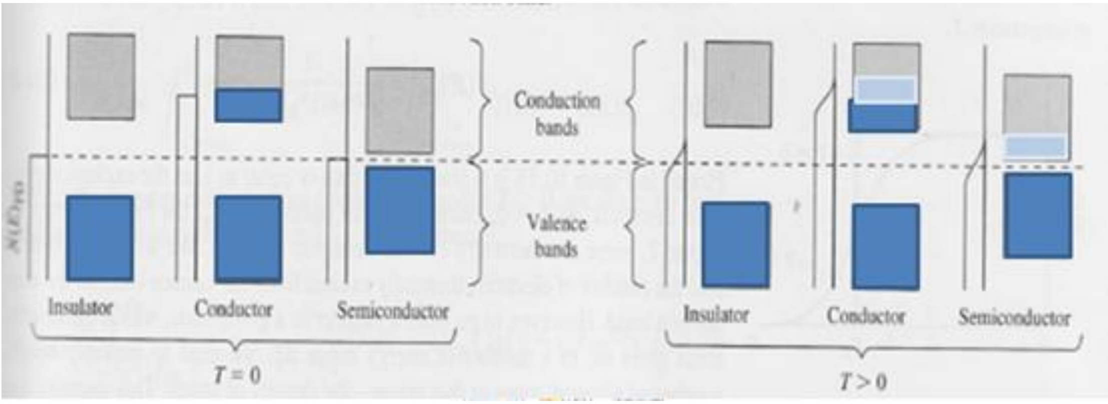
Lecture 13 Topics

- Charge carriers
  - Conduction electrons
  - Valence holes
  - Effective mass
- Extrinsic semiconductors
  - n-type
  - p-type
- Semiconductor devices
  - Diode
  - Transistor
- Superconductivity
  - Characteristics: Perfect conductivity and perfect diamagnetism
  - Type 1, Type 2, and High Tc Superconductors
  - BCS Theory and Cooper Pairs



At T = 0


N(E)f_FD
Conduction bands
Valence bands
Insulator
Conductor
Semiconductor
T = 0
Insulator
Conductor
Semiconductor
T > 0

At T = 0

T > 0

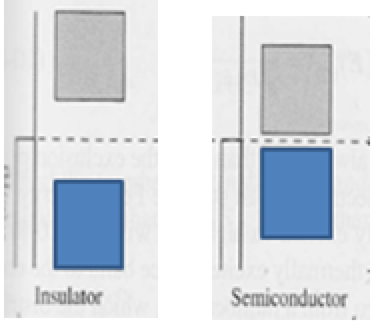
$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_BT}+1} \qquad D(E) = \mathrm{D}$$

$$N_{Valence} = \int_{E_{valence-bottom}}^{E_{valence-top}} \mathcal{N}(E)D(E)dE$$

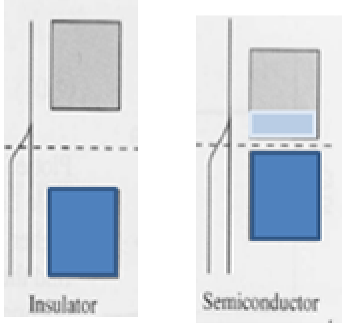
$$= \int_{E_{valence-bottom}}^{E_{valence-top}} 1 \cdot D dE = D\Delta E_{Valence}$$

$$N_{Excited} = \int_{E_F+\frac{1}{2}E_{gap}}^{\infty} \mathcal{N}(E)D(E)dE$$

$$= \int_{E_F+\frac{1}{2}E_{gap}}^{\infty} \frac{D}{e^{(E-E_F)/k_BT}+1} dE = Dk_BT\ln(1+e^{-E_{gap}/2k_BT}) \sim Dk_BTe^{-E_{gap}/2k_BT}$$



T=0



T > 0

$$\frac{N_{Excited}}{N_{Valence}} = \frac{Dk_BTe^{-E_{gap}/2k_BT}}{D\Delta E_{Valence}} = \underbrace{\frac{k_BT}{\Delta E_{Valence}}}_{0.0026\text{ eV}} e^{-E_{gap}/2k_BT}$$

Insulator: 5 eV

~ 10^{-42}

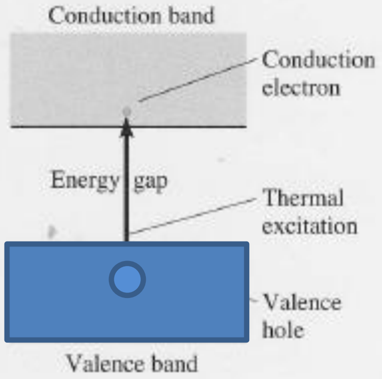
Semiconductor: 1 eV

~ 10^{-8}

Two types of charge carriers

Conduction band electrons

Figure 10.35 Thermal excitation creating a pair of charge carriers.



Two types of charge carriers

Conduction band electrons

Valence band holes

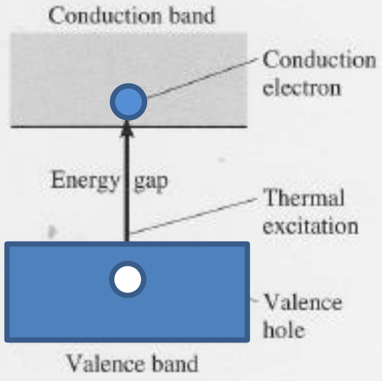


Figure 10.35 Thermal excitation creating a pair of charge carriers.

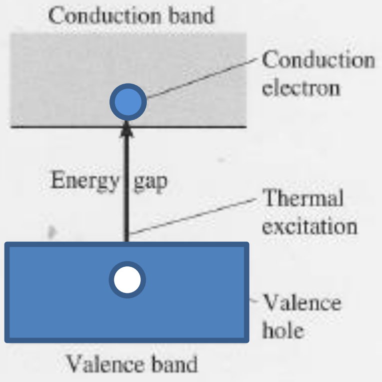
Two types of charge carriers

Conduction band electrons

Valence band holes

- created by electrons left the valence band at T > 0
- Behave like positive charge carriers
- Are free to move in the valence band
- Are always found near the top of the valence band

Figure 10.35 Thermal excitation creating a pair of charge carriers.



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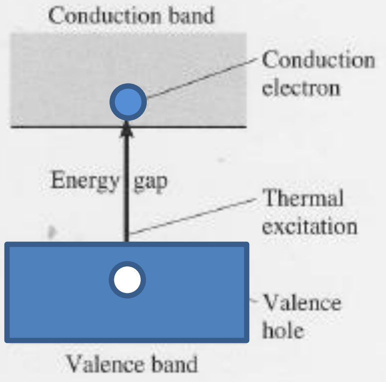
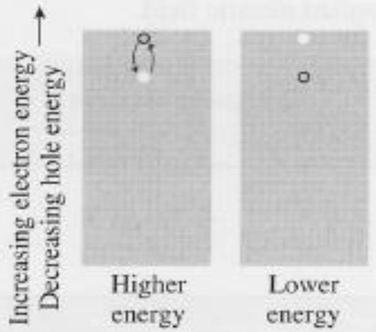


Figure 10.36 Holes float.



Effective Mass

m_{eff} : the ratio between external force and acceleration

$$m_{eff} = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$

For free electrons, $E = \dfrac{\hbar^2 k^2}{2m}$

m_{eff}=

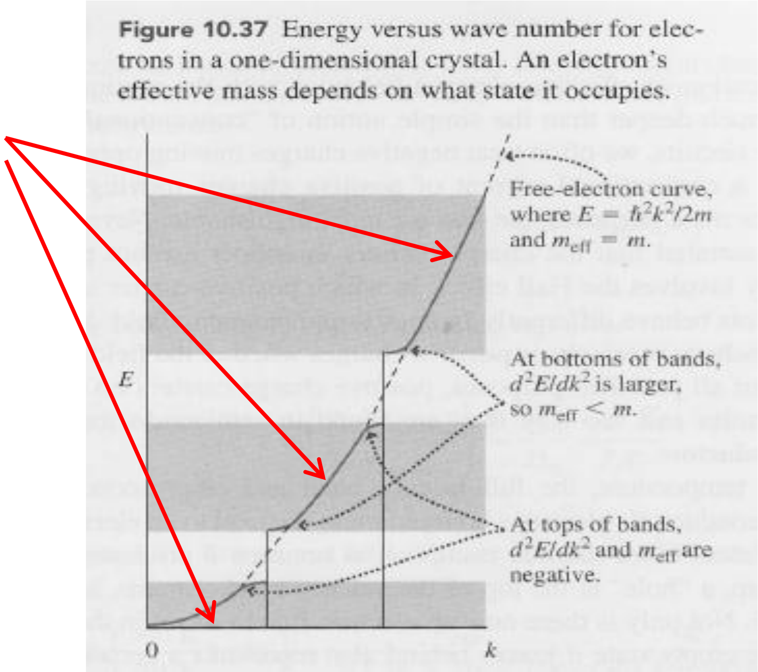


Figure 10.37 Energy versus wave number for electrons in a one-dimensional crystal. An electron's effective mass depends on what state it occupies.

Free-electron curve, where $E = \hbar^2k^2/2m$ and $m_{\text{eff}} = m$.

At bottoms of bands, d^2E/dk^2 is larger, so $m_{\text{eff}} < m$.

At tops of bands, d^2E/dk^2 and m_{eff} are negative.

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At the top of each band

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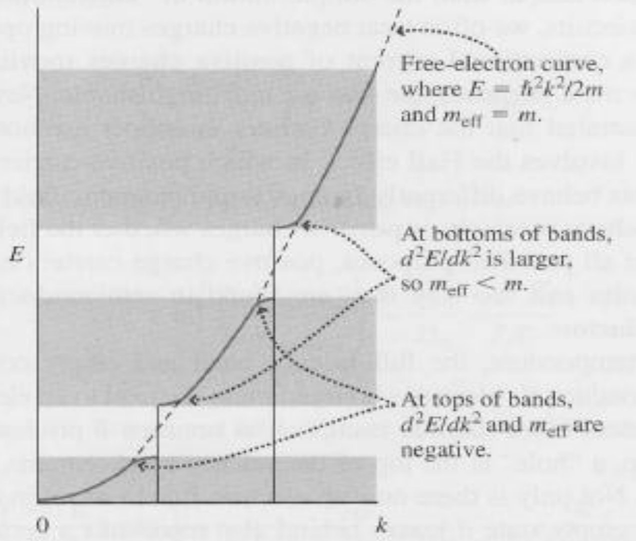

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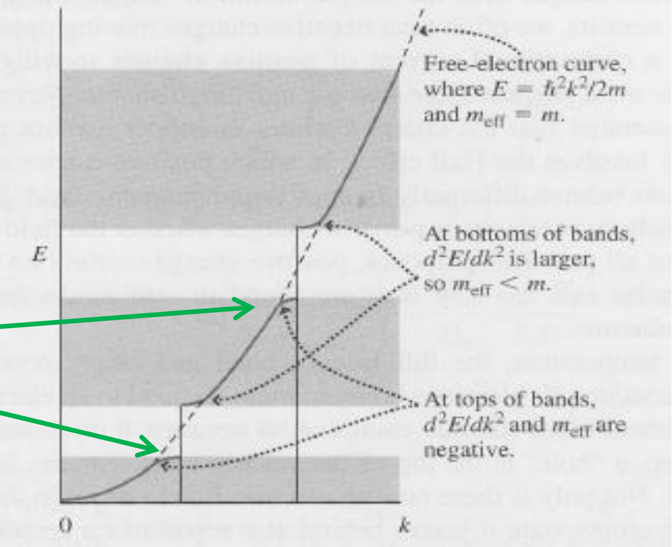
$\frac{d^2E}{dk^2} < 0$ thus , $m_{eff} < 0$



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$m_{eff} = m$

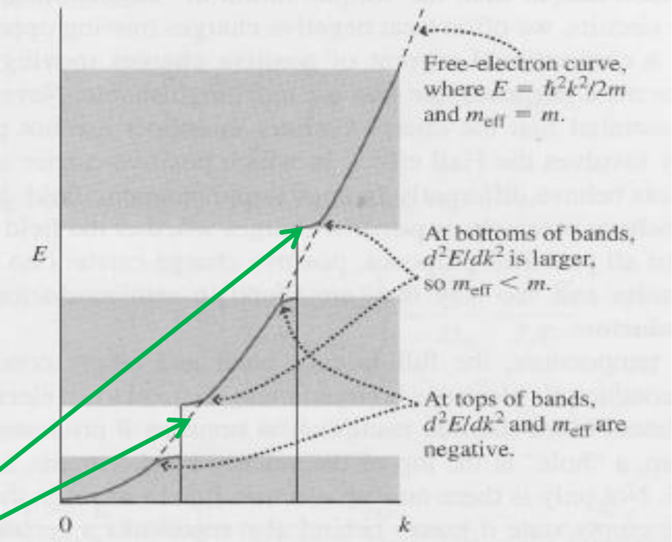
At the top of each band

$$\frac{d^2E}{dk^2} < 0 \quad \text{thus} \, , \ m_{eff} < 0$$

At the bottom of each band

$$\frac{d^2E}{dk^2}\bigg| bottom > \frac{d^2E}{dk^2}\bigg| free\ electron \qquad m_{eff} < m$$

Figure 10.37 Energy versus wave number for electrons in a one-dimensional crystal. An electron's effective mass depends on what state it occupies.



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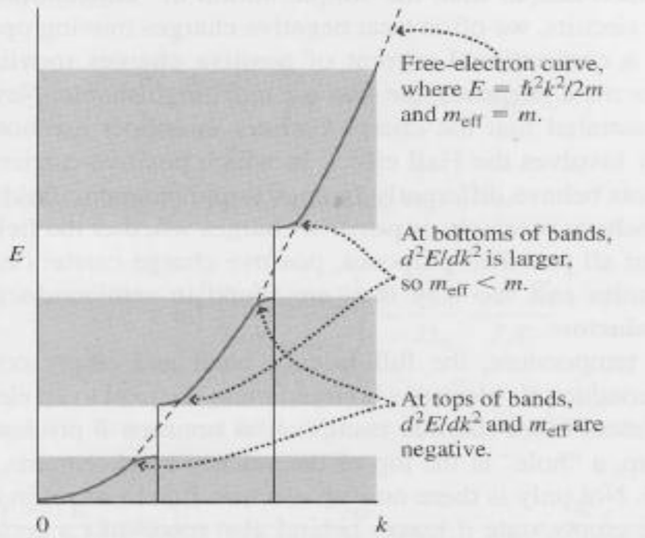

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At bottoms of bands, d^2E/dk^2 is larger, so $m_{eff} < m$.

At tops of bands, d^2E/dk^2 and m_{eff} are negative.

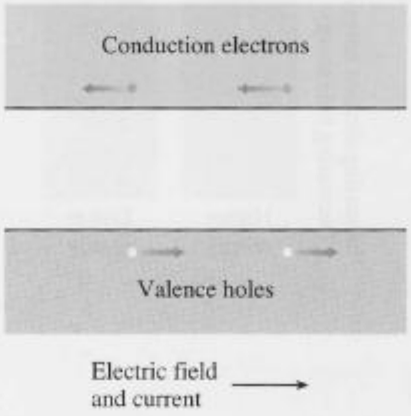
Use Holes to get the positive effective mass!!

At the bottom of each band

$$\left.\frac{d^2E}{dk^2}\right| bottom > \left.\frac{d^2E}{dk^2}\right| free\ electron \qquad m_{eff} < m$$

Current in an E field

Figure 10.38 In a semiconductor at $T > 0$, both holes and electrons contribute to current in the direction of an applied electric field.



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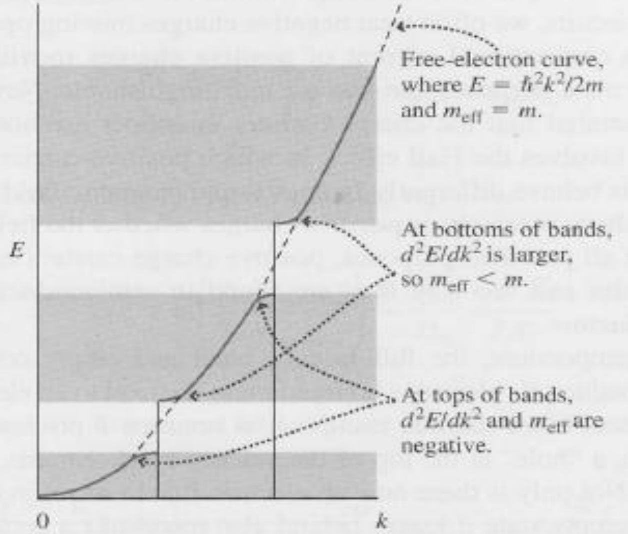
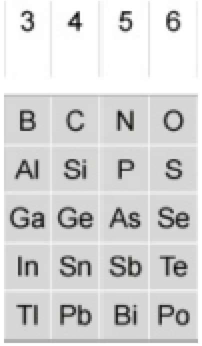


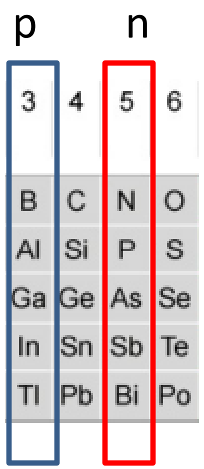
Doping

- Intrinsic semiconductors
- Extrinsic semiconductors:
  - n-type:
    - Use elements with 5 valence electrons as impurities
    - Extra electron per impurity atom
  - p-type:
    - Use elements with 3 valence electrons as impurities
    - A hole per impurity atom



3	4	5	6
B	C	N	O
Al	Si	P	S
Ga	Ge	As	Se
In	Sn	Sb	Te
Tl	Pb	Bi	Po

Doping



- Intrinsic semiconductors
- Extrinsic semiconductors:
  - n-type:
    - Use elements with 5 valence electrons as impurities
    - Extra electron per impurity atom
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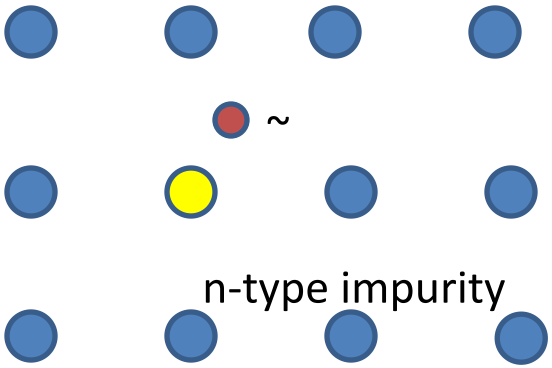
    - A hole per impurity atom

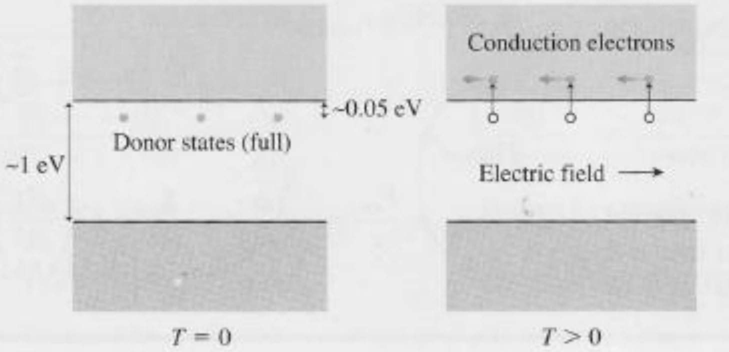
n-type semiconductor

Si: Semiconductor



n-type impurity

Figure 10.39 At $T = 0$, extra electrons in an n-type semiconductor occupy donor states. At $T > 0$, they easily become conduction electrons.



-create donor states below the conduction band.

-electrons in the donor states can be easily excited to be in the conduction band.

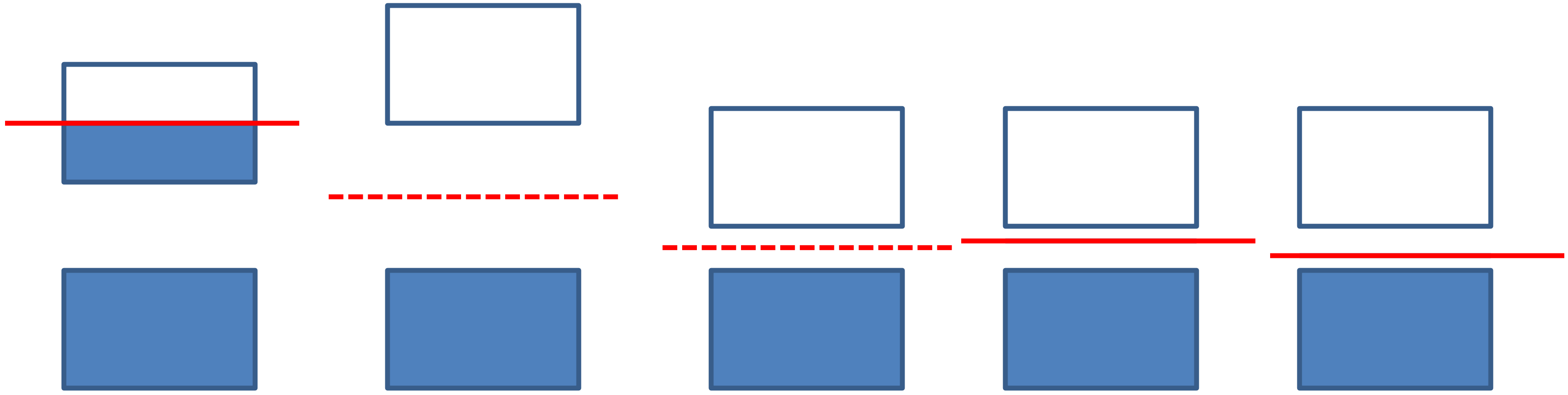
-since impurities are sparse, the donor states do not form energy bands and do not harbor holes.

-in the conduction band, electrons from the donor states are more abundant than those from the valence band

-majority carrier in n-type → conduction band electrons

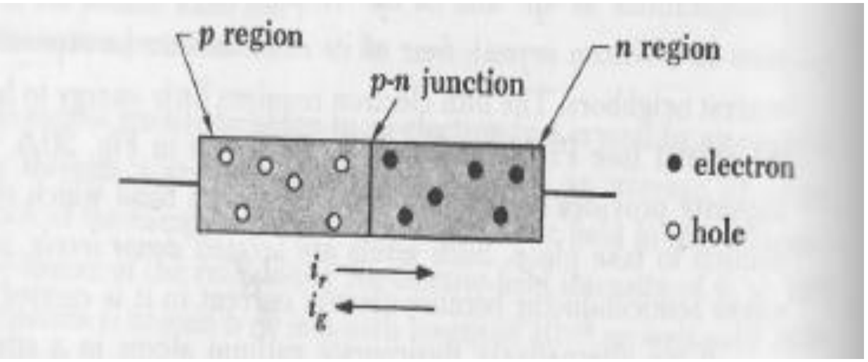
-minority carrier in n-type → holes in valence band

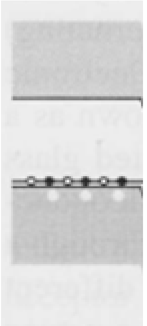
Fermi Energy



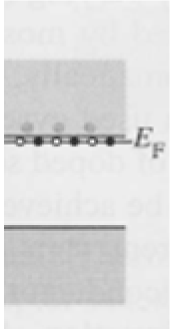
Conductor insulator semiconductor n-type p-type

Unbiased Diode (p-n)



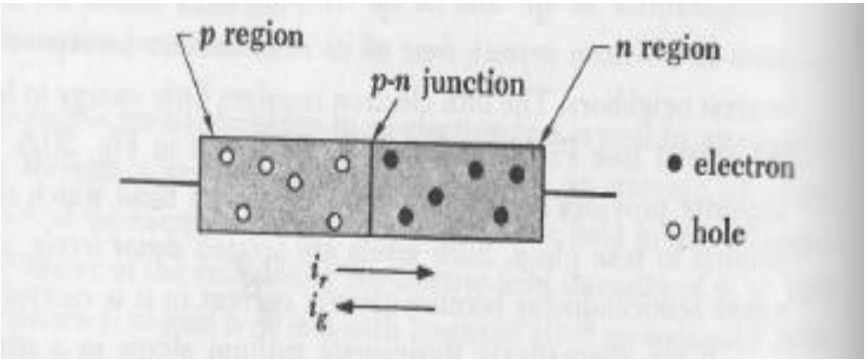


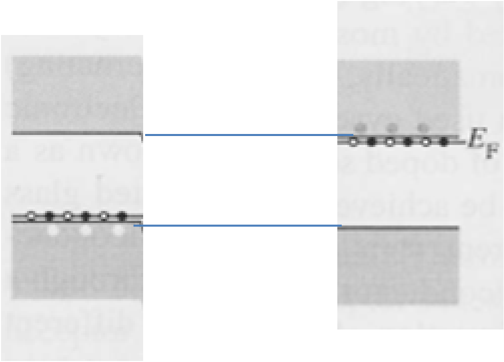
p-type



n-type

Unbiased Diode (p-n)



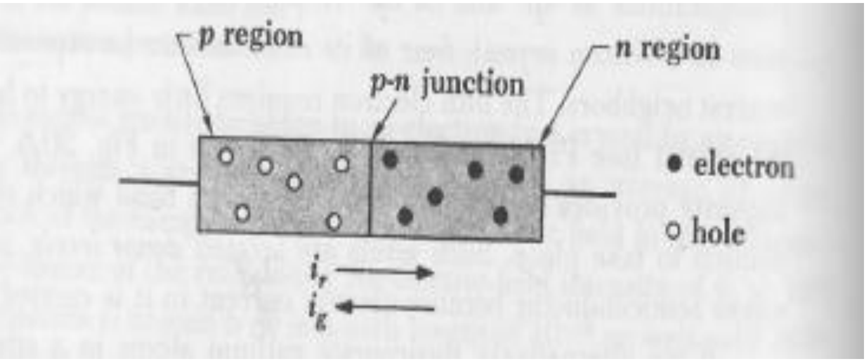


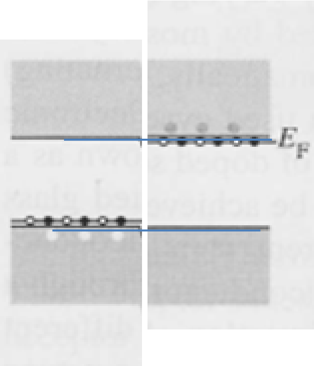
p-type

n-type

What happens when put together?

Unbiased Diode (p-n)





p-type

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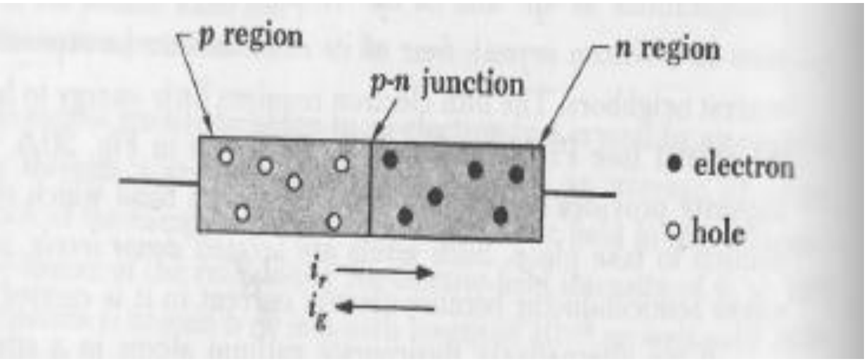
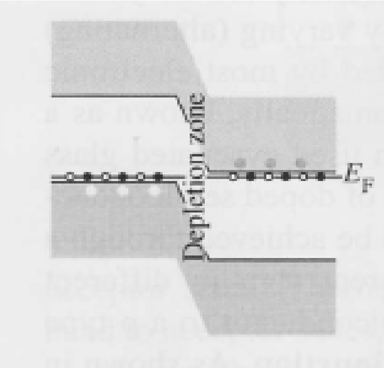
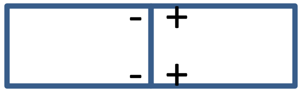






p-type n-type

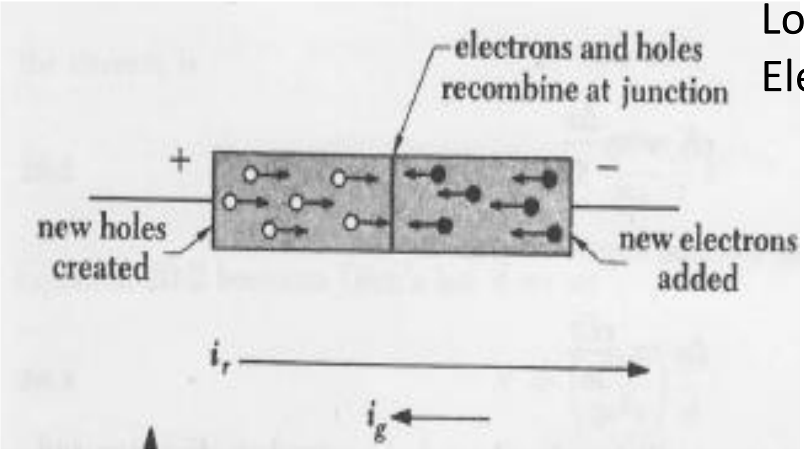
-Diffusion of excessive electrons/holes

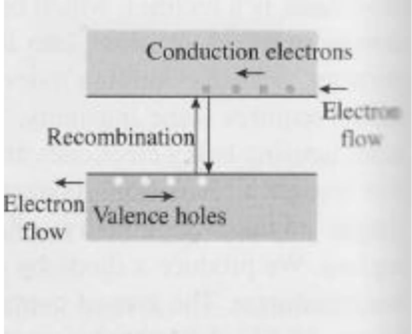
-Coulomb interactions between new charge distributions resulting from diffusion of electrons and holes

-Depletion zone

Forward biased

Higher potential at p end
Lower potential at n-end
Electrons are supplied at n-end

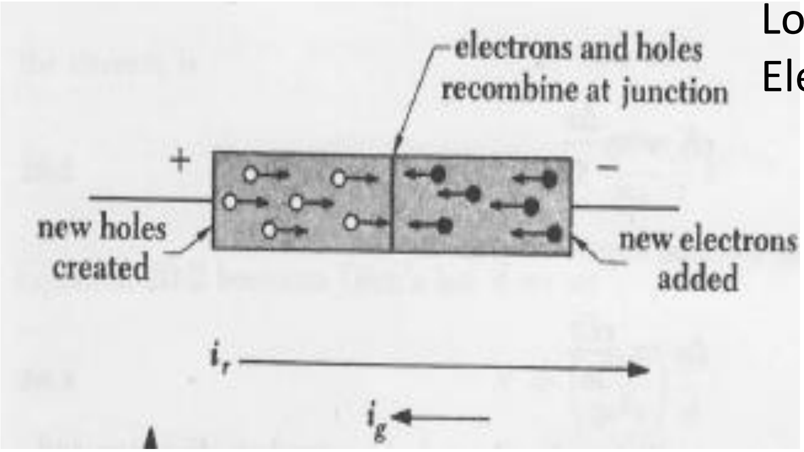


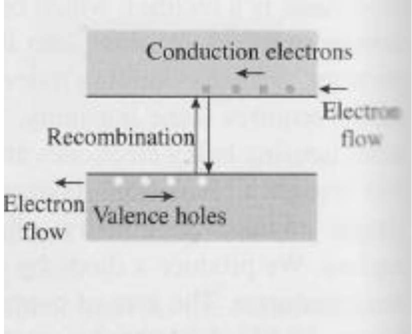


(c) Forward bias

Forward biased

Higher potential at p end
Lower potential at n-end
Electrons are supplied at n-end



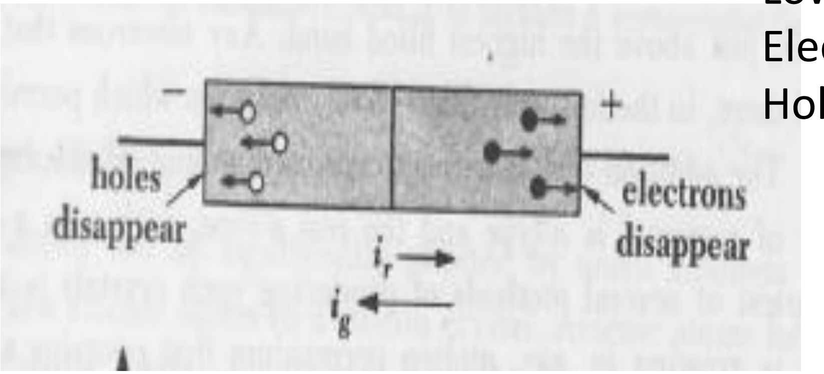


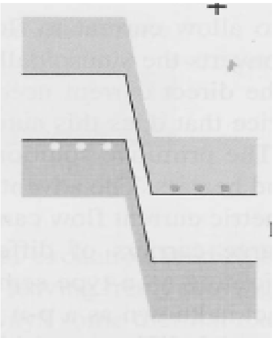
(c) Forward bias

-electrons and holes recombine when they meet
-current flows continuously
-forward-biasing shifts the n-type conduction band upward, allowing higher energy conduction electrons to fill lower energy valence holes. Energy is lost to heat/light

Reverse Biased Diode

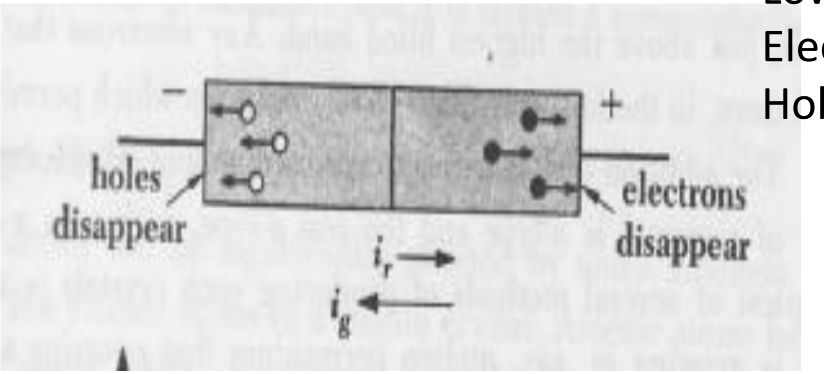
Higher potential at n- end
Lower potential at p-end
Electrons disappear at n-end
Holes disappear at p-end

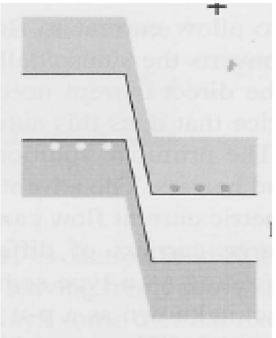



(b) Reverse bias

Reverse Biased Diode

Higher potential at n- end
Lower potential at p-end
Electrons disappear at n-end
Holes disappear at p-end



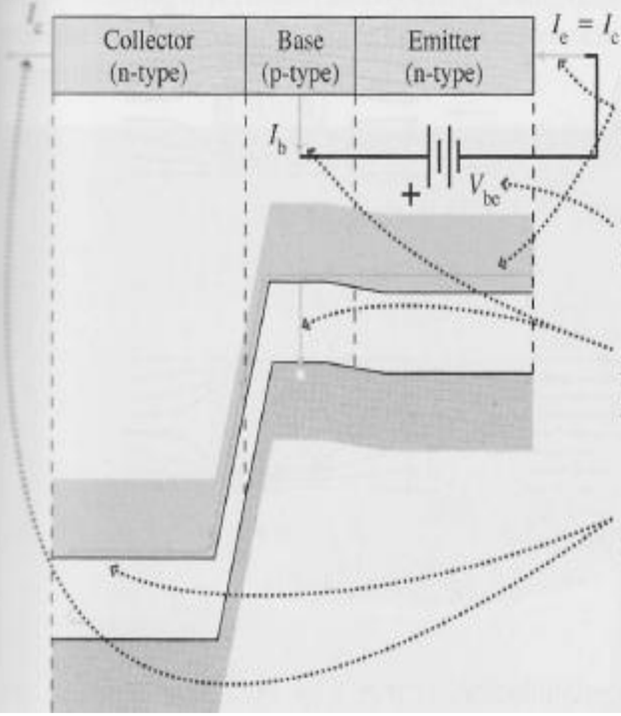


(b) Reverse bias

-depletion region (no free charge carriers)
-current stops immediately
-reverse-biasing makes the potential difference more pronounced
-minor charge carriers (conduction electrons in p and valence holes in n) can recombine at a negligible rate

Transistor (npn)

Figure 10.44 Bands and charge flow in a npn transistor.



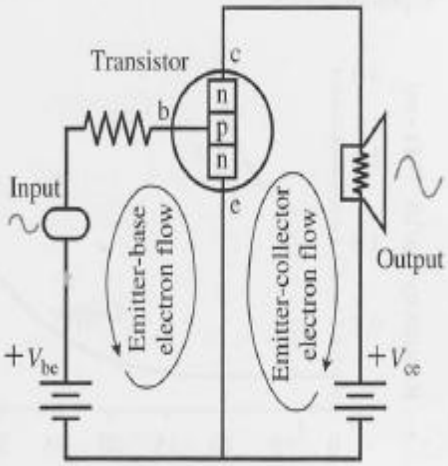
A current of electrons (not conventional current) passing into the base from the emitter . . .

. . . and controlled by the base-emitter bias . . .

. . . divides into I_b, a recombination current flowing out the base, and . . .

. . . the much larger I_c flowing out the collector.

Figure 10.45 The elements of a transistor amplifier.

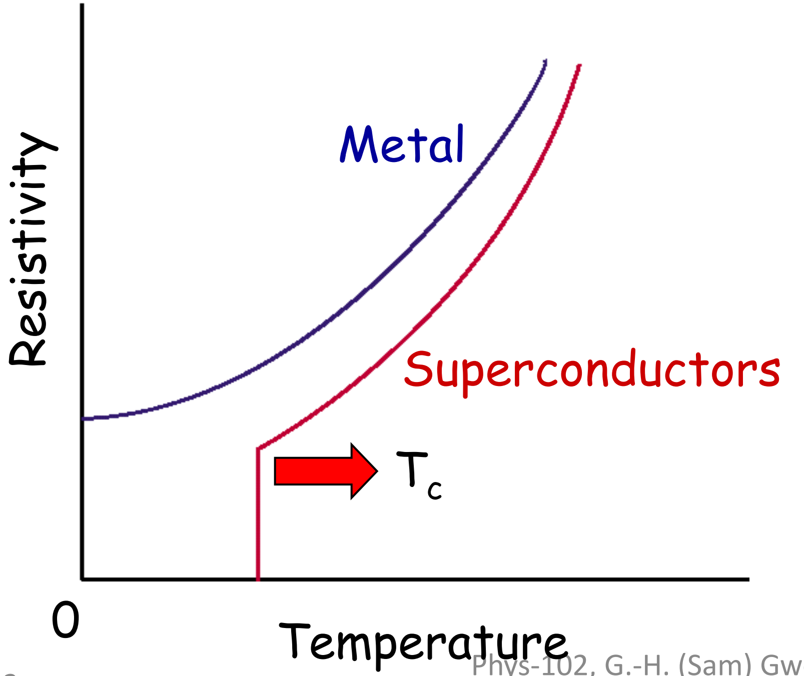


Superconductivity:
Essential Characteristics

1911 **K. Onnes Superconductivity in Hg**

1933 **Meissner effect**

RESISTANCELESS CONDUCTION



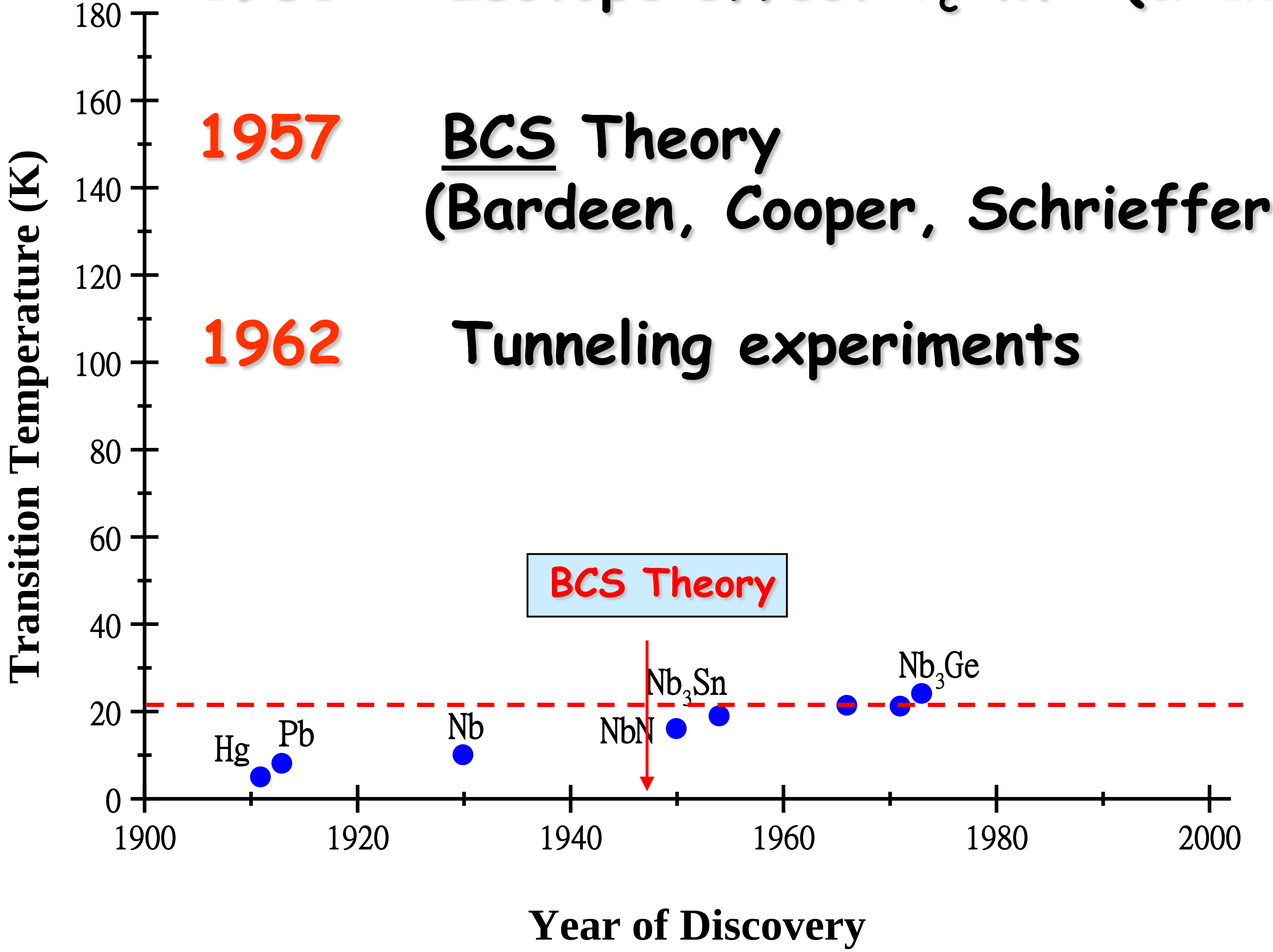
MEISSNER EFFECT:
Perfect diamagnetism

Understanding of Superconductivity

1950 Isotope effect $T_c \sim M^{-\alpha}$ ($\alpha \sim 1/2$)

1957 BCS Theory (Bardeen, Cooper, Schrieffer)

1962 Tunneling experiments



Superconductivity

- Perfect conductivity—critical temperature (Tc)
- Perfect diamagnetism- critical B field (Bc)

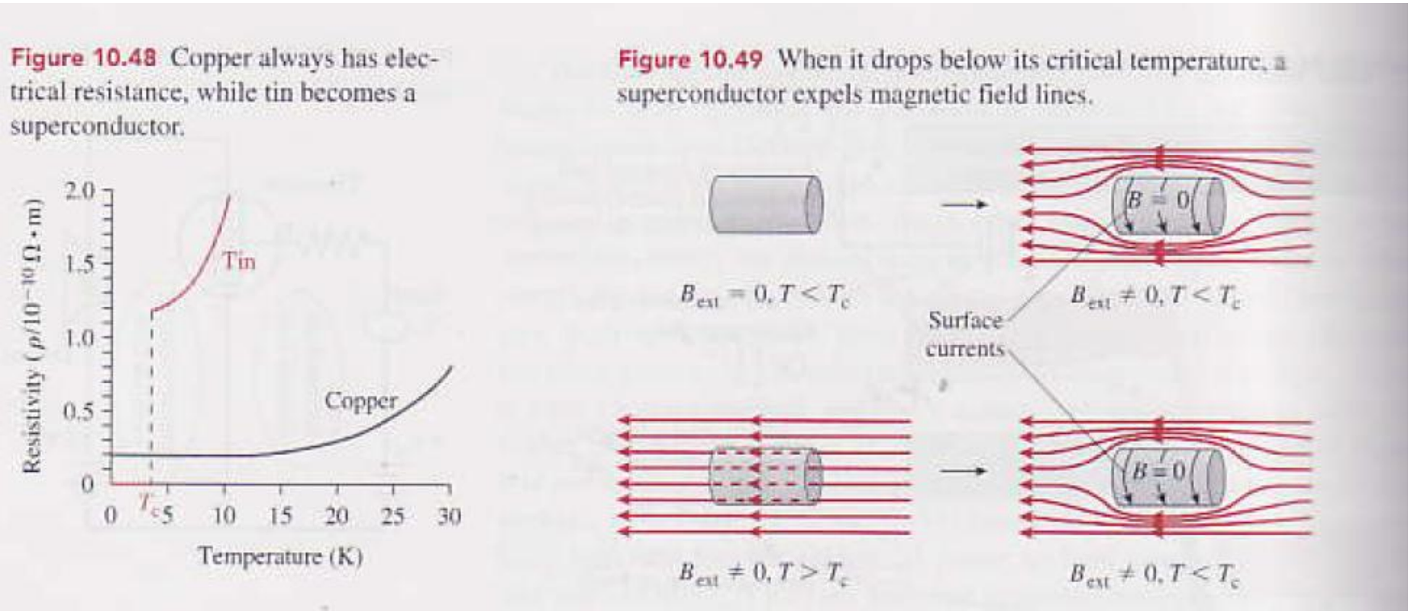


Figure 10.48 Copper always has electrical resistance, while tin becomes a superconductor.

Figure 10.49 When it drops below its critical temperature, a superconductor expels magnetic field lines.

Superconductivity

- Perfect conductivity—critical temperature (Tc)
- Perfect diamagnetism- critical B field (Bc)

Figure 10.48 Copper always has electrical resistance, while tin becomes a superconductor.

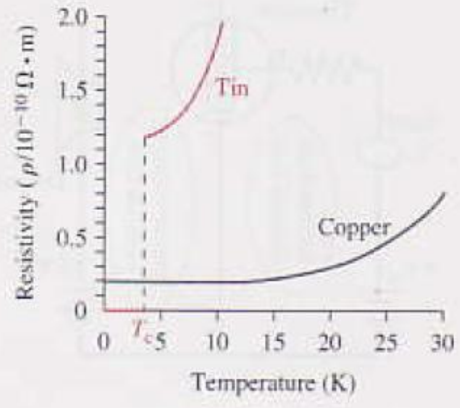
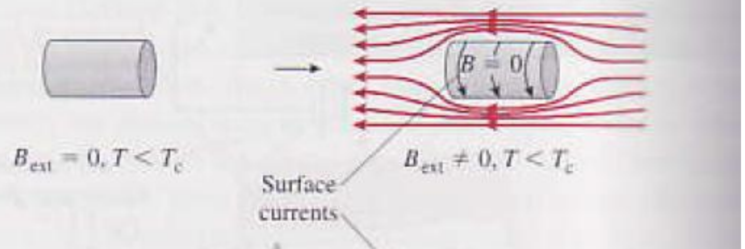
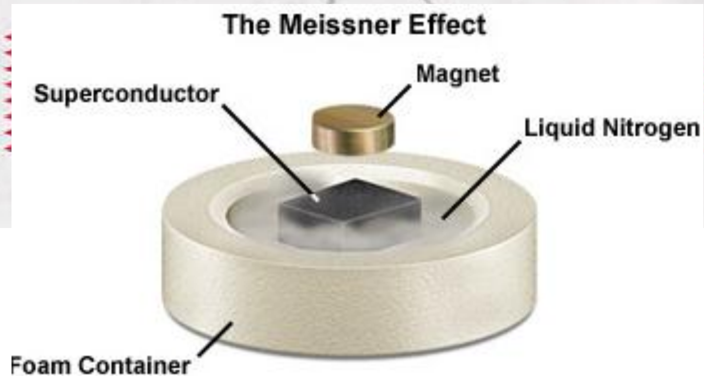


Figure 10.49 When it drops below its critical temperature, a superconductor expels magnetic field lines.

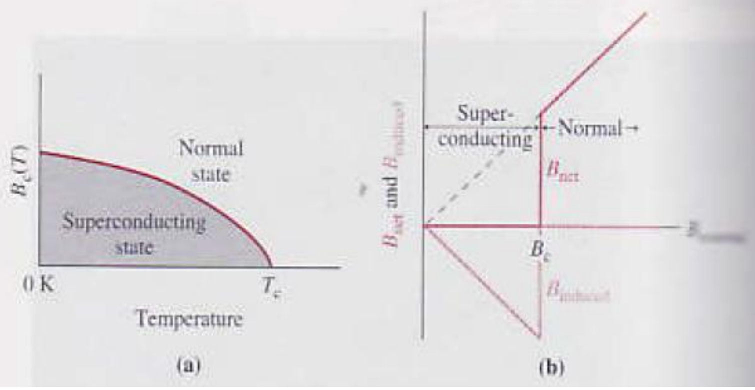




Superconductors repel magnets

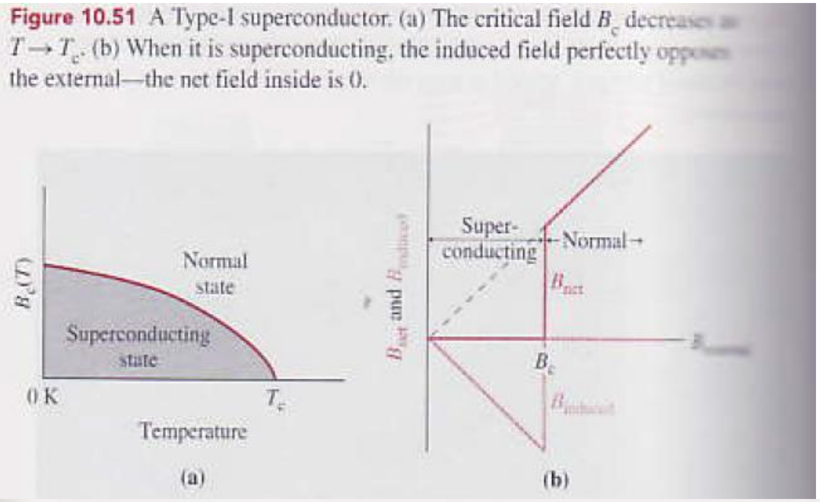
Type 1 Superconductors

Figure 10.51 A Type-I superconductor. (a) The critical field B_c decreases as $T \rightarrow T_c$. (b) When it is superconducting, the induced field perfectly opposes the external—the net field inside is 0.



-Sharp transition at Bc
-No B field in the superconducting material because Bext is cancled by B-induced before Bc
-After Bc, the superconducting material is in a normal state allowing Bext to penetrate the material
-Superconducting elements tend to be Type 1
-Bc is relatively low, 0.01-0.1 T
-Tc is relatively low, 1-9 K

Type 1 vs. 2 Superconductors



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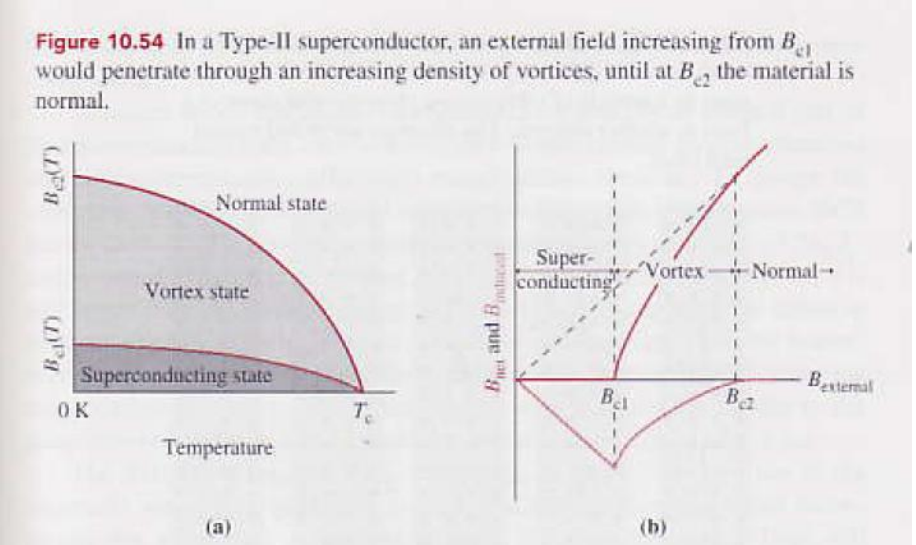


Figure 10.54 In a Type-II superconductor, an external field increasing from B_{c1} would penetrate through an increasing density of vortices, until at B_{c2} the material is normal.

Type 2 Superconductors

Figure 10.52 Magnetic field lines passing through vortices in a type-II superconductor.

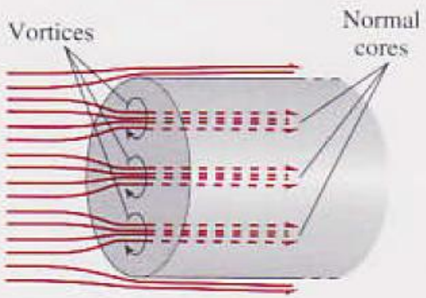


Figure 10.53 Vortices become more dense as field strength increases. (Both represent $B_{c1} < B_{external} < B_{c2}$. See Figure 10.54.)

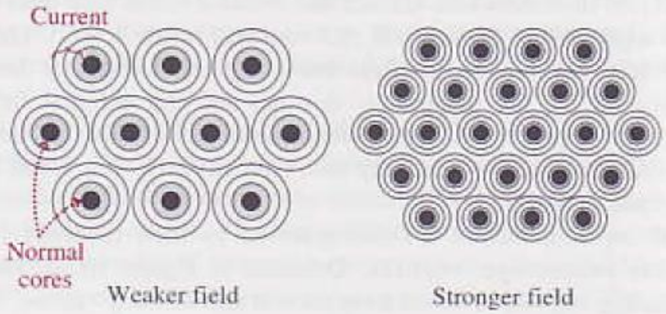
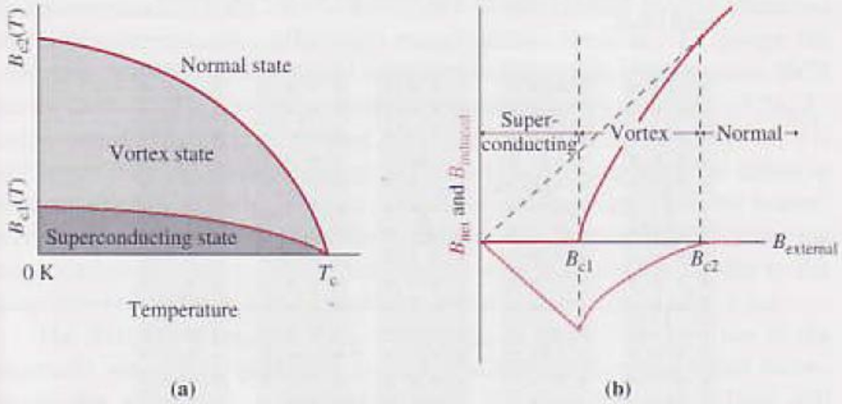


Figure 10.54 In a Type-II superconductor, an external field increasing from B_{c1} would penetrate through an increasing density of vortices, until at B_{c2} the material is normal.



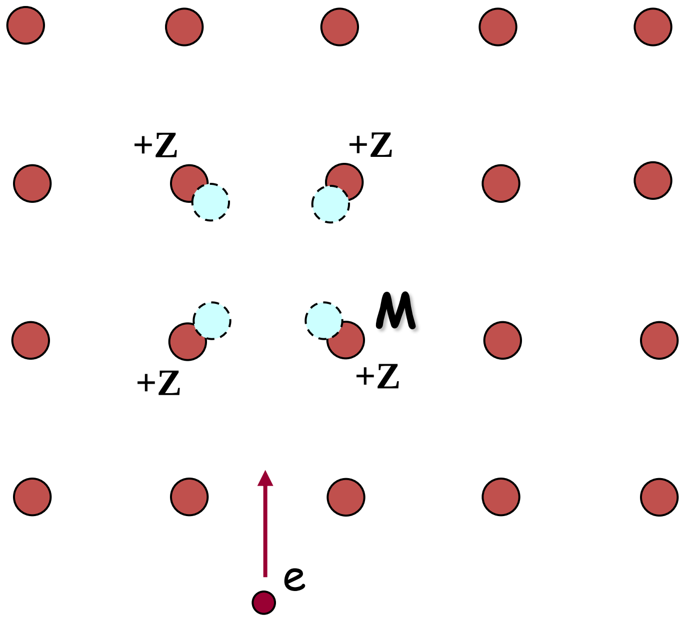
-Vortices: superconducting regions surrounding normal cores
-Stronger field makes vortices become more dense, allowing more field lines to penetrate.
-metallic compounds and alloys
-Bc are 2-3 orders of magnitude grater than type 1
-Tc is twice as high as type 1, around 20K

BCS Theory

- Bardeen, Cooper, and Schrieffer (BCS)
- Explains Type 1 and Type 2 superconductors
- Lattice and electron interactions
- Coherent motion of the electrons in Cooper pairs
  - What is cooper pair?
  - How coherent motion is created?

BCS theory

PHONON MEDIATED PAIRING
(phonon = lattice vibration)

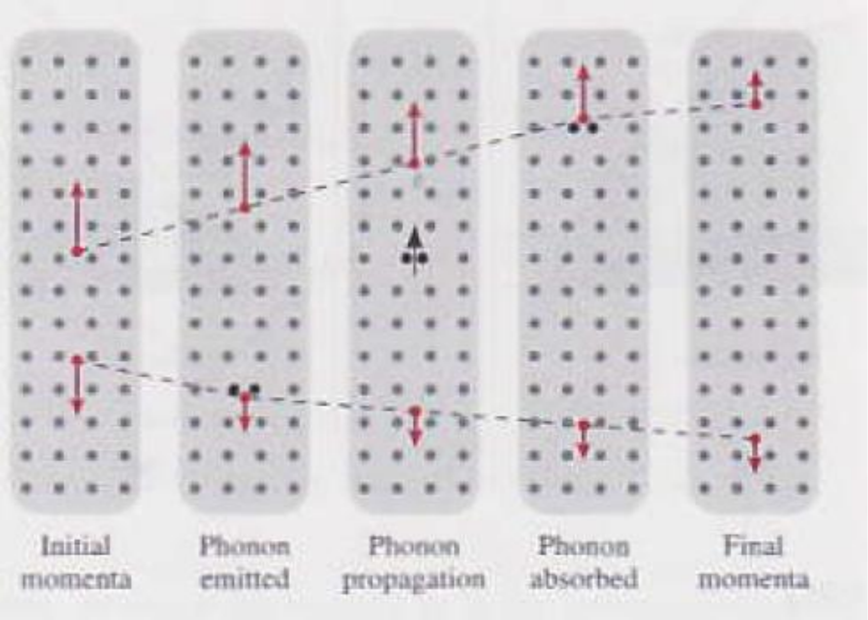


Pairs of electrons: **Cooper pairs**

Superconducting gap: Δ

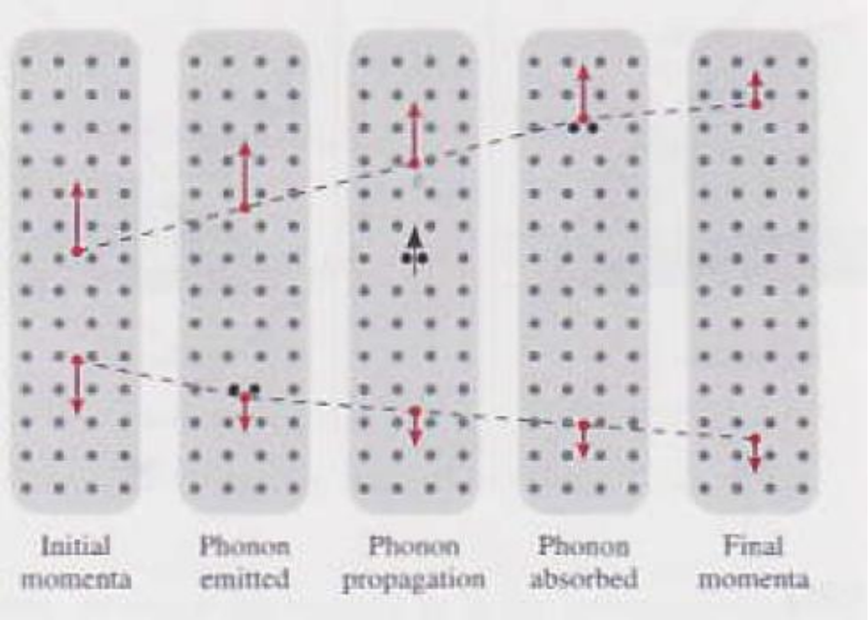
$T_c \sim M^{-1/2}$

Cooper Pairs



Cooper paired electrons
-are bosons as a pair
-have opposite momenta
-opposite spin directions

Cooper Pairs



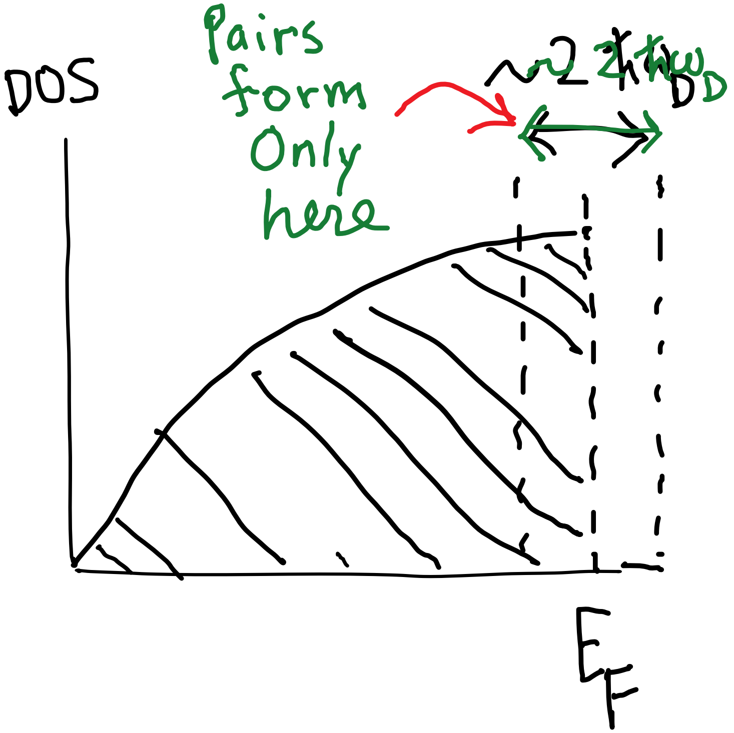
Cooper paired electrons
-are bosons as a pair
-have opposite momenta
-opposite spin directions

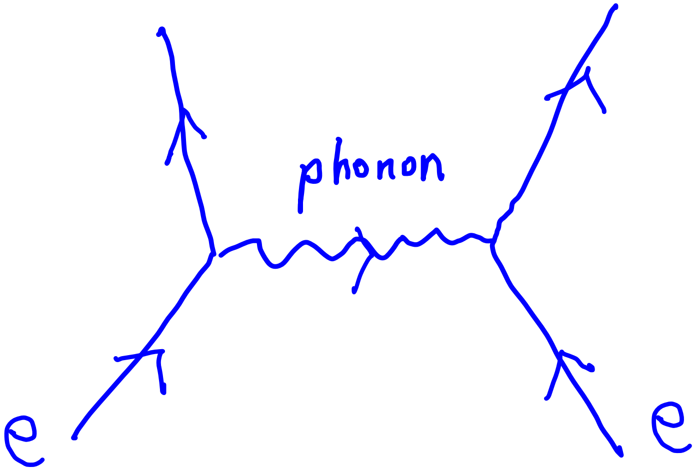
-Paired electrons experience a net attraction by exchanging a phonon
-Lattice is involved

$$T_c \sim M^{-1/2}$$

-This attractive force is small at the order of 0.001 eV which is greater than electron –electron repulsion
-the separation between the two paired electrons is greater than atomic spacing in the lattice

How many electrons participate?

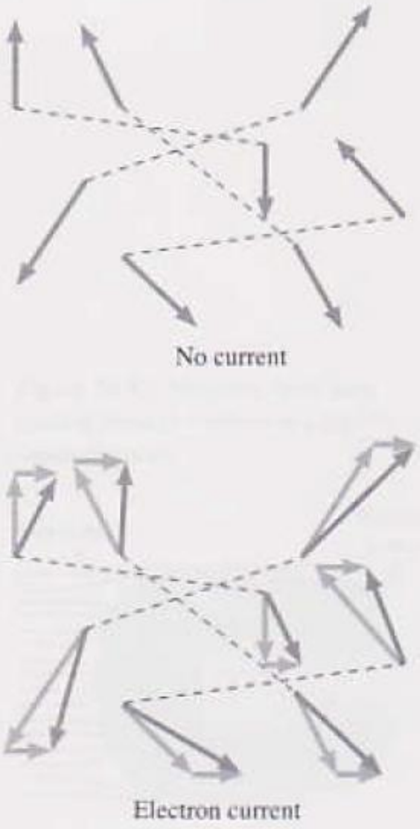




Only those electrons near E_F form Cooper pairs. These are the electrons with energies $E = E_F \pm \hbar\omega_D$ where ω_D is the Debye frequency (typical frequency for phonons). Why? Due to Pauli exclusion principle, electrons cannot scatter by absorbing or emitting a phonon when they are deep inside the Fermi sea ($E \ll E_F$). So, only a fraction, $O\left(\frac{\hbar\omega_D}{E_F}\right)$, of total electrons participate in Cooper pairs.

Coherent motion of Cooper pairs

Figure 10.56 All Cooper pairs have the same momentum: zero when current is zero; nonzero otherwise.



Cooper paired electrons
-are bosons as a pair
-have opposite momenta
-opposite spin directions
-binding energy between electrons in a Cooper pair is greater than typical thermal energy between electron and lattice ion at low temperatures
-thus, collisions are not going to stop electrons in cooper-pairs to move coherently
-Type 1 superconductors have longer mean free paths than Type 2 superconductors