Lecture 10 Topics

- Bose-Einstein Statistics Applications
  - Blackbody radiation by photons
  - Laser: Light amplification by the stimulated emission of radiation

Three distributions

Distribution	Occupation index	Particles	Identical particles?	Spin	Distingui-shable?	Exclusion principle?
Boltzman	$\frac{1}{Be^{E/k_BT}}$	Classical	No	Any spin	Yes	No
Bose-Einstein	$\frac{1}{Be^{E/k_BT}-1}$	Bosons	Yes	0 or integer spin	No	No
Fermi-Dirac	$\frac{1}{Be^{E/k_BT}+1}$	Fermions	Yes	1/2	No	Yes

Hadrons

TABLE 12.2 Commonly produced hadrons

Baryons	Mass (MeV/c^2)	Spin	Strange-ness	I, I_3	Lifetime, τ (or width $\hbar/\tau$)
p (uud)	938	$\frac{1}{2}$	0	$\frac{1}{2}, +\frac{1}{2}$	$>10^{32}$ yr
n (udd)	940	$\frac{1}{2}$	0	$\frac{1}{2}, -\frac{1}{2}$	889 s
Σ^+ (uus)	1189	$\frac{1}{2}$	−1	1, +1	8.0×10^{-11} s
Σ^0 (uds)	1193	$\frac{1}{2}$	−1	1, 0	7.4×10^{-20} s
Λ^0 (uds)	1116	$\frac{1}{2}$	−1	0, 0	2.6×10^{-10} s
Σ^- (dds)	1197	$\frac{1}{2}$	−1	1, −1	1.5×10^{-10} s
Ξ^0 (uss)	1315	$\frac{1}{2}$	−2	$\frac{1}{2}, -\frac{1}{2}$	2.9×10^{-10} s
Ξ^- (dss)	1321	$\frac{1}{2}$	−2	$\frac{1}{2}, -\frac{1}{2}$	1.6×10^{-10} s
Δ^{++} (uuu)	1232	$\frac{3}{2}$	0	$\frac{3}{2}, +\frac{3}{2}$	120 MeV
Δ^+ (uud)	1232	$\frac{3}{2}$	0	$\frac{3}{2}, +\frac{1}{2}$	120 MeV
Δ^0 (udd)	1232	$\frac{3}{2}$	0	$\frac{3}{2}, -\frac{1}{2}$	120 MeV
Δ^- (ddd)	1232	$\frac{3}{2}$	0	$\frac{3}{2}, -\frac{3}{2}$	120 MeV
Σ^{*+} (uus)	1383	$\frac{3}{2}$	−1	1, +1	~40 MeV
Σ^{*0} (uds)	1384	$\frac{3}{2}$	−1	1, 0	~40 MeV
Σ^{*-} (dds)	1387	$\frac{3}{2}$	−1	1, −1	~40 MeV
Ξ^{*0} (uss)	1532	$\frac{3}{2}$	−2	$\frac{1}{2}, +\frac{1}{2}$	~10 MeV
Ξ^{*-} (dss)	1535	$\frac{3}{2}$	−2	$\frac{1}{2}, -\frac{1}{2}$	~10 MeV
Ω^- (sss)	1672	$\frac{3}{2}$	−3	0, 0	8.2×10^{-11} s

Mesons	Mass (MeV/c^2)	Spin	Strange-ness	I, I_3	Lifetime, τ (or width $\hbar/\tau$)
$\pi^+(u\bar{d})$	140	0	0	1, +1	2.6×10^{-8} s
$\pi^0(u\bar{u} + d\bar{d})$	135	0	0	1, 0	8.4×10^{-17} s
$\pi^-(d\bar{u})$	140	0	0	1, −1	2.6×10^{-8} s
$K^+(u\bar{s})$	494	0	+1	$\frac{1}{2}, +\frac{1}{2}$	1.2×10^{-8} s
$K^0_S(d\bar{s}, s\bar{d})$	498	0	mix	$\frac{1}{2}$, mix	8.9×10^{-11} s
$K^0_L(d\bar{s}, s\bar{d})$	498	0	mix	$\frac{1}{2}$, mix	5.2×10^{-8} s
$K^-(s\bar{u})$	494	0	−1	$\frac{1}{2}, -\frac{1}{2}$	1.2×10^{-8} s
$\rho^+(u\bar{d})$	769	1	0	1, +1	151 MeV
$\rho^0(u\bar{u} + d\bar{d})$	769	1	0	1, 0	151 MeV
$\rho^-(d\bar{u})$	769	1	0	1, −1	151 MeV
$K^{*+}(u\bar{s})$	892	1	+1	$\frac{1}{2}, +\frac{1}{2}$	50 MeV
$K^{*0}(d\bar{s})$	896	1	+1	$\frac{1}{2}, -\frac{1}{2}$	51 MeV
$\bar{K}^{*0}(s\bar{d})$	896	1	−1	$\frac{1}{2}, +\frac{1}{2}$	51 MeV
$K^{*-}(s\bar{u})$	892	1	−1	$\frac{1}{2}, -\frac{1}{2}$	50 MeV
Heavy mesons—containing quarks beyond the strange					
J/ψ ($c\bar{c}$)	3100	1	0	0, 0	87 keV
Υ ($b\bar{b}$)	9460	1	0	0, 0	~50 keV

TABLE 12.1 Fundamental forces and particles

Force	Gravitation	Electroweak			Strong	Residual
Property	Mass/energy	Charge/weak charge			Color charge	
Strength	$\sim 10^{-39}$	$\sim 10^{-2}$	$\sim 10^{-6}$		1	
Range	$1/r^2$	$1/r^2$	10^{-3} fm		short	1 fm
Mediating Bosons	Graviton?	Photon, γ	W^+, W^-	Z^0	Gluon	$\pi^{\pm}, \pi^0$
Spin	2?	1	1	1	1	0
Mass	0?	$< 6 \times 10^{-22}$	80.4×10^3	91.2×10^3	< 10	140, 135
Charge	—	0	$+1, -1$	0	0	$\pm 1, 0$
Color charge	—	—	—	—	r, g, or b + $\bar{r}$, $\bar{g}$, or $\bar{b}$	Neutral

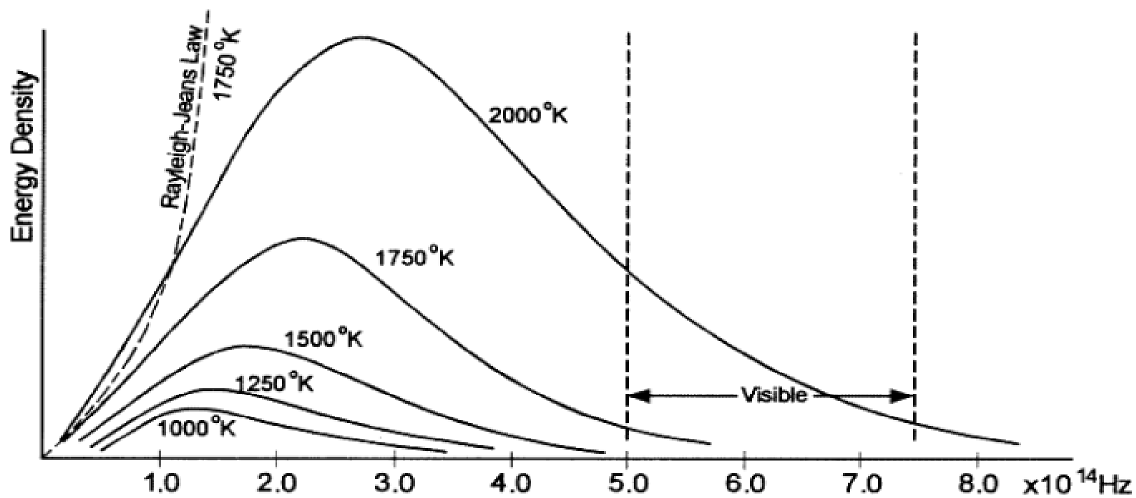
Quarks

Participants in gravitation, electroweak, and strong

	Spin	Mass	Charge	Color charge
Up, u	$\frac{1}{2}$	~5	$+\frac{2}{3}$	r, g, b
Down, d	$\frac{1}{2}$	~10	$-\frac{1}{3}$	r, g, b
Strange, s	$\frac{1}{2}$	~100	$-\frac{1}{3}$	r, g, b
Charm, c	$\frac{1}{2}$	$\sim 1.3 \times 10^3$	$+\frac{2}{3}$	r, g, b
Bottom, b	$\frac{1}{2}$	$\sim 4.5 \times 10^3$	$-\frac{1}{3}$	r, g, b
Top, t	$\frac{1}{2}$	$\sim 180 \times 10^3$	$+\frac{2}{3}$	r, g, b

Blackbody radiation

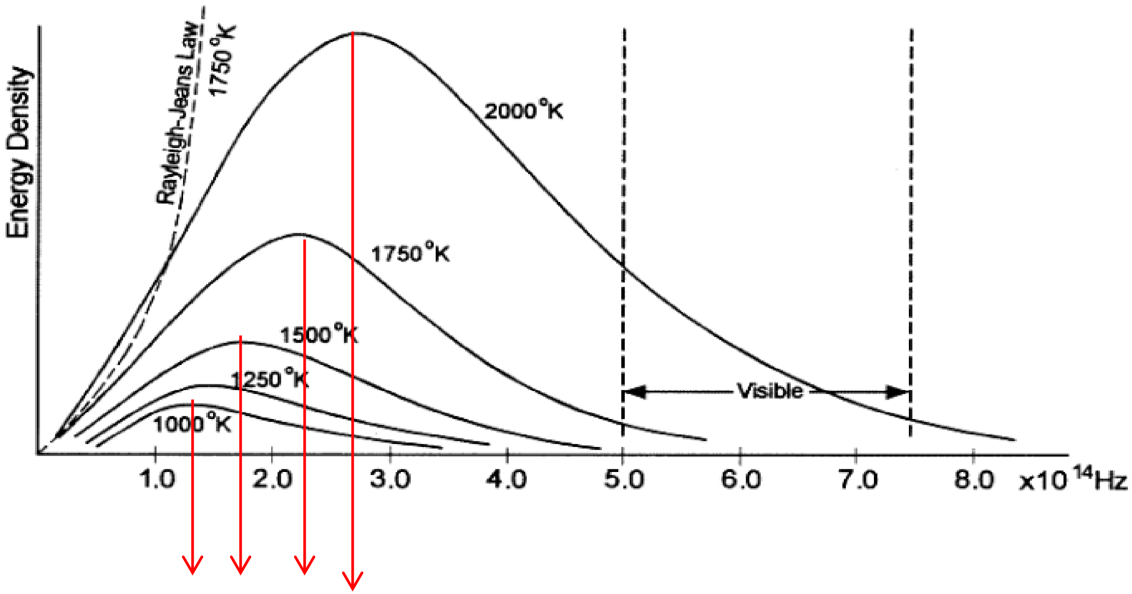
Figure 6.24



Stefan-Boltzmann law: The intensity of radiation $\propto T^4$

Blackbody radiation

Figure 6.24



Stefan-Boltzmann law: The intensity of radiation $\propto T^4$

Wien's law: The frequency associated with the maximum energy intensity
Is proportional to T

Rayleigh-Jeans Approach

Consider a cubic cavity of L x L x L

The number of permissible states can be written as:

$$E_{(n_x,n_y,n_z)} = (n_x^2 + n_y^2 + n_z^2)\frac{\pi^2\hbar^2}{2mL^2}$$

Rayleigh-Jeans Approach

Consider a cubic cavity of L x L x L

The number of permissible states can be written as:

$$n_x = \frac{2L}{\lambda} = 1, 2, 3, \dots$$

$$n_y = \frac{2L}{\lambda} = 1, 2, 3, \dots$$

$$n_z = \frac{2L}{\lambda} = 1, 2, 3, \dots$$

Rayleigh-Jeans Approach

Consider a cubic cavity of L x L x L

The number of permissible states can be written as:

$$n_x = \frac{2L}{\lambda} = 1, 2, 3, \ldots$$

$$n_y = \frac{2L}{\lambda} = 1, 2, 3, \ldots$$

$$n_z = \frac{2L}{\lambda} = 1, 2, 3, \ldots$$

$$n^2 = n_x^2 + n_y^2 + n_z^2 = \left(\frac{2L}{\lambda}\right)^2 \quad where \begin{cases} n_x = 0, 1, 2 \ldots \\ n_y = 0, 1, 2 \ldots \\ n_z = 0, 1, 2 \ldots \\ except\ when\ all\ n_x n_y n_z\ are\ zero \end{cases}$$

Rayleigh-Jeans Approach

Consider a cubic cavity of L x L x L

The number of permissible states can be written as:

$$n_x = \frac{2L}{\lambda} = 1, 2, 3, \dots$$

$$n_y = \frac{2L}{\lambda} = 1, 2, 3, \dots$$

$$n_z = \frac{2L}{\lambda} = 1, 2, 3, \dots$$

$$n^2 = n_x^2 + n_y^2 + n_z^2 = (\frac{2L}{\lambda})^2 \quad where \begin{cases} n_x = 0, 1, 2 \dots \\ n_y = 0, 1, 2 \dots \\ n_z = 0, 1, 2 \dots \\ except\ when\ all\ n_x n_y n_z\ are\ zero \end{cases}$$

$$n = \frac{2L}{\lambda} \rightarrow \quad dn = -\frac{2L}{\lambda^2} d\lambda$$

The number of permissible states can be written in terms of λ

$$N(n)dn = 2\text{x}\frac{1}{8} 4\pi n^2 dn =$$

Rayleigh-Jeans Approach

Consider a cubic cavity of L x L x L

The number of permissible states can be written as:

$$n_x = \frac{2L}{\lambda} = 1, 2, 3, \ldots$$

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$$n = \frac{2L}{\lambda} \rightarrow \quad dn = -\frac{2L}{\lambda^2} d\lambda$$

The number of permissible states can be written in terms of λ

$$N(n)dn = 2\text{x}\frac{1}{8}4\pi n^2 dn = \pi n^2 dn = \pi \left(\frac{2L}{\lambda}\right)^2 \left(-\frac{2L}{\lambda^2} d\lambda\right) = -\frac{8\pi L^3 d\lambda}{\lambda^4} = -\frac{8\pi V d\lambda}{\lambda^4} \equiv -N(\lambda)d\lambda$$

Rayleigh-Jeans Approach

Since

$$\lambda = \frac{c}{\nu} \quad and \; d\lambda = -\frac{c\ d\nu}{\nu^2}$$

The number of permissible states in terms of ν

$$N(n)dn = -\frac{8\pi V d\lambda}{\lambda^4} ==$$

Rayleigh-Jeans Approach

Since

$$\lambda = \frac{c}{\nu} \quad and \; d\lambda = -\frac{c\, d\nu}{\nu^2}$$

The number of permissible states in terms of ν

$$N(n)dn = -\frac{8\pi V d\lambda}{\lambda^4} == -\frac{8\pi V}{(\frac{c}{\nu})^4}\left(-\frac{c\, d\nu}{\nu^2}\right) = \frac{8\pi V}{c^3}\nu^2 d\nu \equiv N(\nu)d\nu$$

, the energy radiation rate can be expressed in terms of ν:

$$u(\nu)d\nu = k_B T N(\nu)d\nu = \frac{8\pi V k_B T}{c^3}\nu^2 d\nu$$

Planck's Hypothesis

$$E_n = nh\nu$$

$$\bar{E} = \sum E_n P(E_n) = \frac{\sum_n E_n e^{-\frac{E_n}{k_B T}}}{\sum_n e^{-\frac{E_n}{k_B T}}}$$

Planck's Hypothesis

$$E_n = nh\nu$$

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Since $0 < e^{-\frac{nh\nu}{k_B T}} < 1$ and consider $e^{-\frac{nh\nu}{k_B T}} = x$

Planck's Hypothesis

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Since $0 < e^{-\frac{nh\nu}{k_B T}} < 1$ and consider $e^{-\frac{nh\nu}{k_B T}} = x$

$$\bar{E} = h\nu \frac{\sum_n n x^n}{\sum_n x^n}$$

Planck's Hypothesis

$$E_n = nh\nu$$

$$\bar{E} = \sum E_n P(E_n) = \frac{\sum_n E_n e^{-\frac{E_n}{k_B T}}}{\sum_n e^{-\frac{E_n}{k_B T}}} := \frac{\sum_n nh\nu\, e^{-\frac{nh\nu}{k_B T}}}{\sum_n e^{-\frac{nh\nu}{k_B T}}}$$

Since $0 < e^{-\frac{nh\nu}{k_B T}} < 1$ and consider $e^{-\frac{nh\nu}{k_B T}} = x$

$$\bar{E} = h\nu \frac{\sum_n nx^n}{\sum_n x^n} :$$

Using the following sums when $|x| < 1$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \;;\; \sum_{n=0}^{\infty} nx^n = \frac{x}{(1-x)^2}$$

Planck's Hypothesis

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$$\bar{E} = h\nu \frac{\sum_n nx^n}{\sum_n x^n} :$$

Using the following sums when $|x| < 1$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \; ; \sum_{n=0}^{\infty} nx^n = \frac{x}{(1-x)^2}$$

$$\bar{E} = h\nu \frac{\sum_n nx^n}{\sum_n x^n} = h\nu \frac{\frac{x}{(1-x)^2}}{\frac{1}{1-x}} = h\nu \frac{x}{1-x} = h\nu \frac{1}{x^{-1}-1} = h\nu \frac{1}{e^{\frac{h\nu}{k_B T}}-1}$$

Planck's Hypothesis

$$E_n = nh\nu$$

$$\bar{E} = \sum E_n P(E_n) = \frac{\sum_n E_n e^{-\frac{E_n}{k_B T}}}{\sum_n e^{-\frac{E_n}{k_B T}}} = \frac{\sum_n nh\nu\, e^{-\frac{nh\nu}{k_B T}}}{\sum_n e^{-\frac{nh\nu}{k_B T}}}$$

Since $0 < e^{-\frac{nh\nu}{k_B T}} < 1$ and consider $e^{-\frac{nh\nu}{k_B T}} = x$

$$\bar{E} = h\nu \frac{\sum_n n x^n}{\sum_n x^n}$$

Using the following sums when $|x| < 1$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-}$$

Boltzman	$\frac{1}{Be^{E/k_B T}}$
Bose-Einstein	$\frac{1}{Be^{E/k_B T} - 1}$
Fermi-Dirac	$\frac{1}{Be^{E/k_B T} + 1}$

Energy

$$\bar{E} = h\nu \frac{\sum_n n x^n}{\sum_n x^n} = h\nu \frac{\frac{x}{(1-x)^2}}{\frac{1}{1-x}} = h\nu \frac{x}{1-x} = h\nu \frac{1}{x^{-1}-1} = h\nu \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$$

Planck's Hypothesis

$$E_n = nh\nu$$

The number of permissible states can be rewritten in terms of energy

$$E = h\nu \text{ and } \nu = \frac{E}{h} \text{ and } d\nu = \frac{1}{h} dE$$

The total number of permissible states in terms of E would be

$$N(E)dE = \frac{8\pi V}{c^3 h^3} E^2 dE$$

Therefore, Density of States would be

$$D(E) = \frac{8\pi V}{c^3 h^3} E^2$$

$$E = \int_0^\infty E\mathcal{N}(E)D(E)dE \quad = \int_0^\infty \frac{E}{e^{E/k_B T}-1}\left(\frac{8\pi V}{h^3 c^3}E^2\right)dE$$

Use $E \equiv k_B T x$

Planck's Hypothesis

$$E_n = nh\nu$$

$$D(E) = \frac{m^{3/2} V \sqrt{2}}{\pi^2 \hbar^3} \sqrt{E}$$

$$E = \int_0^\infty E\mathcal{N}(E)D(E)dE \quad = \int_0^\infty \frac{E}{e^{E/k_BT}-1}\left(\frac{8\pi V}{h^3c^3}E^2\right)dE$$

Use $E \equiv k_BTx$

$$= \frac{8\pi V}{h^3c^3}\int_0^\infty (k_BT)^4 \frac{x^3}{e^x-1}dx$$

Planck's Hypothesis

$$E_n = nh\nu$$

$$D(E) = \frac{m^{3/2} V \sqrt{2}}{\pi^2 \hbar^3} \sqrt{E}$$

$$E = \int_0^\infty E \mathcal{N}(E) D(E) dE = \int_0^\infty \frac{E}{e^{E/k_B T} - 1} \left(\frac{8\pi V}{h^3 c^3} E^2 \right) dE$$

Use $E \equiv k_B T x$

$$= \frac{8\pi V}{h^3 c^3} \int_0^\infty (k_B T)^4 \frac{x^3}{e^x - 1} dx = \frac{8\pi V (k_B T)^4}{h^3 c^3} \frac{\pi^4}{15} = \frac{8 V \pi^5 k_B^4}{h^3 c^3} T^4$$

Planck's Hypothesis

$E_n = nh\nu$

$$D(E) = \frac{m^{3/2} V \sqrt{2}}{\pi^2 \hbar^3} \sqrt{E}$$

$$E = \int_0^\infty E \mathcal{N}(E) D(E) dE \quad = \int_0^\infty \frac{E}{e^{E/k_B T} - 1} \left(\frac{8\pi V}{h^3 c^3} E^2 \right) dE$$

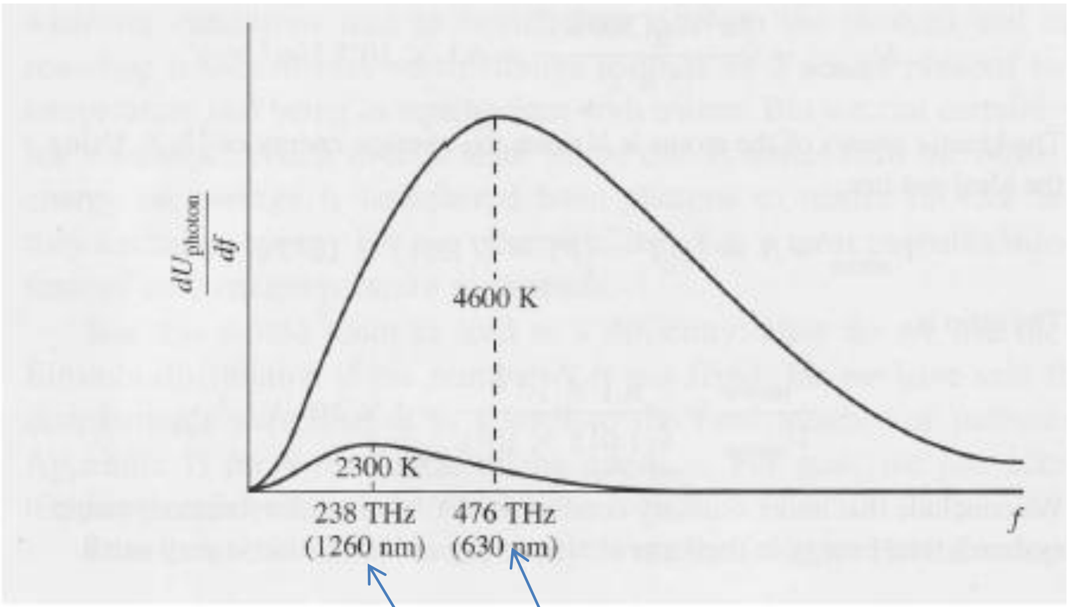
Use $E \equiv k_B T x$

$$= \frac{8\pi V}{h^3 c^3} \int_0^\infty (k_B T)^4 \frac{x^3}{e^x - 1} dx = \frac{8\pi V (k_B T)^4}{h^3 c^3} \frac{\pi^4}{15} = \frac{8 V \pi^5 k_B^4}{h^3 c^3} T^4$$

Stefan-Boltzman's law

Planck's Hypothesis

$$dE = \frac{h\nu^3}{e^{h\nu/k_B T} - 1}\left(\frac{8\pi V}{c^3}\right) d\nu$$



Wien's Law

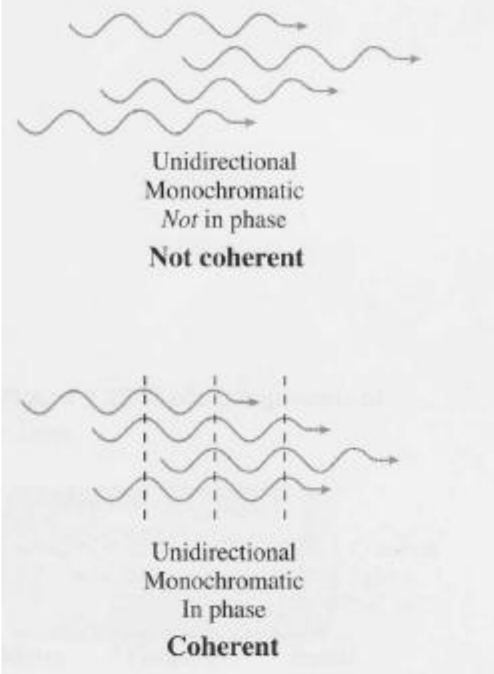
LASER

Light Amplification by the Stimulated Emission of Radiation

Coherent Light

→Moving in one direction
→Of a single wavelength
→In phase
→A lot of them

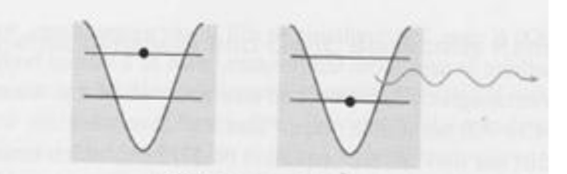
Figure 9.21 Coherent versus incoherent light.



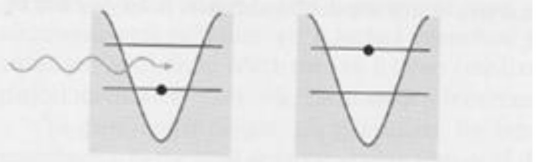
Einstein's theory

Spontaneous Emission:

$$R_{spo} = A_{spo} N_2$$



Spontaneous emission

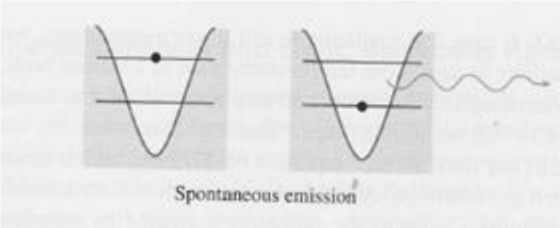


Absorption

Einstein's theory

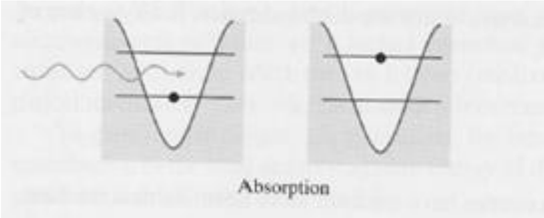
Spontaneous Emission:

$$R_{spo} = A_{spo} N_2$$



Absorption:

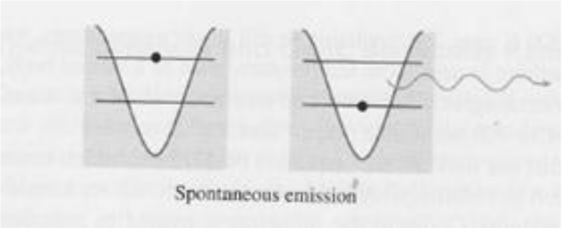
$$R_{abs} = B_{abs} N_1 Y(\Delta E)$$



Einstein's theory

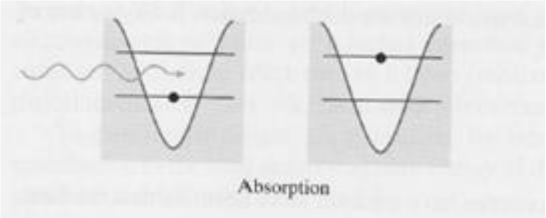
Spontaneous Emission:

$$R_{spo} = A_{spo} N_2$$



Absorption:

$$R_{abs} = B_{abs} N_1 \underline{Y(\Delta E)}$$

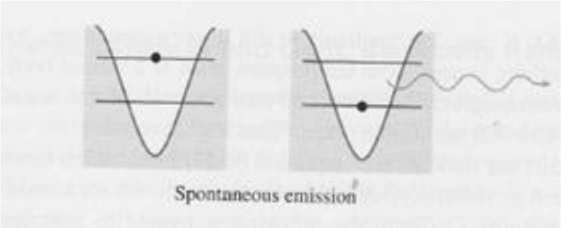


The number of photons with the energy difference

Einstein’s theory

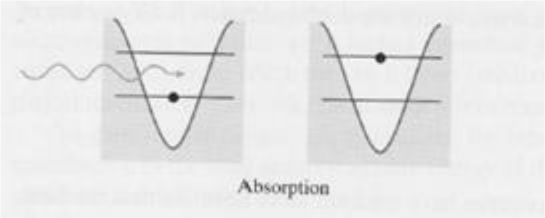
Spontaneous Emission:

$$R_{spo} = A_{spo} N_2$$



Absorption:

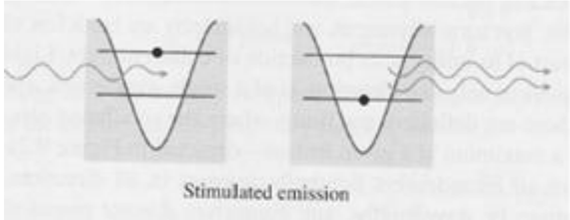
$$R_{abs} = B_{abs} N_1 \underline{Y(\Delta E)}$$



The number of photons with the energy difference

Stimulated Emission:

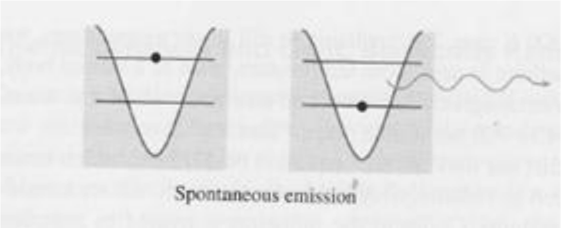
$$R_{sti} = B_{sti} N_2 Y(\Delta E)$$



Einstein's theory

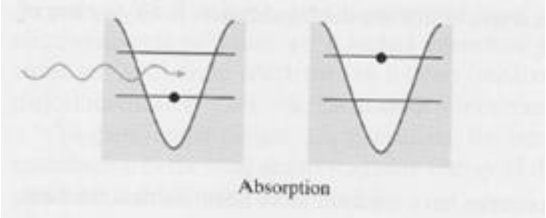
Spontaneous Emission:

$$R_{spo} = A_{spo} N_2$$



Absorption:

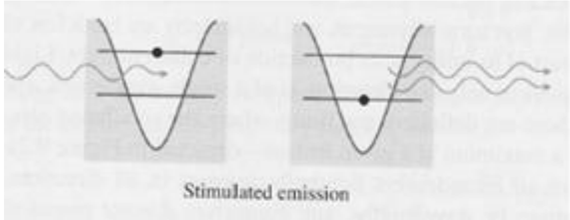
$$R_{abs} = B_{abs} N_1 Y(\Delta E)$$



The number of photons with the energy difference

Stimulated Emission:

$$R_{sti} = B_{sti} N_2 Y(\Delta E)$$



Emission = Absorption

Einstein’s theory

Emission = Absorption

$$R_{spo} + R_{sti} = R_{abs}$$

Einstein's theory

Emission = Absorption

$$R_{spo} + R_{sti} = R_{abs}$$

$$A_{spo}N_2 + B_{sti}N_2Y(\Delta E) = B_{abs}N_1Y(\Delta E)$$

Einstein's theory

Emission = Absorption

$$R_{spo} + R_{sti} = R_{abs}$$

$$A_{spo}N_2 + B_{sti}N_2Y(\Delta E) = B_{abs}N_1Y(\Delta E)$$

$$(B_{abs}N_1 - B_{sti}N_2)Y(\Delta E) = A_{spo}N_2$$

$$Y(\Delta E) = \frac{A_{spo}N_2}{(B_{abs}N_1 - B_{sti}N_2)} = \frac{A_{spo}/B_{abs}}{\frac{N_1}{N_2} - \frac{B_{sti}}{B_{abs}}} = \frac{A_{spo}/B_{abs}}{e^{\Delta E/k_BT} - \frac{B_{sti}}{B_{abs}}}$$

$$\text{Since } N_1 \propto e^{-\frac{E_1}{k_BT}} \text{ and } N_2 \propto e^{-\frac{E_2}{k_BT}}$$

Einstein's theory

Emission = Absorption

$$R_{spo} + R_{sti} = R_{abs}$$

$$A_{spo}N_2 + B_{sti}N_2Y(\Delta E) = B_{abs}N_1Y(\Delta E)$$

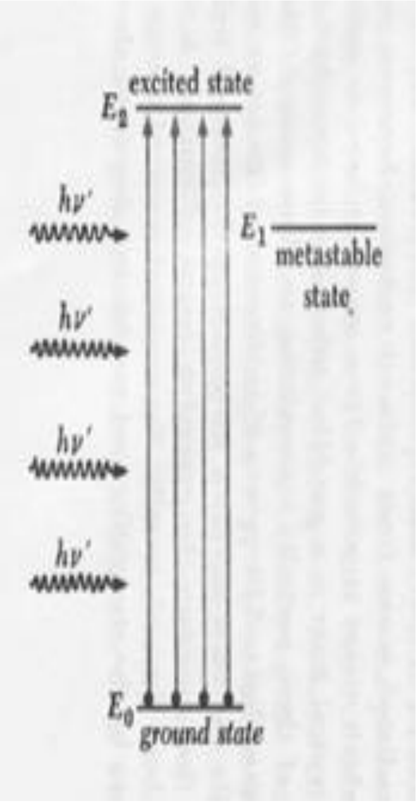
$$(B_{abs}N_1 - B_{sti}N_2Y)Y(\Delta E) = A_{spo}N_2$$

$$Y(\Delta E) = \frac{A_{spo}N_2}{(B_{abs}N_1 - B_{sti}N_2Y)} = \frac{A_{spo}/B_{abs}}{\frac{N_1}{N_2} - \frac{B_{sti}}{B_{abs}}} = \frac{A_{spo}/B_{abs}}{e^{\Delta E/k_BT} - \frac{B_{sti}}{B_{abs}}}$$

= 1

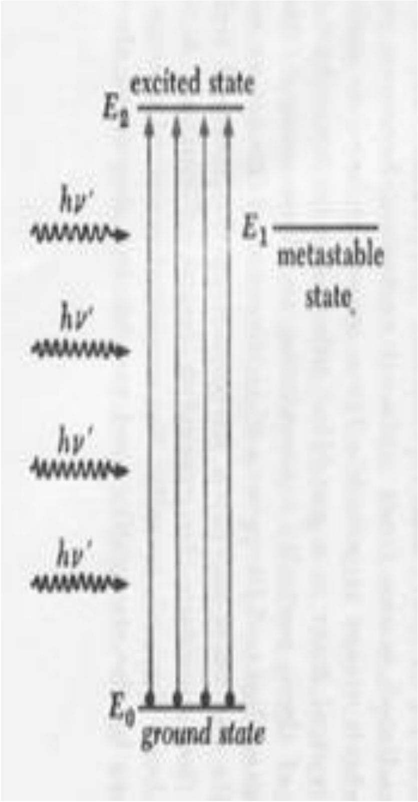
If $N_1 = N_2$

Three level Laser

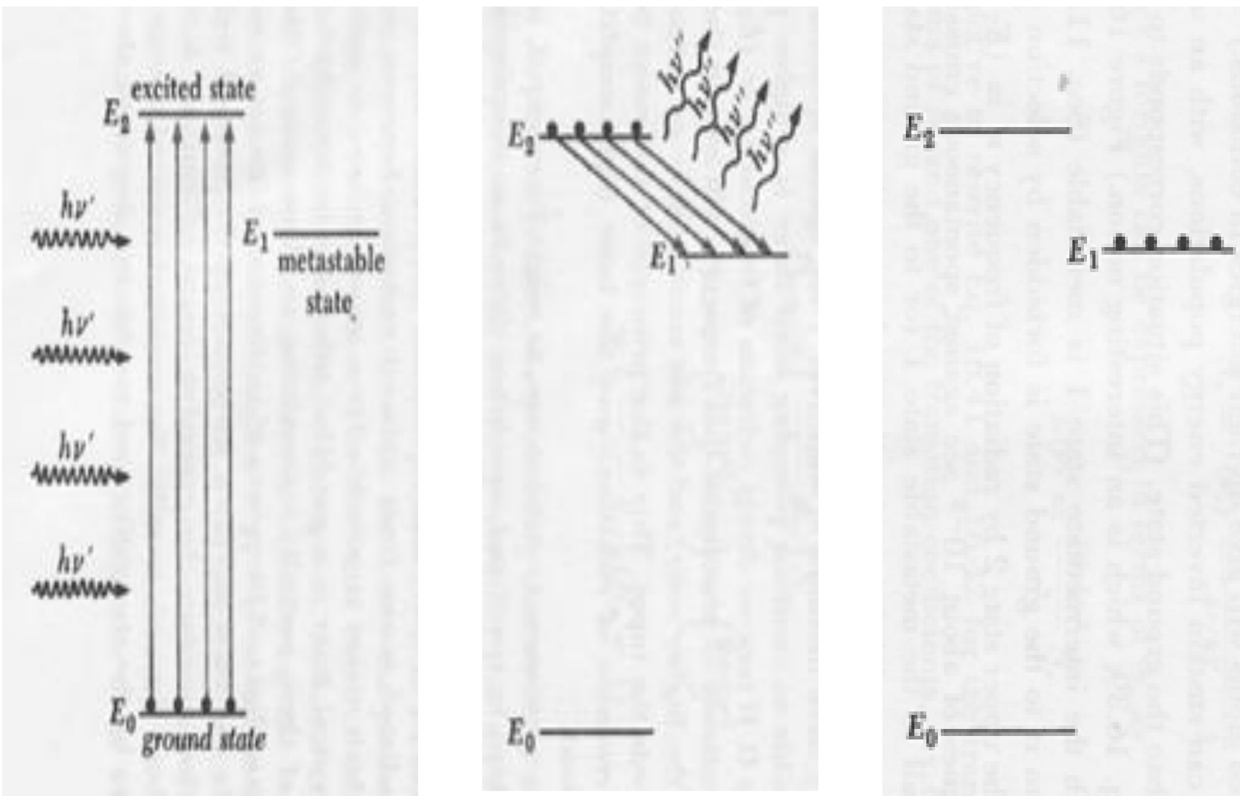


Optical pumping

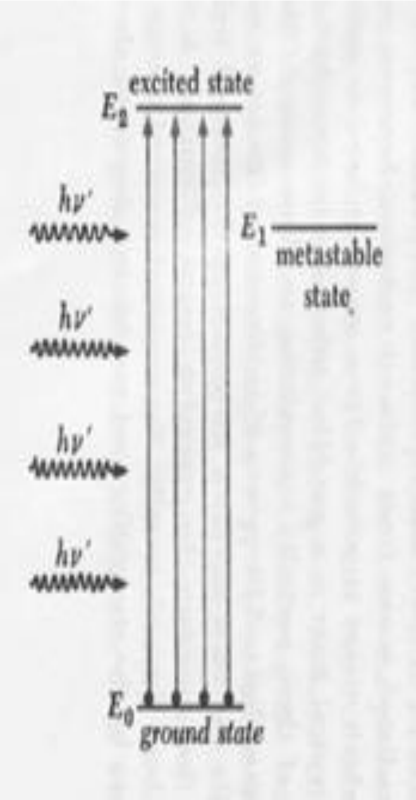
Three level Laser



Three level Laser



Three level Laser



Population Inversion!!!

Three level Laser