

**PH102: Modern Physics Homework 3 (Due: 2/08/2013, Friday 5 pm)**

- 1. (5 points)** Textbook: Harris, Chapter 9 Conceptual Question #2
- 2. (5 points)** Textbook: Harris, Chapter 9 Conceptual Question #6
- 3. (10 points)** For a free electron inside an infinite potential well, energy can be defined as

$$E = \frac{\pi^2 \hbar^2}{2mL^2} n^2 \quad (\text{See the textbook p.375})$$

where  $L$  is the size of a one dimensional infinite potential well.

- (a) (3 points) Show that the density of states of a free electron in one dimension is

$$D(E) = \frac{L}{\pi} \sqrt{\frac{2m}{\hbar^2}} \frac{1}{\sqrt{E}}$$

- (b) (3 points) Show that in two dimensions, the density of states of a free electron is

$$D(E) = \frac{Am}{\pi \hbar^2} \quad \text{where } A = L^2$$

- (c) (3 points) Show that in three dimensions, the density of states becomes

$$D(E) = \frac{m^{3/2} V \sqrt{2}}{\pi^2 \hbar^3} \sqrt{E} \quad \text{where } V = L^3$$

- (d) (1 point) Sketch the density of states for each of the three dimensions as a function of  $E$ .

- 4. (10 points)** Textbook: Harris, Chapter 9 Exercises #60

**5. (10 points)** Consider a system of one dimensional harmonic oscillators. The probability that such an oscillator have the energy  $E$  at the temperature  $T$  is given by the Boltzman factor  $e^{-E/k_B T}$ , and so its average energy is found by integration  $E e^{-E/k_B T}$  over all possible energies and then dividing by the integral of  $e^{-E/k_B T}$  in order to normalize the result. The total energy of the oscillator at any time is the sum of its instantaneous kinetic and potential energies

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

Where  $v$  is the particle's speed and  $x$  is its displacement from the equilibrium position.

- (a) Show that the average energy is  $k_B T$ , using the following formula:

$$\bar{E} = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E e^{-\frac{E}{k_B T}} dv dx}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{E}{k_B T}} dv dx}$$

(Hint: do the integral separately for  $dv$  and  $dx$ .)

- (b) At high temperature, the average energy of a classical one-dimensional oscillator is  $k_B T$  as you proved above, and for an atom in a monatomic ideal gas, it is  $\frac{3}{2} k_B T$ . Explain the difference, using the equipartition theorem (Chapter 9 conceptual question #18).