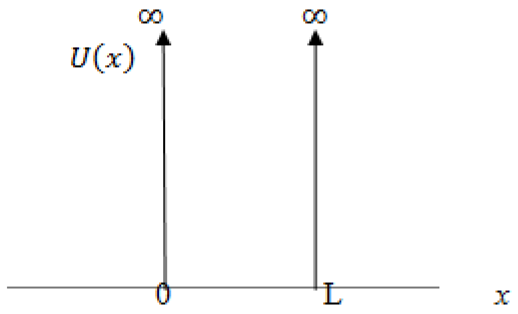
PH 102: Interactive Lecture 4 Topics

- Particle in a 1-D infinite potential well
- Particle in a 3-D infinite potential well
  - Schrodinger Equation
  - Separation of Variables
  - Energy quantization
  - Wave functions
  - Energy degeneracy
  - Energy split
- Hydrogen atom spectral lines
  - Empirical Formula
  - Hydrogen atom models and spectral lines
- Clarification on HW4 and HW5

1-D infinite potential well

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = \mathrm{E}\,\psi(x)$$

$$U(x) = \begin{cases} \infty & x \le 0 \\ 0 & 0 < x < L \\ \infty & x \ge L \end{cases}$$



1-D infinite potential well

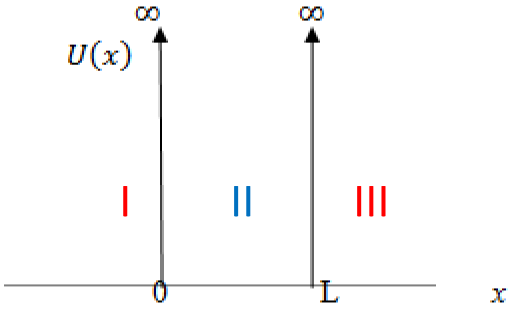
$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = \mathrm{E}\,\psi(x)$$

$$U(x) = \begin{cases} \infty & x \le 0 \\ 0 & 0 < x < L \\ \infty & x \ge L \end{cases} \quad \begin{matrix} \longrightarrow \\ \longrightarrow \\ \longrightarrow \end{matrix}$$

Region I

Region II

Region III



1-D infinite potential well

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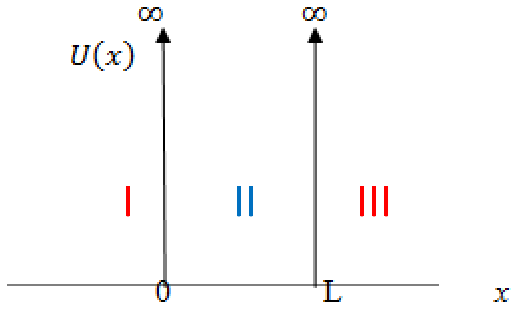
$$U(x) = \begin{cases} \infty & x \leq 0 \\ 0 & 0 < x < L \\ \infty & x \geq L \end{cases} \quad \begin{array}{l} \rightarrow \text{Region I} \\ \rightarrow \text{Region II} \\ \rightarrow \text{Region III} \end{array}$$



Regions I and III, no wave function can exist →

$$\psi_{x\leq 0}(x) = 0$$
$$\psi_{x\geq L}(x) = 0$$

1-D infinite potential well

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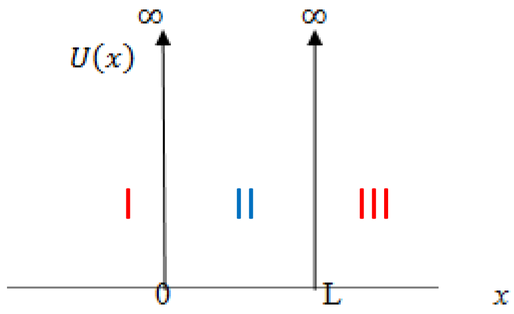

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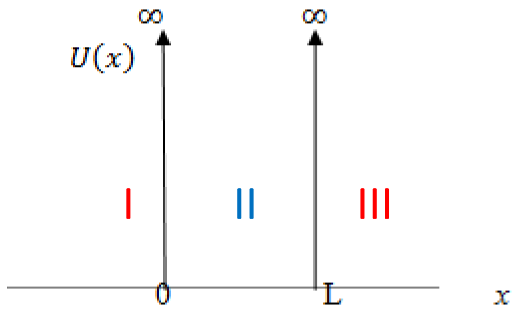
Region II, $\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = \mathrm{E}\,\psi(x)$ → $\psi_{0<x<L}(x) = A\sin(kx)$; $k = \sqrt{\frac{2mE}{\hbar^2}}$

1-D infinite potential well

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = \mathrm{E}\,\psi(x)$$

$$U(x) = \begin{cases} \infty & x \le 0 \longrightarrow \\ 0 & 0 < x < L \longrightarrow \\ \infty & x \ge L \longrightarrow \end{cases}$$

Region I
Region II
Region III



Regions I and III, no wave function can exist → $\psi_{x\le 0}(x) = 0$
$\psi_{x\ge L}(x) = 0$

Region II, $\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = \mathrm{E}\,\psi(x)$ → $\psi_{0<x<L}(x) = A\sin(kx)$; $k = \sqrt{\frac{2mE}{\hbar^2}}$

$\psi_{x\ge L}(x=0) = 0$ → $\psi_{0<x<L}(x=L) = \sin(kL) = 0$

$$kL = \sqrt{\frac{2mE}{\hbar^2}}L = n\pi$$ where n =1, 2, 3, etc.

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$ Energy quantization

1-D infinite potential well

Normalization:

$$\psi_{0<x<L}(x) = A\sin(kx) = \mathrm{A}\ \sin(\frac{n\pi x}{L})$$

$$\int_0^L |\psi_{0<x<L}(x)|^2 = 1 = \mathrm{A}^2 \int_0^L \sin\left(\frac{n\pi x}{L}\right)^2 dx = \mathrm{A}^2 \frac{\mathrm{L}}{2} \rightarrow \mathrm{A} = \sqrt{\frac{2}{L}}$$

Wave function: $\psi_{0<x<L}(x) = \sqrt{\frac{2}{L}}\ \sin(\frac{n\pi x}{L})$

Energy $E = \frac{n^2\ \pi^2\ \hbar^2}{2mL^2}$

1-D infinite potential well

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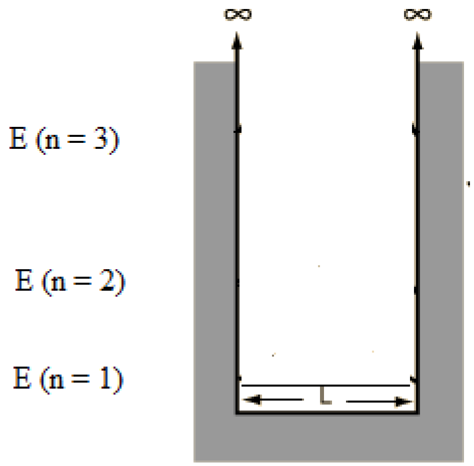
Energy $E = \frac{n^2\,\pi^2\,\hbar^2}{2mL^2}$

Obtain the first three energy levels ($\frac{\pi^2\,\hbar^2}{2mL^2}$) and draw their associated wave functions



1-D infinite potential well

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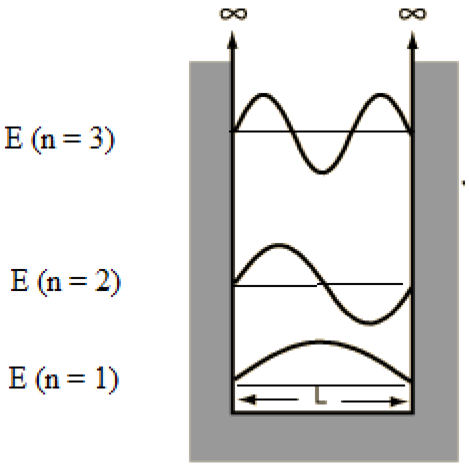
Wave function: $\psi_{0<x<L}(x) = \sqrt{\frac{2}{L}}\,\sin(\frac{n\pi x}{L})$

Energy $E = \frac{n^2\,\pi^2\,\hbar^2}{2mL^2}$

$$E\ (n=3) = 9\,\frac{\pi^2\,\hbar^2}{2mL^2}$$

$$E\ (n=2) = 4\,\frac{\pi^2\,\hbar^2}{2mL^2}$$

$$E\ (n=1) = 1\,\frac{\pi^2\,\hbar^2}{2mL^2}$$

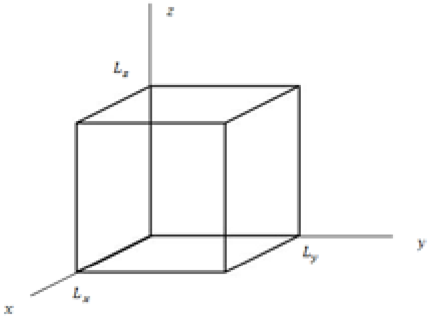


Particle in a 3-d infinite well

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\vec{x}) + U(\vec{x})\psi(\vec{x}) = E\,\psi(\vec{x})$$

In (x, y, z) coordinates, $\vec{x} = (x, y, z)$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi(x, y, z) = -\frac{2m}{\hbar^2}(E - U(x, y, z))\psi(x, y, z)$$



$$U(\vec{x}) = \begin{cases} 0 & 0 < x < L_x, 0 < y < L_y, 0 < z < L_z \\ \infty & otherwise \end{cases}$$

Wave functions exist only inside the 3-d infinite well.

Particle in a 3-d infinite well

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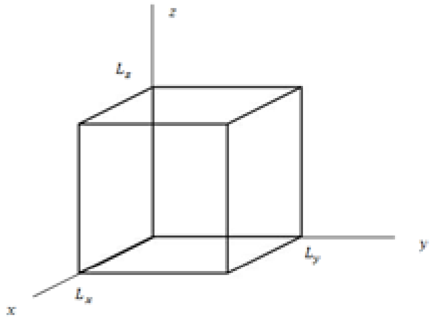

$$U(\vec{x}) = \begin{cases} 0 & 0 < x < L_x, 0 < y < L_y . 0 < z < L_z \\ \infty & otherwise \end{cases}$$

Wave functions exist only inside the 3-d infinite well.

Separation of variables: $\psi(\vec{x}) = \psi(x, y, z) = F(x)G(y)H(z)$

$$\frac{1}{F(x)}\frac{\partial^2 F(x)}{\partial x^2} + \frac{1}{G(y)}\frac{\partial^2 G(y))}{\partial y^2} + \frac{1}{H(z)}\frac{\partial^2 H(z)}{\partial z^2} = -\frac{2mE}{\hbar^2}$$

Particle in a 3-d infinite well

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\vec{x}) + U(\vec{x})\psi(\vec{x}) = E\,\psi(\vec{x})$$

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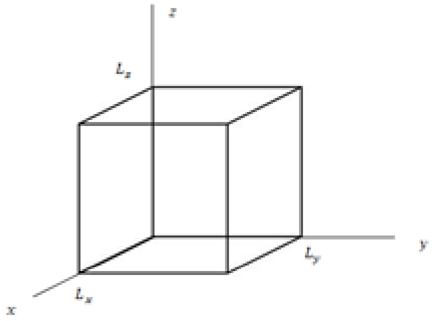

$$U(\vec{x}) = \begin{cases} 0 & 0 < x < L_x, 0 < y < L_y, 0 < z < L_z \\ \infty & otherwise \end{cases}$$

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Separation of variables: $\psi(\vec{x}) = \psi(x, y, z) = F(x)G(y)H(z)$

$$\frac{1}{F(x)}\frac{\partial^2 F(x)}{\partial x^2} + \frac{1}{G(y)}\frac{\partial^2 G(y))}{\partial y^2} + \frac{1}{H(z)}\frac{\partial^2 H(z)}{\partial z^2} = -\frac{2mE}{\hbar^2}$$

=constant

constant=Cx **Constant=Cy** **Constant=Cz**

$$Cx + Cy + Cz = -\frac{2mE}{\hbar^2}$$

Particle in a 3-d infinite well

1-D solutions:

$$\frac{\partial^2 \psi(x)}{\partial x^2} = \frac{-2m\,E}{\hbar^2}\,\psi(x)$$

Wave function: $\psi_{0<x<L}(x) = \sqrt{\frac{2}{L}}\,\sin(\frac{n\pi x}{L})$

Energy $E = \frac{n^2\,\pi^2\,\hbar^2}{2mL^2}$

$$\begin{cases} \frac{d\,F(x)}{dx^2} = C_x F(x) \\ \frac{d\,G(y)}{dy^2} = C_y G(y) \\ \frac{d\,H(z)}{dz^2} = C_z H(z) \end{cases}$$

Particle in a 3-d infinite well

1-D solutions:

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Wave function: $\psi_{0<x<L}(x) = \sqrt{\frac{2}{L}}\,\sin(\frac{n\pi x}{L})$

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3-D solutions: wave functions

$$\begin{cases} \frac{d\,F(x)}{dx^2} = C_x F(x) \\ \frac{d\,G(y)}{dy^2} = C_y G(y) \\ \frac{d\,H(z)}{dz^2} = C_z H(z) \end{cases}$$

$$\rightarrow F(x) = A_x \sin\frac{n_x \pi x}{L_x}$$

$$\rightarrow G(y) = A_y \sin\frac{n_y \pi y}{L_y}$$

$$\rightarrow H(z) = A_z \sin\frac{n_z \pi z}{L_z}$$

Particle in a 3-d infinite well

1-D solutions:

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3-D solutions: wave functions

$$\begin{cases} \frac{d\,F(x)}{dx^2} = C_x F(x) & \rightarrow F(x) = A_x \sin\frac{n_x \pi x}{L_x} \\ \frac{d\,G(y)}{dy^2} = C_y G(y) & \rightarrow G(y) = A_y \sin\frac{n_y \pi y}{L_y} \\ \frac{d\,H(z)}{dz^2} = C_z H(z) & \rightarrow H(z) = A_z \sin\frac{n_z \pi z}{L_z} \end{cases}$$

Cx = $-\frac{n_x^2\pi^2}{{L_x}^2}$ Cy = $-\frac{n_y^2\pi^2}{{L_y}^2}$ Cz= $-\frac{n_z^2\pi^2}{{L_z}^2}$

Cx + Cy + Cz = $-\frac{2mE}{\hbar^2}$ = $-\frac{n_x^2\pi^2}{{L_x}^2} - \frac{n_y^2\pi^2}{{L_y}^2} - \frac{n_z^2\pi^2}{{L_z}^2}$

Particle in a 3-D infinite well

$$E_{(n_x,n_y,n_z)} = \left(\frac{n_x^2}{L_x^{\,2}} + \frac{n_y^2}{L_y^{\,2}} + \frac{n_z^2}{L_z^{\,2}}\right)\frac{\pi^2\hbar^2}{2m}$$

$$\psi\,(x,y,z) = F(x)G(y)H(z) = A\,\sin\frac{n_x\pi x}{L_x}\sin\frac{n_y\pi y}{L_y}\sin\frac{n_z\pi z}{L_z}$$

Lowest energy state =

Wave function for the lowest energy state =

Particle in a 3-D infinite well

$$E_{(n_x,n_y,n_z)} = \left(\frac{n_x^2}{{L_x}^2} + \frac{n_y^2}{{L_y}^2} + \frac{n_z^2}{{L_z}^2}\right)\frac{\pi^2\hbar^2}{2m}$$

$$\psi\,(x,y,z) = F(x)G(y)H(z) = A\ \sin\frac{n_x\pi x}{L_x}\sin\frac{n_y\pi y}{L_y}\sin\frac{n_z\pi z}{L_z}$$

Lowest energy state = $E_{(1,1,1)} = \left(\frac{1^2}{{L_x}^2} + \frac{1^2}{{L_y}^2} + \frac{1^2}{{L_z}^2}\right)\frac{\pi^2\hbar^2}{2m}$

Wave function for the lowest energy state = $\psi_{(1,1,1)} = A\ \sin\frac{\pi x}{L_x}\sin\frac{\pi y}{L_y}\sin\frac{\pi z}{L_z}$

Particle in a 3-D infinite well

$$E_{(n_x,n_y,n_z)} = \left(\frac{n_x^2}{{L_x}^2} + \frac{n_y^2}{{L_y}^2} + \frac{n_z^2}{{L_z}^2}\right)\frac{\pi^2\hbar^2}{2m}$$

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When $L_x = L_y = L_z = L$

Lowest energy state =

Wave function for the lowest energy state =

Second lowest energy state(s) =

Wave functions for the second lowest energy state(s) =

Particle in a 3-D infinite well

$$E_{(n_x,n_y,n_z)} = \left(\frac{n_x^2}{{L_x}^2} + \frac{n_y^2}{{L_y}^2} + \frac{n_z^2}{{L_z}^2}\right)\frac{\pi^2\hbar^2}{2m}$$

$$\psi\,(x,y,z) = F(x)G(y)H(z) = A\,\sin\frac{n_x\pi x}{L_x}\sin\frac{n_y\pi y}{L_y}\sin\frac{n_z\pi z}{L_z}$$

When $L_x = L_y = L_z = L$

Lowest energy state = $E = 3\,\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$

Wave function for the lowest energy state = $\psi_{(1,1,1)} = A\,\sin\frac{\pi x}{L}\sin\frac{\pi y}{L}\sin\frac{\pi z}{L}$

Second lowest energy state(s) = $E_{(2,1,1)} = E_{(1,2,1)} = E_{(1,1,2)} = (2^2 + 1^2 + 1^2)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 6\,\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$

Wave functions for the second lowest energy state(s) = $\begin{cases} \psi_{(2,1,1)} = A\,\sin\frac{2\pi x}{L}\sin\frac{\pi y}{L}\sin\frac{\pi z}{L} \\ \psi_{(1,2,1)} = A\,\sin\frac{\pi x}{L}\sin\frac{2\pi y}{L}\sin\frac{\pi z}{L} \\ \psi_{(1,1,2)} = A\,\sin\frac{\pi x}{L}\sin\frac{\pi y}{L}\sin\frac{2\pi z}{L} \end{cases}$

An electron in a cubit 3d infinite well of 1 nm at the E(2,1,1) state

$$E_{(2,1,1)} = 6\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = (2^2 + 1^2 + 1^2)\frac{\pi^2(1.055 \text{ x } 10^{-34}\text{ J sec})^2}{2(9.11 \text{ x } 10^{-31}\text{kg})(10^{-9}\text{ m})^2}$$

$$= 3.62 \text{ x } 10^{-19}\text{ J} = 2.26 \text{ eV} \ \text{(the same as } E_{(1,2,1)} = E_{(1,1,2)})$$

$$\text{Where}\begin{cases}\text{electron mass} = 9.11 \text{ x } 10^{-31}\text{kg} \\ \hbar = 1.055 \text{ x } 10^{-34}\text{ J sec} \\ L = 10^{-9}\text{ m}\end{cases} \quad \text{and } 1 \text{ eV} = 1.602 \text{ x } 10^{-19}\text{ J}$$

Where is a particle with the value most likely to be found?

Probability density

- $|\psi_{(2,1,1)}|^2 = A^2\left(\sin\frac{2\pi x}{L}\right)^2\left(\sin\frac{\pi y}{L}\right)^2\left(\sin\frac{\pi z}{L}\right)^2$

,

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$$\text{Where}\begin{cases}\text{electron mass} = 9.11 \text{ x } 10^{-31}\text{kg}\\ h = 1.055 \text{ x } 10^{-34}\text{ J sec}\\ L = 10^{-9}\text{ m}\end{cases} \qquad \text{and } 1 \text{ eV} = 1.602 \text{ x } 10^{-19}\text{ J}$$

Where is a particle with the value most likely to be found?

Probability density

- $|\psi_{(2,1,1)}|^2 = A^2\left(\sin\frac{2\pi x}{L}\right)^2\left(\sin\frac{\pi y}{L}\right)^2\left(\sin\frac{\pi z}{L}\right)^2$

Since the value of $(\sin\theta)^2$ *is highest when* $\theta = \frac{1}{2}\pi, \frac{3}{2}\pi$, *etc.*, the probability

$$\begin{cases}x = \frac{L}{4}, \frac{3L}{4}\\ y = \frac{L}{2}\\ z = \frac{L}{2}\end{cases}$$

Energy Split

Consider (that is, a slightly non-symmetric box along the z axis)

$$L_x = L_y = L_z = L \longrightarrow L_x = L_y = L,\ L_z = .9\,L$$

$$E_{(1,1,1)} = (2^2 + 1^2 + 1^2)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 3\,\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

$$E_{(2,1,1)} = E_{(1,2,1)} = E_{(1,1,2)} = (2^2 + 1^2 + 1^2)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 6\,\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

Energy Split

Consider (that is, a slightly non-symmetric box along the z axis)

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$$E_{(1,1,1)} = \left(\frac{1^2}{L^2} + \frac{1^2}{L^2} + \frac{1^2}{.9^2L^2}\right)\left(\frac{\pi^2\hbar^2}{2m}\right) = (1 + 1 + 1.11)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 3.11\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

$$E_{(2,1,1)} = E_{(1,2,1)} = E_{(1,1,2)} = (2^2 + 1^2 + 1^2)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 6\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

$$E_{(2,1,1)} = E_{(1,2,1)} = \left(\frac{2^2}{L^2} + \frac{1^2}{L^2} + \frac{1^2}{.9^2L^2}\right)\left(\frac{\pi^2\hbar^2}{2m}\right) = (4 + 1 + 1.23)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 6.23\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

$$E_{(1,1,2)} = \left(\frac{1^2}{L^2} + \frac{1^2}{L^2} + \frac{2^2}{.9^2L^2}\right)\left(\frac{\pi^2\hbar^2}{2m}\right) = (1 + 1 + 4.92)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 6.92\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

Energy Split

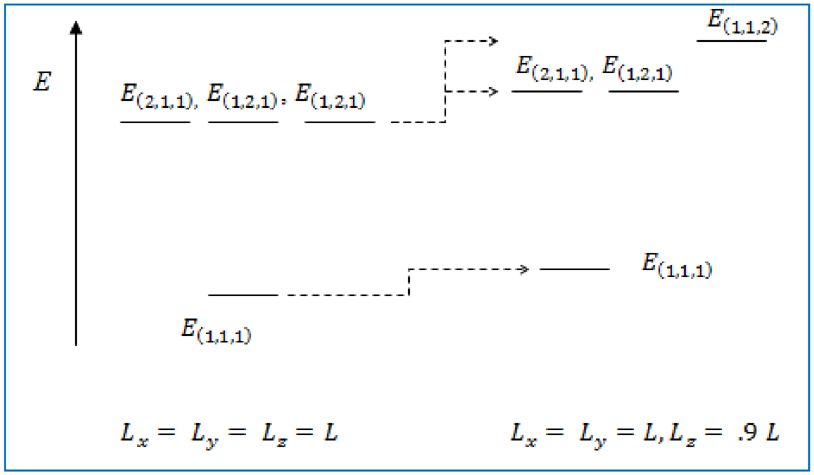
Consider (that is, a slightly non-symmetric box along the z axis)

$$L_x = L_y = L_z = L \xrightarrow{\qquad\qquad} L_x = L_y = L,\ L_z = .9\,L$$

$$E_{(1,1,1)} = (2^2 + 1^2 + 1^2)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 3\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

$$E_{(1,1,1)} = \left(\frac{1^2}{L^2} + \frac{1^2}{L^2} + \frac{1^2}{.9^2L^2}\right)\left(\frac{\pi^2\hbar^2}{2m}\right) = (1 + 1 + 1.11)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 3.11\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

$$E_{(2,1,1)} = E_{(1,2,1)} = E_{(1,1,2)} = (2^2 + 1^2 + 1^2)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 6\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$



$$E_{(2,1,1)} = E_{(1,2,1)} = \left(\frac{2^2}{L^2} + \frac{1^2}{L^2} + \frac{1^2}{.9^2L^2}\right)\left(\frac{\pi^2\hbar^2}{2m}\right) = (4 + 1 + 1.23)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 6.23\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

$$E_{(1,1,2)} = \left(\frac{1^2}{L^2} + \frac{1^2}{L^2} + \frac{2^2}{.9^2L^2}\right)\left(\frac{\pi^2\hbar^2}{2m}\right) = (1 + 1 + 4.92)\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 6.92\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

Hydrogen atom spectral lines

$$E_n = -\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}\frac{1}{n^2} = -\frac{e^2}{8\pi\varepsilon_0}\frac{me^2}{4\pi\varepsilon_0\hbar^2}\frac{1}{n^2} = -\frac{e^2}{8\pi\varepsilon_0 a_0}\frac{1}{n^2} = (-13.6\text{ eV})\frac{1}{n^2}$$

$$\text{Where } a_0(Bohr\ Radius) = \frac{me^2}{4\pi\varepsilon_0\hbar^2} = 0.0529\text{ nm} = 0.529\text{ A}$$

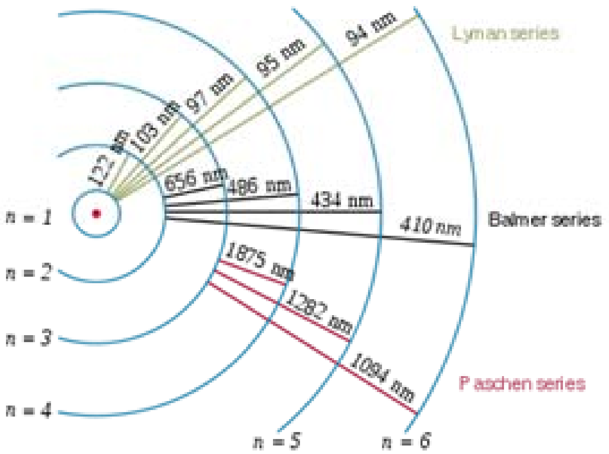
Hydrogen atom spectral lines

$$E_n = -\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}\frac{1}{n^2} = -\frac{e^2}{8\pi\varepsilon_0}\frac{me^2}{4\pi\varepsilon_0\hbar^2}\frac{1}{n^2} = -\frac{e^2}{8\pi\varepsilon_0 a_0}\frac{1}{n^2} = (\text{-13.6 eV})\frac{1}{n^2}$$

$$\text{Where } a_0(Bohr\ Radius) = \frac{me^2}{4\pi\varepsilon_0\hbar^2} = 0.0529\ \text{nm} = 0.529\ \text{A}$$

$$E_{photon} = \frac{hc}{\lambda} = E_{initial} - E_{final} = -\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}\left(\frac{1}{n_{final}^2} - \frac{1}{n_{initial}^2}\right)$$

$$\frac{1}{\lambda} = \frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2 hc}\left(\frac{1}{n_{final}^2} - \frac{1}{n_{initial}^2}\right) = 1.097\ x\ 10^7\ m^{-1}\left(\frac{1}{n_{final}^2} - \frac{1}{n_{initial}^2}\right)$$

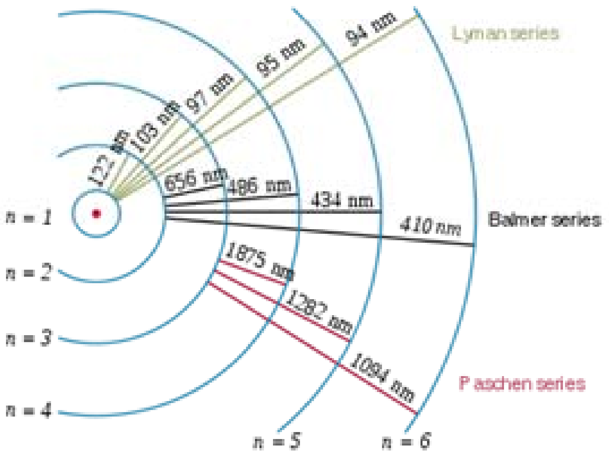


Hydrogen atom spectral lines

$$E_n = -\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}\frac{1}{n^2} = -\frac{e^2}{8\pi\varepsilon_0}\frac{me^2}{4\pi\varepsilon_0\hbar^2}\frac{1}{n^2} = -\frac{e^2}{8\pi\varepsilon_0 a_0}\frac{1}{n^2} = (\text{-13.6 eV})\frac{1}{n^2}$$

$$\text{Where } a_0(Bohr\ Radius) = \frac{me^2}{4\pi\varepsilon_0\hbar^2} = \text{ 0.0529 nm} = \text{0.529 A}$$

$$E_{photon} = \frac{hc}{\lambda} = E_{initial} - E_{final} = -\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}\left(\frac{1}{n_{final}^2} - \frac{1}{n_{initial}^2}\right)$$



$$\frac{1}{\lambda} = \frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2 hc}\left(\frac{1}{n_{final}^2} - \frac{1}{n_{initial}^2}\right) = 1.097 \times 10^7\, m^{-1}\left(\frac{1}{n_{final}^2} - \frac{1}{n_{initial}^2}\right)$$

- When $n_{final} = 1$, *Lyman Series* (discovered between 1905-1914)
- When $n_{final} = 2$, *Balmer Series* (in 1885)
- When $n_{final} = 3$, *Pachen Series* (in 1908)
- When $n_{final} = 4$, *Brackett Series* (in 1922)
- When $n_{final} = 5$, *Pfund Series* (in 1924)
- When $n_{final} = 6$, *Humphreys Series* (in 1953)

Spectral Lines

Lyman Series ($n_{final} = 1$)

$n_{initial}$	**λ (nm)**
2	**122**
3	**103**
4	**97.3**
5	**95.0**
6	**93.8**
∞	**91.2**

Balmer Series ($n_{final} = 2$)

$n_{initial}$	**λ (nm)**
3	**656**
4	**486**
5	**434**
6	**410**
7	**397**
∞	**365**

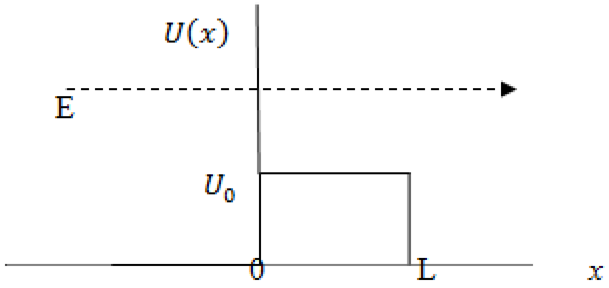
Pachen Series (($n_{final} = 3$)

$n_{initial}$	**λ (nm)**
4	**1870**
5	**1280**
6	**1090**
7	**1020**
8	**954**
∞	**820**

HW5: Resonant Transmission

1.4.3. Potential Barrier

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & 0 \le x \le L \\ 0 & x > L \end{cases}$$



IF E > U_0

Using the results from the step potential example, the following wave functions can be considered as solutions to the Schrodinger Equation under the potential barrier:

- $\psi_{x<0}$ = Incoming wave function + Reflected wave function

  $= \quad A\, e^{+ikx} \quad + \quad B\, e^{-ikx} \qquad$ where $k = \sqrt{\frac{2mE}{\hbar^2}}$

- $\psi_{0 \le x \le L}$ = Incoming wave function + Reflected wave function

  $= \quad C\, e^{+ik'x} \quad + \quad D\, e^{-ik'x} \qquad$ where $k' = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$

- $\psi_{x>L}$ = Transmitted wave function = Fe^{ikx} $\qquad$ where $k = \sqrt{\frac{2mE}{\hbar^2}}$

- Transmission probability $= \dfrac{\text{transmitted particle flux}}{\text{incoming particle flux}} = \dfrac{|\psi_{trans}|^2 k}{|\psi_{incoming}|^2 \; k} = \dfrac{F*F}{A^*A}$

$$= \frac{\frac{4k'^2k^2}{(k^2-k'^2)^2}}{sin^2(k'L)+\frac{4k'^2k^2}{(k^2-k'^2)^2}}$$

- Reflection probability $= \dfrac{\text{reflected particle flux}}{\text{incoming particle flux}} = \dfrac{|\psi_{reflected}|^2 k}{|\psi_{incoming}|^2 \; k} = \dfrac{B^*B}{A^*A}$

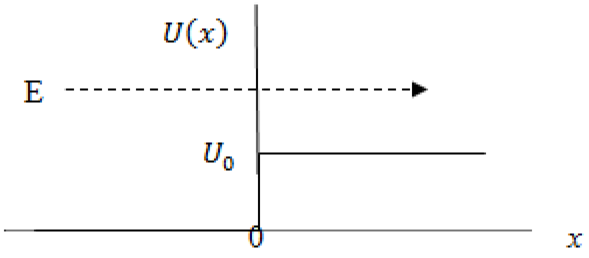
$$= \frac{sin^2(k'L)}{sin^2(k'L)+\frac{4k'^2k^2}{(k^2-k'^2)^2}}$$

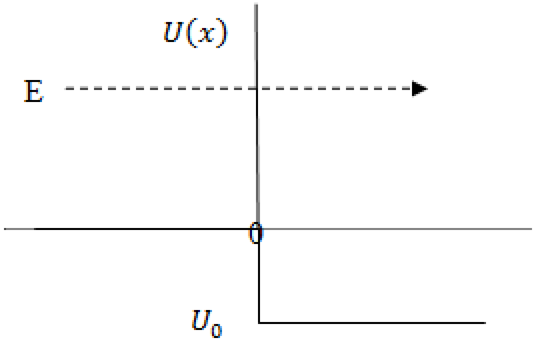
(Note. Sine dependence → Resonnant Transmission $k'L = \sqrt{\frac{2m(E-U_0)}{\hbar^2}} = n\pi$)

HW4. Potential Step

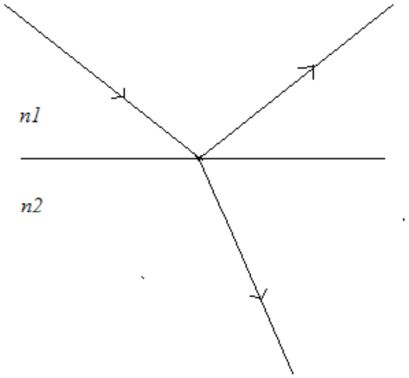
1.4.2. Potential Step

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & x \geq 0 \end{cases}$$





??? n1 > n2 vs. n1 < n2



- $\psi_{x<0}$ = Incoming wave function + Reflected wave function
  = A e^{+ikx} + B e^{-ikx}